Monetary Policy in a Schumpeterian Growth Model with Two R&D Sectors

Chien-Yu Huang∗ Yibai Yang†
Nanjing Audit University University of Macau
Zhijie Zheng‡
Southwestern University of Finance and Economics

November 13, 2019

Abstract

This study investigates the welfare effect of monetary policy in a Schumpeterian economy with an upstream and a downstream sector in which R&D investment of these sectors is subject to a cash-in-advance (CIA) constraint. A higher nominal interest rate generates two effects on welfare: the manufacturing-R&D reallocation effect and the cross-R&D-sector effect, where the former depresses the contribution of growth to welfare by shifting labor from R&D to production, whereas the latter enhances it by shifting labor from a more cash-constrained R&D sector to a less-constrained one. To analyze the optimal monetary policy, we examine the necessary and sufficient conditions for the (sub)optimality of the Friedman rule by relating the underinvestment and overinvestment of R&D in the decentralized equilibrium. We find that this relationship is crucially determined by the presence of CIA constraints, the relative productivity between upstream R&D and downstream R&D, and the strength of markup.

JEL classification: O30; O40; E41.

Keywords: CIA constraint, Endogenous growth; Monetary policy; Two R&D sectors

∗Institute of Urban Development, Nanjing Audit University, Nanjing, Jiangsu, China. Email address: chuang4@ncsu.edu.
†Department of Economics, University of Macau, Taipa, Macao, China. Email address: yibai.yang@hotmail.com.
‡Research Institute of Economics and Management, Southwestern University of Finance and Economics, Chengdu, Sichuan, China. Email address: zhengzhijie1919@gmail.com.
1 Introduction

In this study, we explore the optimality and welfare implications of monetary policy in a Schumpeterian economy with two vertically related sectors, in which the R&D investment of these sectors is subject to a cash-in-advance (CIA) constraint. It is well known that Friedman (1969) proposed a monetary policy rule according to which the optimal nominal interest rate is zero.\(^1\) Since then, there has been a large body of literature that have analyzed the optimality of the Friedman rule in different economic environments.\(^2\) Moreover, a growing number of studies have attempted to incorporate money demand into R&D-based growth models to address this stylized fact by finding the condition for the (sub)optimality of the Friedman rule, such as Chu and Cozzi (2014) and Hori (2019). Therefore, one major purpose of this study is to explore the long-run effects of monetary policy on social welfare by comparing our results to the Friedman rule within a realistic vertical-industry structure for R&D activities.

There are two main reasons for considering a vertical-industry structure for CIA-constrained R&D activities in an endogenous growth framework. First, existing evidence strongly supports the fact that both downstream and upstream firms invest in R&D activities to develop innovations that upgrade their technologies.\(^3\) For example, an early study by Nelson (1986) finds that both upstream and downstream industries have significant contributions to the US R&D intensity. The importance of vertical innovation is also confirmed by survey evidence from Germany and the US in Harabi (1998) and Vonortas (2012) (pp.125-141), respectively. At the industrial level, studies, such as McLaren (1999) and Banerjee and Lin (2003), have well documented that innovations in the automobile sector in Japan and the US are conducted by both auto makers and auto parts suppliers. A recent study by Pillai (2013) shows that upstream semiconductor equipment firms like Nikon, Canon, Applied Materials, and ASML invent new generations of capital equipment to allow microprocessor firms like Intel and AMD to invent higher performance microprocessors. More recently, Yang (2019) shows that in the US smartphone market, upstream chipset firms (such as Qualcomm) invest to develop innovations, with which downstream handset makers (such as Samsung) innovate to improve their hardware. Therefore, the feature of R&D sectors for vertical industries needs to be stressed in the R&D-based growth model.

Second, another important feature of the latest literature on money and R&D is that R&D investment is heavily affected by liquidity requirements. Brown et al. (2012) find that the increase in corporate cash flow in the 1990s was the result of firms’ smoothing of R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Falato and Sim (2014) use US firm-level data to demonstrate that firms hold cash to finance their R&D investment due to the presence of financing frictions. Brown and Petersen (2015) reveal that firms tend to use cash to finance investment in R&D but not in capital. A recent study by Lyandres and Palazzo (2016) shows that the sharp increase in the average cash-to-assets ratio for US firms since the mid-1980s

---

\(^1\)See Huang et al. (2017) for a low or zero level of nominal interest rate target in large economies such as the US, Japan, and China during 2008–2015.

\(^2\)See the discussion regarding the optimality of the Friedman rule in, for example, Ho et al. (2007) for inflation taxation and Gahvari (2012) for an overlapping generations model.

\(^3\)In the well-established tradition of R&D-based endogenous growth, such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), firms engage in research activities in either the downstream sector (i.e., the final-goods sector) or the upstream sector (i.e., the intermediate-goods sector). In their line of argument, policy instruments can only affect the resource allocation in R&D for either of these sectors with manufacturing.
is driven almost only by firms which invest heavily in R&D. Therefore, the presence of a CIA 
constraint on R&D also needs to be emphasized in the R&D-based growth model.

Therefore, to analyze the effects of monetary policy, we take into account the above features 
of R&D. We build up a scale-invariant version of the quality-ladder growth model in which both 
the upstream and downstream sectors devote resources to R&D activities and money demand 
is incorporated into the model through a CIA constraint on R&D investments in these vertically 
related sectors. Under the CIA constraints on the upstream and downstream R&D, there are two 
effects of an increase in the nominal interest rate on resource reallocation. First, a higher nominal 
interest rate generates the manufacturing-R&D-reallocation effect through shifting the resources (i.e., 
labor in this study) from the R&D sector to the manufacturing sector given that the cost of 
borrowing for R&D investments rises, and this effect retards economic growth. Second, a higher 
nominal interest rate generates the cross-R&D-sector effect through reallocating the resources from 
the more cash-constrained R&D sector to the less constrained one, and this effect will enhance 
economic growth with a large markup. We show that as long as the productivity of the less 
constrained R&D sector is large relative to the more constrained one, the growth-enhancing effect 
will dominate the growth-retarding effect at low levels of nominal interest rate. Nevertheless, 
as the nominal interest rate increases, this domination will be gradually mitigated and finally 
reversed. Hence, the nominal interest rate will yield an overall inverted-U effect on economic 
growth, as documented in recent empirical studies, such as López-Villavicencio and Mignon 
(2011) and Eggoh and Khan (2014).4

More importantly, to address the recent fact that nominal interest rates have been low, we 
explore the long-run welfare implications of monetary policy by contrasting our results with the 
Friedman rule monetary policy (i.e., zero nominal interest rate targeting). Specifically, this study 
reexamines the necessary and sufficient conditions for the (sub)optimality of the Friedman rule 
by relating the underinvestment and overinvestment of R&D in the decentralized economy where 
the industries that conduct R&D activities are vertically related. We find that this relationship 
is crucially determined by the presence of CIA constraints, the relative productivity between 
upstream R&D and downstream R&D, and the strength of markup. In particular, in the presence 
of the CIA constraint on only one R&D sector, when the relative productivity of this sectoral 
R&D is low (high), R&D overinvestment (underinvestment) is sufficient but not necessary for the 
Friedman rule to be suboptimal (optimal). Nonetheless, in the presence of the CIA constraints 
with equal strength for both sectoral R&D, the overinvestment (underinvestment) in R&D is 
sufficient (necessary) for the Friedman rule to be suboptimal (optimal), regardless of whether the 
relative productivity of the sectoral R&D is low or high. In other words, R&D underinvestment 
may also lead the Friedman rule to be suboptimal in the above cases. Moreover, the strength 
of markup plays a role in determining the degree of R&D over- or under-investment that could 
generate the suboptimality of the Friedman rule.

In addition, we perform a quantitative analysis to evaluate the growth and welfare effects of 
monetary policy by calibrating this two-R&D-sector model to the US data. The benchmark case 
shows that a zero-interest-rate policy (i.e., the Friedman rule) maximizes economic growth and 
social welfare, and the welfare gain by optimizing the nominal interest rate from the decentral-

4See Chu et al. (2017) for an analysis of the CIA constraint on R&D in a Schumpeterian growth model with 
endogenous entry of heterogeneous firms, which also yields an inverted-U effect of monetary policy on economic 
growth.
ized equilibrium can be as large as 1% of consumption. Furthermore, to be consistent with more empirical evidence and to test the robustness of the analytical results, we conduct several sensitivity checks by varying the CIA parameters and the markup level to demonstrate the cases in which the growth-maximizing or welfare-maximizing rate of nominal interest becomes positive (e.g., the Friedman rule can be suboptimal).

Previous studies on economic-growth monetary policy have examined the welfare effect of inflation and the use of the Friedman rule, but the impact of the two-R&D-sector structure is relatively less explored in this literature. Therefore, the novel contribution of this study is to (a) review the welfare-maximizing design for monetary policy in a dynamic general equilibrium framework in which both vertically-related industries feature R&D activities; and (b) provide a new rationale for the relationship between R&D investment and the (sub)optimality of the Friedman rule.

This study contributes to the growth-theoretic literature on R&D and monetary policy that features CIA requirements. The pioneering work by Marquis and Reffett (1994) firstly introduces a CIA constraint on consumption to the Romer (1990) type variety-expansion growth model to investigate the effects of monetary policy, which proves that the Friedman rule is optimal. Unlike their model, the current study considers a quality-ladder growth model and analyzes the effects of monetary policy via a CIA constraint on R&D.\(^5\) Our study is closely related to Chu and Cozzi (2014), who show that the overinvestment of R&D in the market economy is the necessary and sufficient condition for the suboptimality of the Friedman rule in a Schumpeterian growth model with a CIA constraint on R&D.\(^6\) Their result is based on the setting with R&D activities in the intermediate-goods sector (i.e., the upstream sector in this study), and so raising the nominal interest rate yields only a reallocating effect on resources from R&D to production. The current study complements their interesting study by considering a more realistic setting in which R&D activities are engaged in both the intermediate-goods and final-goods sectors. Under this setting, in addition to the manufacturing-R&D reallocation effect as identified in Chu and Cozzi (2014), raising the nominal interest rate generates an additional cross-R&D-sector effect between the upstream and downstream R&D sectors, which makes both R&D overinvestment and underinvestment possible, thereby leading the optimal nominal interest rate to be positive. Therefore, to the best of our knowledge, this is the first study that analyzes monetary policy in a growth-theoretic framework featuring R&D activities for vertical industries in a cash-in-advance economy.

Additionally, this study contributes to a small but growing literature that investigates the effects of government policy in endogenous growth models with two R&D sectors. For example, as for R&D subsidies, Li (2000) analyzes the effectiveness of R&D subsidies in stimulating economic growth in a two-R&D-sector model with both fully endogenous growth and semi-endogenous growth, whereas Segerstrom (2000) completely characterizes the long-run growth effects of R&D subsidies in an endogenous growth model with both vertical R&D and horizontal R&D. As for patent policy, Goh and Olivier (2002) explore optimal patent protection in a

---

\(^5\)Other endogenous growth models with CIA constraints on R&D activities include Chu et al. (2015), Zheng et al. (2019), and Gil and Iglesias (2019).

\(^6\)Hori (2019) extends the model of Chu and Cozzi (2014) by considering heterogeneity in R&D firms’ productivity. He finds that if R&D firms are heterogeneous (homogenous), the Friedman rule can be suboptimal (is always optimal) under a severe financial constraint.
variety-expansion growth model where firms in both the upstream and downstream sectors engage in R&D, whereas Chu (2011) addresses a similar issue in a quality-ladder growth model where firms in two horizontal final-goods sectors (i.e., the downstream sectors) engage in R&D. Consequently, our paper complements the above interesting studies by focusing on the role of monetary policy in a two-R&D-sector economy.\footnote{Huang et al. (2015) and Zheng et al. (2019) also examine the growth and welfare implications of monetary policy in a two-R&D-sector Schumpeterian growth model with and without endogenous market structure, respectively. Nevertheless, R&D investments are conducted to develop vertical and horizontal innovations in their study instead of upstream and downstream industries in the current study.}

The rest of this paper is organized as follows. Section 2 presents the model setup. Section 3 characterizes the decentralized equilibrium and analyzes the growth effect of monetary policy. Section 4 explores the welfare effects of monetary policy and derives the optimal monetary policy. Section 5 provides a quantitative analysis. Section 6 concludes the study.

2 Model

In this section, we present the monetary Schumpeterian growth model. We extend a version of the quality-ladder model in Grossman and Helpman (1991) by allowing firms to invest in R&D to develop innovations in both upstream (i.e., intermediate-goods) and downstream (i.e., final-goods) sectors as in Goh and Olivier (2002) and by introducing money demand via a CIA constraint on R&D investments as in Chu and Cozzi (2014) and Huang et al. (2017). The nominal interest rate serves as the monetary policy instrument and the effects of monetary policy are examined by considering the implications of altering the rate of nominal interest on economic growth and social welfare.

2.1 Households

At time $t$, each household has a population size of $N_t$, which grows at the rate of $n \geq 0$ such that $\dot{N}_t = nN_t$. There is a unit continuum of identical households, and the lifetime utility function of each member is given by

$$U = \int_0^\infty e^{-\rho t}c_t dt,$$

where $\rho > 0$ represents the discount rate, and $c_t$ is the consumption good, which is a composite of a unit continuum of differentiated final goods $y_t(j)$ indexed by $j \in [0, 1]$ such that

$$c_t = \exp \left( \int_0^1 \ln y_t(j) dj \right).$$

The law of motion for assets of each household member (expressed in real terms) is

$$\dot{a}_t + \dot{m}_t = (r_t - n)a_t + \dot{w}_t + i_tb_t + \tau_t - c_t - (\pi_t + n)m_t,$$

where $a_t$ is the real asset value, $r_t$ is the real interest rate, and each individual inelastically supplies one unit of labor at the real wage rate $w_t$. $\tau_t$ denotes the real lump-sum transfer from
the government, \( \pi_t \) is the inflation rate that reflects the cost of holding money, and \( m_t \) is the real money balance that the household member holds in order to facilitate entrepreneurs’ loans \( b_t \), which finance the R&D investment with a return rate of \( i_t \). Therefore, the cash-in-advance (CIA) constraint is given by \( b_t \leq m_t \).

There are two stages for utility maximization. In the first stage, each household member decides the allocation of expenditure across the unit measure of final products. Solving the static optimization problem gives rise to the demand for the differentiated final goods \( y_{j,t} \) given by

\[
y_{t}(j) = \frac{c_t}{p_{y,t}(j)},
\]

where \( p_{y,t}(j) \) is the price of \( y_{t}(j) \). In the second stage, the household member allocates her expenditure across the planning horizon. The optimal problem is to maximize the discounted utility in (1) subject to the budget constraint in (3) and the CIA constraint. Solving the standard dynamic optimization yields the familiar Euler equation such that

\[
\frac{\dot{c}_t}{c_t} = r_t - \rho - n.
\]

Finally, by using the optimality condition for real money balances \( m_t \), we can derive the Fisher equation such that \( i_t = \pi_t + r_t \).

### 2.2 Final Goods

In this study, the final-goods sector is referred to as the downstream sector. As for each differentiated final good \( j \), the total demand equals its supply such that

\[
Y_t(j) = N_i y_{t}(j),
\]

given that members in the same household are identical. In addition, the aggregate amount of final goods in this economy is given by a unit continuum of differentiated final goods \( Y_t(j) \) such that

\[
Y_t = \exp \left( \int_0^1 \ln Y_t(j) dj \right).
\]

The differentiated final goods in each industry \( j \) are produced by a monopolistic leader, who holds a patent on the latest innovation and uses a unit continuum of intermediate goods indexed by \( k \in [0, 1] \). This leader’s products are replaced by the ones of a new entrant who has a more advanced innovation due to the Arrow replacement effect. The current leader’s production function is given by

\[
Y_t(j) = z^{q_{y,t}(j)} \exp \left( \int_0^1 \ln x_t(j,k) dk \right),
\]

where the parameter \( z > 1 \) measures the step size of each quality improvement, \( q_{y,t}(j) \) denotes the number of innovations between time 0 and \( t \), and \( x_t(j,k) \) is the quantity of intermediate good
$k$ used for final good $j$. Thus, the marginal cost of producing final good $j$ is

$$mc_{y,t}(j) = \frac{P_{x,t}}{z^{q_{x,t}(j)}}$$

(9)

where $P_{x,t} \equiv \exp \left( \int_0^1 \ln p_{x,t}(k) \, dk \right)$ is the price index for the intermediate goods and $p_{x,t}(k)$ is the price of intermediate good $k$.

In each differentiated final goods industry, the current and previous leaders engage in Bertrand competition. Following previous studies such as Goh and Olivier (2002), this model assumes that intellectual property rights protect inventions in the form of incomplete patent breadth. The degree of patent breadth, which is exogenously set by the policy of patent authority, determines the markup $\mu > 1$ that each monopolist can charge over its marginal cost. The profit-maximizing price is

$$p_{y,t}(j) = \mu \frac{P_{x,t}}{z^{q_{y,t}(j)}}.$$  

(10)

The monopolistic profit of each differentiated final-goods producer is identical and is given by

$$\Pi_{y,t} = \Pi_{y,t}(j) = (\mu - 1) Y_t(j) \frac{p_{y,t}(j)}{\mu} = \left( \frac{\mu - 1}{\mu} \right) c_t N_t,$$

(11)

where the second equality and the third equality are obtained by using (10) and (4), respectively. (11) implies that given consumption expenditure $c_t N_t$, a final-goods producer’s profit $\Pi_{y,t}$ is increasing in the markup $\mu$. Then, using the definition of price index $P_{x,t}$ along with (9), (10), and (11) yields the demand for intermediate good $k$ such that

$$x_t = \int_0^1 x_t(j,k) \, dj = \frac{c_t N_t}{\mu p_{x,t}(k)}.$$  

(12)

### 2.3 Intermediate Goods

In this study, the intermediate-goods sector is referred to as the *upstream sector*. The environment of the intermediate-goods sector is similar to that of the final-goods sector. Each industry in this sector is temporarily dominated by a monopolist holding the latest innovation, and the industry leadership is replaced by an entrant who holds a new invention. However, the production structure of this sector is different from that of the previous sector since intermediate goods are produced by manufacturing labor. Specifically, the production function for the current intermediate-goods producer in industry $k$ is given by

$$x_t(k) = z^{q_{x,t}(k)} L_{x,t}(k),$$

(13)

where the step size of quality improvement is assumed to be identical to that in the final-goods sector, $q_{x,t}(k)$ is the number of innovations as of time $t$, and $L_{x,t}(k)$ is the employment level of

---

8Although the step size $z$ of quality improvement is the same between the final-goods and intermediate-goods sectors, the numbers of innovations in these two sectors are different. Hence, as will be shown in Subsection 3.1, the levels of state-of-the-art technology in these sectors are also different.
production in industry $k$. Given the pricing strategy of the current leaders in this sector, the profit-maximizing price is again a constant markup over the marginal cost such that

$$p_{x,t}(k) = \mu mc_{x,t}(k) = \frac{w_t}{z_{y,t}(k)}. \quad (14)$$

Accordingly, the monopolistic profit for each intermediate-goods producer is

$$\Pi_{x,t} = \Pi_{x,t}(k) = \left(\frac{\mu - 1}{\mu}\right) w_t x_t(k) = \left(\frac{\mu - 1}{\mu}\right) \frac{c_t N_t}{\mu}, \quad (15)$$

where the second equality and the third one are obtained by using (14)-(15) and (12), respectively. Given $c_t N_t$, an increase in $\mu$ has a non-monotonic effect on the intermediate-goods producer’s profit $\Pi_{x,t}$; increasing $\mu$ raises $\Pi_{x,t}$ through the term $1 - 1/\mu$ due to a larger market power, whereas it also lowers $\Pi_{x,t}$ through the term $1/\mu$ due to a smaller demand for the intermediate goods in (12) with the exercise of the final-goods producer’s market power.\footnote{The presence of $(1 - 1/\mu)/\mu$ in (15) captures the double-marginalization problem as in the traditional industrial organization literature. Specifically, Chapter 17 in Belleflamme and Peitz (2015) claims that “in a market with firms operating only at one level of a vertical supply chain, ... a downstream firm applies a margin to the wholesale price which includes the margin of an upstream firm. The inefficiency arises because the retailer does not take into account the externality exerted on the upstream firm by changing the retail price.”}

Moreover, the production-labor income in the industry for intermediate good $k$ is

$$w_t L_{x,t}(k) = \frac{1}{\mu} p_{x,t}(k) x_t(k) = \frac{c_t N_t}{\mu^2}, \quad (16)$$

and therefore the labor demand for intermediate good $k$ is given by

$$L_{x,t}(k) = \frac{c_t N_t}{\mu^2 w_t}. \quad (17)$$

### 2.4 Innovations and R&D

The environments for R&D firms that conduct innovations for the final-goods and intermediate-goods sectors are the same as follows. The expected value of owning the most recent innovation in industry $y$ $(k)$ in the final- (intermediate-) goods sector is denoted as $v_{y,t}(j)$ $(v_{x,t}(k))$. Following the standard literature, we focus on a symmetric equilibrium (see, for example, Cozzi et al. (2007)). This implies that given $\Pi_{y,t}(j) = \Pi_{y,t}(k) = \Pi_{x,t}$, it follows that $v_{y,t}(j) = v_{y,t}(k) = v_{x,t}$. Denote by $\lambda_{y,t}$ $(\lambda_{x,t})$ the aggregate-level Poisson arrival rate of innovations for final-(intermediate-) goods. Then, the Hamilton-Jacobi-Bellman (HJB) equation for $v_{y,t}$ $(v_{x,t})$ is

$$r_t v_{s,t} = \Pi_{s,t} + \dot{v}_{s,t} - \lambda_{s,t} v_{s,t}, \quad (18)$$

which is the no-arbitrage condition for the value of the asset in sector $s = \{y, x\}$, respectively. In equilibrium, the return on the asset $r_t v_{s,t}$ equals the sum of the flow profits $\Pi_{s,t}$, the capital gain $\dot{v}_{s,t}$, and the potential losses $\lambda_{s,t} v_{s,t}$ when creative destruction takes place.

New innovations in each industry in the final- and intermediate-goods sectors are generated by a unit continuum of R&D firms indexed by $\theta \in [0, 1]$ and $\vartheta \in [0, 1]$, respectively, and each of
the R&D firms in sectors \(y\) and \(x\) employs R&D labor \(L^y_{r,t}(\theta)\) and \(L^x_{r,t}(\theta)\) for producing inventions. This study follows the existing literature, such as Chu and Cozzi (2014) and Huang et al. (2017), to incorporate a CIA constraint on R&D investment at time \(t\), such that households lend the \(\theta\)-th (\(\theta\)-th) entrepreneur an amount \(B^y_{t}(\theta) = b^y_{t}(\theta)N_t\) (\(B^x_{t}(\theta) = b^x_{t}(\theta)N_t\)) of money, which finances the wage payment for the downstream-R&D labor \(w_tL^y_{r,t}(\theta)\) (for upstream-R&D labor \(w_tL^x_{r,t}(\theta)\)) with the extra burden of an interest payment on the nominal interest rate \(i_t\). Thus, the expected profit of the \(\chi\)-th R&D firm, where \(\chi = \{\theta, \theta\}\), is

\[
\Pi^\chi_{r,t}(\chi) = v_{s,t}\lambda_{s,t}(\chi) - (1 + \xi_{s}i_t)w_tL^s_{r,t}(\chi),
\]

where \(\xi_{s} = \{\xi_y, \xi_x\} \in [0,1]\) is the strength of the CIA constraint on downstream (upstream) R&D. Moreover, the firm-level arrival rate of innovations \(\lambda_{s,t}(\chi)\) is formulated by

\[
\lambda_{s,t}(\chi) = \varphi_{s,t}L^s_{r,t}(\chi) = \frac{\varphi_{s}}{N_t}L^s_{r,t}(\chi),
\]

where the specification \(\varphi_{s,t} = \varphi_{s}/N_t\) captures the dilution effect that removes scale effects as in Chu and Cozzi (2014) and \(\varphi_{s} = \varphi_{y}, \varphi_{x}\) is the productivity parameter for downstream and upstream R&D, respectively. In equilibrium, the aggregate-level arrival rate of innovations is thus given by \(\lambda_{s,t} = \int^1_0 \lambda_{s,t}(\chi)d\chi = \varphi_{s}L^s_{r,t}/N_t\), where \(L^s_{r,t} = \int^1_0 L^s_{r,t}(\chi)d\chi\) is the aggregate labor devoted to the \(s = \{y, x\}\) R&D sector. Then, free entry into the R&D sectors implies the following zero-expected-profit condition:

\[
v_{s,t}\lambda_{s,t} = (1 + \xi_{s}i_t)w_tL^s_{r,t}.
\]

This equation is a condition pinning down the allocation of labor in the R&D sectors.

### 2.5 Monetary Authority

Denote the nominal money supply by \(M_t\) and its growth rate by \(\Phi_t = \dot{M}_t/M_t\), respectively. Accordingly, the real money balance is given by \(m_tN_t = M_t/p_t\), where \(p_t\) is the price of consumption. Then, consider that the growth rate of money supply \(\Phi_t\) serves as a policy instrument that can be controlled by the monetary authority. In this case, the rate of inflation is endogenously determined by \(\pi_t = \Phi_t - m_t/M_t - n\). Additionally, combining this condition with the Fisher equation (i.e., \(i_t = \pi_t + r_t\)) yields the one-to-one relationship between the nominal interest rate and the nominal money supply, such that

\[
i_t = \Phi_t + \rho.
\]

Given this result, throughout the rest of this study, we will use \(i_t\) to represent the instrument of monetary policy for simplicity. Finally, the monetary authority redistributes the increase in money supply (i.e., the seigniorage revenue) as a lump-sum transfer to the households, namely, \(\tau_tN_t = \dot{M}_t/p_t = \Phi_t m_t N_t = [(\pi_t + n)m_t + \dot{m}_t]N_t\).

---

\[\text{On the balanced growth path, which will be shown in Section 3.1, } c_t \text{ and } m_t \text{ grow at the same rate of } r_t - \rho - n \text{ according to the Euler equation.}\]
3 Decentralized Equilibrium

An equilibrium consists of a sequence of allocations \([c_t, m_t, Y_t(j), y_t(j), x_t(k), L_{x,t}(k), L^y_{r,t} (\theta), L^x_{r,t} (\theta)]_{t=0}^{\infty}, k, \theta, \theta \in [0,1]\) and a sequence of prices \([r_t, p_{y,t}(j), p_{x,t}(k), w_t, v_{y,t}, v_{x,t}]_{t=0}^{\infty}, k, \theta \in [0,1]\). Moreover, at each instance of time,

- households produce \([c_t]\) using \([y_t(j)]\) as inputs to maximize payoffs taking \([p_{y,t}(j)]\) as given;
- households choose \([c_t]\) to maximize their utility taking \([r_t, i_t, w_t]\) as given;
- monopolistic leaders for final goods produce \([Y_t(j)]\) and choose \([p_{y,t}(j)]\) to maximize profits taking \([p_{x,t}(k)]\) as given;
- monopolistic leaders for intermediate goods produce \([x_t(k)]\) and choose \([p_{x,t}(k), L_{x,t}(k)]\) to maximize profits taking \([w_t]\) as given;
- competitive downstream-R&D firms choose \([L^y_{r,t}(\theta)]\) to maximize profits taking \([w_t, v_{y,t}]\) as given;
- competitive upstream-R&D firms choose \([L^x_{r,t}(\theta)]\) to maximize profits taking \([w_t, v_{x,t}]\) as given;
- the consumption-goods market clears such that \(c_t N_t = Y_t\);
- the labor market clears such that \(L_{x,t} + L^y_{r,t} + L^x_{r,t} = N_t\);
- the innovations value adds up to households’ asset value such that \(v_{x,t} + v_{y,t} = a_t N_t\);
- the R&D entrepreneurs finance their wage payments through borrowing such that \(\xi_y w_t L^y_{r,t} + \xi_x w_t L^x_{r,t} = b_t N_t\) and
- the monetary authority balances its budget such that \(\tau_t N_t = (i_t - \rho) m_t N_t\).

3.1 Balanced Growth Path

This section characterizes the decentralized equilibrium for this model and shows that the economy grows along a unique and stable balanced growth path (BGP). To facilitate this result, we first derive the growth rate of aggregate technology \(g_t\). Using (8), (9), and (14) yields \(Y_t = Z_{y,t} Z_{x,t} L_{x,t}\), where \(Z_{y,t}\) and \(Z_{x,t}\) are defined as the level of technology in the downstream sector and in the upstream sector, where \(Z_{y,t} = \exp \left( \ln z \int_0^1 q_t(j) dj \right) = \exp \left( \ln z \int_0^1 \lambda y_t, d\lambda \right)\) and \(Z_{x,t} = \exp \left( \ln z \int_0^1 q_t(k) dk \right) = \exp \left( \ln z \int_0^1 \lambda x_t, d\lambda \right)\), respectively, and the second equalities are obtained by the law of large numbers. Then, differentiating these two equations with respect to time yields the growth rate of aggregate technology given by

\[
g_t = \frac{\dot{Z}_{y,t}}{Z_{y,t}} + \frac{\dot{Z}_{x,t}}{Z_{x,t}} = (\varphi_y \dot{L}^y_{r,t} + \varphi_x \dot{L}^x_{r,t}) \ln z, \tag{23}
\]

where the second equality is obtained by using (20) and \(\dot{L}^y_{r,t} \equiv L^y_{r,t}/N_t\) and \(\dot{L}^x_{r,t} \equiv L^x_{r,t}/N_t\) are defined as downstream-R&D labor per capita and upstream-R&D labor per capita, respectively. Similarly, \(L_{x,t} \equiv L_{x,t}/N_t\) is defined as manufacturing labor per capita.

For an arbitrary path of the nominal interest rate \([i_t]_{t=0}^{\infty}\), we obtain the following result:

**Proposition 1.** Holding constant \(i\), the economy jumps to a unique and stable balanced growth path.

**Proof.** See Appendix A.

\[\square\]
3.2 Equilibrium Allocations and the Growth Effect

As implied by Proposition 1, given a constant $i$, the equilibrium labor allocations $\{l_x^*, l_y^*, l_r^*, l_r^v, l_r^x\}$ are stationary along the BGP. Using the zero-expected-profit condition for upstream R&D (21) and the production-labor income in the intermediate-goods sector (16) yields $v_{y,t} \lambda_{y,t} = (1 + \xi_y i) N t c_t / (\mu^2 L_{x,t})$ implying $v_{y,t} / v_{y,t} = c_t / c_t + n$. Combining this result with (6) and (18) and imposing the BGP implies $\Pi_y / v_y = \rho + \lambda_y$. Then, substituting this equation into (21) and applying (11) and (20) derives the relationship between $l_r^v$ and $l_x$ such that

$$l_r^v = \frac{\mu (\mu - 1) l_x}{1 + \xi_y i} - \frac{\rho}{\phi_y},$$

which is the first equation to solve for $\{l_x, l_y^*, l_r^*, l_r^v, l_r^x\}$. Following a similar logic, we can use (6), (15), (16), (18), (20), and (21) to derive the second equation, which is the relationship between $l_r^*$ and $l_x$ given by

$$l_r^* = \frac{(\mu - 1) l_x}{1 + \xi_y i} - \frac{\rho}{\phi_x}. \tag{25}$$

The last equation is the labor-market-clearing condition such that

$$l_x + l_r^* + l_r^v = 1. \tag{26}$$

Thus, solving (24)-(26) yields the equilibrium labor allocations as follows:

$$l_x = \frac{1 + \frac{\rho}{\phi_x} + \frac{\rho}{\phi_y}}{1 + \frac{\mu - 1}{1 + \xi_y i} + \mu (\mu - 1) / (1 + \xi_y i)} \tag{27}$$

$$l_r^v = \frac{\mu (\mu - 1)}{1 + \xi_y i} \left( \frac{1 + \frac{\rho}{\phi_x} + \frac{\rho}{\phi_y}}{1 + \frac{\mu - 1}{1 + \xi_y i} + \mu (\mu - 1) / (1 + \xi_y i)} \right) - \frac{\rho}{\phi_y}. \tag{28}$$

$$l_r^x = \frac{\mu - 1}{1 + \xi_y i} \left( \frac{1 + \frac{\rho}{\phi_x} + \frac{\rho}{\phi_y}}{1 + \frac{\mu - 1}{1 + \xi_y i} + \mu (\mu - 1) / (1 + \xi_y i)} \right) - \frac{\rho}{\phi_x}. \tag{29}$$

In these equilibrium labor allocations, (27) shows that production labor $l_x$ is increasing in the nominal interest rate $i$, because a higher $i$ raises the cost of borrowing for R&D investment, which reallocates the labor from R&D to manufacturing. Nevertheless, (28) reveals that there are two effects of $i$ on the downstream-R&D labor $l_r^v$. On the one hand, $i$ has a negative effect on $l_r^v$ due to the reallocation of labor to production as aforementioned (i.e., the manufacturing-R&D reallocation effect). On the other hand, $i$ has a negative (positive) effect on $l_r^v$ if $\xi_y$ is greater (smaller) than $\xi_x$, namely, downstream R&D is more (less) bound by the CIA constraint than upstream R&D. This creates another reallocation effect of labor between the two R&D sectors (i.e., the cross-R&D-sector effect). Whether a higher $i$ increases or decreases $l_r^v$ depends on the relative magnitudes of $\xi_y$ and $\xi_x$ and the level of markup $\mu$. Specifically, when $\xi_y \geq \xi_x$, the cross-R&D-sector effect is negative and thus reinforces the manufacturing-R&D reallocation effect, causing $l_r^v$ to be decreasing in $i$. 


When $\zeta_y < \zeta_x$, the cross-R&D-sector effect becomes positive. In this case, if $\mu$ is sufficiently large (small), the markup strengthens (dampens) the positive cross-R&D-sector effect and induces it to dominate (to be dominated by) the negative manufacturing-R&D reallocation effect. As a result, $l^y_r$ is increasing (decreasing) in $i$. Furthermore, (29) shows that $i$ also has these two effects on the upstream-R&D labor $l^r$, and the analysis of the overall impact is analogous to that for the impact of $i$ on $l^y$.

Therefore, the above result and (23) imply a mixed effect of the nominal interest rate $i$ on the equilibrium growth rate of technology $g = \ln z(\phi_y l^y_r + \phi_x l^x_r)$, and this effect is determined by the impacts of $i$ on the levels of R&D labor $l^y_r$ and $l^x_r$ in addition to the productivity parameters $\phi_y$ and $\phi_x$. Using (27)-(29) and differentiating (23) with respect to $i$ yields

$$\frac{\partial g}{\partial i} = (\phi_y \frac{\partial l^y_r}{\partial i} + \phi_x \frac{\partial l^x_r}{\partial i}) \ln z$$

where $\Lambda \equiv (1 + \zeta_y i)(1 + \zeta_x i) + (\mu - 1)(1 + \zeta_x i) + \mu(\mu - 1)(1 + \zeta_x i)$. It can be seen that $\partial g / \partial i$ is decreasing in $i$, namely, $g$ is a concave function of $i$. In particular, if $\partial g / \partial i|_{i=0} = \mu \phi_y[\mu(\mu - 1)\zeta_x - \mu \zeta_y] + \phi_x \left[\mu(\mu - 1)\zeta_y - (\mu^{2} - \mu + 1)\zeta_x\right] > 0$, then the growth rate of aggregate technology $g$ and the nominal interest rate $i$ exhibit an inverted-U relationship. Intuitively, R&D labor in the less CIA constrained sector tends to be increasing in the nominal interest rate $i$ with the support of a large markup. With a large sectoral productivity parameter, the less constrained sector yields a sufficiently large positive growth effect that can dominate the two negative growth effects caused by (i) the reallocation of labor from R&D to production and (ii) the more constrained sector where R&D labor is decreasing in $i$. Thus, monetary policy is growth-enhancing in this case. Notwithstanding, as $i$ increases, the above domination of the less constrained sector becomes increasingly weaker as compared to the effects (i) and (ii). Then, monetary policy turns to become growth-retarding. Notice that if the markup is small and/or the productivity parameter in the less constrained sector is small, the growth-increasing part of monetary policy will be invalid; $g$ can only be decreasing in $i$.

**Proposition 2.** Suppose that the condition that $\mu \phi_y[\mu(\mu - 1)\zeta_x - \mu \zeta_y] + \phi_x \left[\mu(\mu - 1)\zeta_y - (\mu^{2} - \mu + 1)\zeta_x\right] > (\leq)0$ holds. Then the nominal interest rate $i$ generates an inverted-U (a monotonically negative) effect on the equilibrium growth rate $g$.

**Proof.** Proven in the text.

### 3.3 Socially Optimal Allocations

Imposing balanced growth on (i) yields

$$U = \frac{1}{\rho} \left( \ln c_0 + \frac{g}{\rho} \right)$$

\textsuperscript{11}When $\zeta_x > \zeta_y$, $l^r_y$ is decreasing in $i$, whereas when $\zeta_x < \zeta_y$, $l^r_y$ is increasing (decreasing) in $i$ if $\mu$ is sufficiently large (small).
where \( c_0 = Z_{x,0}Z_{y,0}l_x \) and \( g = \ln(\varphi_y l^y_r + \varphi_x l^x_r) \). Dropping the exogenous terms \( Z_{x,0} \) and \( Z_{y,0} \) and maximizing (31) subject to the resource constraint for labor \( l_x + l^y_r + l^x_r = 1 \) yields the first-best allocations denoted by a superscript asterisk:

\[
\begin{align*}
    l^*_x &= \min \left\{ \frac{\rho}{\varphi_y \ln z}, \frac{\rho}{\varphi_x \ln z} \right\}, \\
    l^*_y &= \max \left\{ 1 - \frac{\rho}{\varphi_y \ln z}, 0 \right\}, \\
    l^*_x &= \max \left\{ 1 - \frac{\rho}{\varphi_x \ln z}, 0 \right\}.
\end{align*}
\]

The socially optimal outcome implies that technology advances in the downstream and upstream sectors are perfectly substitutable. Therefore, a corner solution arises for the first-best labor allocations in the sense that the social optimum only allocates labor to the R&D sector with a higher level of productivity. Specifically, suppose \( \varphi_y > \varphi_x \), that is, innovative activities in the downstream sector are more productive, so that devoting all R&D labor to this sector is socially optimal. As a result, the labor in the upstream R&D sector is zero, implying that economic growth in the social optimum only depends on downstream innovations. By contrast, for \( \varphi_y < \varphi_x \), the situation reverses such that R&D labor is only allocated to the upstream sector but not at all to the downstream sector.

4 Optimal Monetary Policy and the Friedman Rule

In this section, we analyze the optimal monetary policy and examine the conditions under which the Friedman rule is (sub)optimal. In Sections 4.1 and 4.2, we consider the cases in which the CIA constraint is only imposed on downstream R&D or upstream R&D, respectively. In Section 4.3, we consider the case in which the model features an equal CIA constraint on both R&D sectors. We use \( i^* \) to denote the optimal nominal interest rate, which maximizes social welfare.

Furthermore, it is useful to note that in the following analysis, the relative R&D productivity between downstream R&D and upstream R&D (\( \varphi_y < \varphi_x \) or \( \varphi_y > \varphi_x \)) is crucial in determining the social optimal sectoral labor allocations in R&D. For saving space, throughout the remaining study, we mainly focus on the analysis of a high relative productivity of upstream R&D (or a low relative productivity of downstream R&D), i.e., \( \varphi_y < \varphi_x \), as the analysis of \( \varphi_y > \varphi_x \) is analogous to that of \( \varphi_y < \varphi_x \).13

---

12 Suppose that the productivity is identical between the two R&D sectors such that \( \varphi_x = \varphi_y = \varphi \). This then implies that the first-best production labor \( L^*_x = \rho/(\varphi \ln z) \). In this case, one condition is lost to determine the relationship between the two first-best R&D labor allocations \( L^*_x \) and \( L^*_y \). In other words, any combination of \( \{L^*_y, L^*_x\} \) satisfying \( L^*_y + L^*_x = 1 - \rho/(\varphi \ln z) \) can be a solution. Without loss of generality, it is assumed that \( \varphi_y \) and \( \varphi_x \) differ to facilitate the welfare analysis that follows, and thus, the possibility of \( \varphi_y = \varphi_x \) is excluded.

13 The numerical analysis shown in subsection 5.1 indicates that the situation of \( \varphi_y < \varphi_x \) is more likely to occur. The analytical results of \( \varphi_y > \varphi_x \) are available upon request.
4.1 CIA Constraint on Downstream R&D

In this subsection, we consider the optimal monetary policy with the CIA constraint only applying to downstream R&D, which is captured by $\xi_x = 0$. For simplicity, we also normalize the CIA constraint on downstream R&D to unity (i.e., $\xi_y = 1$) in this case.

By imposing $\xi_x = 0$ and $\xi_y = 1$, the equilibrium labor allocations are simplified to

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{\mu + \frac{\mu(\mu - 1)}{1+i}}$$  \hspace{1cm} (35)

$$l^y_r = \frac{\mu(\mu - 1)}{1+i} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) - \frac{\rho}{\varphi_y}$$  \hspace{1cm} (36)

$$l^x_r = \frac{(\mu - 1) \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right)}{\mu + \frac{\mu(\mu - 1)}{1+i}} - \frac{\rho}{\varphi_x}.$$  \hspace{1cm} (37)

From (35)-(37), it is easy to see that production labor $l_x$ is increasing in the nominal interest rate $i$, whereas downstream- (upstream-) R&D labor is decreasing (increasing) in $i$. Given the fact that downstream R&D is more constrained by the CIA constraint than upstream R&D (i.e., $\xi_y = 1 > \xi_x = 0$), the effect of $i$ now operates through the CIA constraint on the downstream R&D sector so that a higher $i$ increases the cost of downstream R&D relative to upstream R&D, leading to a labor reallocation from downstream R&D to upstream R&D and manufacturing.

Accordingly, substituting (35)-(37) into (31) yields the lifetime utility $U$ along the BGP. Then, to examine the (sub)optimality of the Friedman rule in this case, differentiating $U$ with respect to $i$ and evaluating $\partial U/\partial i$ at $i = 0$ yields

$$\text{sign} \left( \frac{\partial U}{\partial i} \bigg|_{i=0} \right) = \text{sign} \left\{ 1 - \frac{\ln{z}}{\mu^2 \rho} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) \left[ \varphi_x + \mu(\varphi_y - \varphi_x) \right] \right\},$$  \hspace{1cm} (38)

which can be either positive or negative depending on the parameter values. Given that downstream R&D is less productive than upstream R&D, namely, $\varphi_y < \varphi_x$, we compare the first-best production labor (32) and the equilibrium production labor (35) evaluated at $i = 0$. If the equilibrium at $i = 0$ features R&D overinvestment (i.e., $l^*_x |_{i=0} + l^*_y |_{i=0} > l^x_r + l^y_r$ or $1 - \varphi_x \ln{z} (1 + \rho/\varphi_x + \rho/\varphi_y)/(\mu^2 \rho) > 0$), the right-hand side of (38) becomes

$$\frac{\partial U}{\partial i} |_{i=0} = 1 - \frac{\varphi_x \ln{z}}{\mu^2 \rho} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) - \frac{\ln{z}}{\mu^2 \rho} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) \mu(\varphi_y - \varphi_x) > 0.$$  \hspace{1cm} (39)

This implies that R&D overinvestment in equilibrium is sufficient (but not necessary) for the Friedman rule to be suboptimal. Intuitively, by raising $i$ to be positive, on the one hand, the manufacturing-R&D reallocation effect pushes down the excess growth (i.e., reducing $l^*_x + l^*_y$ toward $l^*_{rx} + l^*_{ry}$) in exchange for an efficient increase in consumption to approximate its optimal level (i.e., moving $l_x$ closer to $l^*_x$), which yields a layer of welfare gain; on the other hand, the cross-R&D-sector effect stimulates technology growth through shifting R&D labor from the less
productive sector to the more productive sector (i.e., shifting \( l_i^U \) to \( l_i^c \)), which yields another layer of welfare gain. Therefore, the welfare level increases unambiguously.

Nevertheless, if R&D underinvestment occurs in equilibrium (i.e., \( 1 - \varphi_x \ln z(1 + \rho/\varphi_x + \rho/\varphi_y)/(\ln^2 \rho) < \ln z(1 + \rho/\varphi_x + \rho/\varphi_y) (\varphi_y - \varphi_x)/(\mu \rho) < 0 \)), \( \partial U/\partial i \rvert_{i=0} \) also may be positive, implying that R&D underinvestment could lead the Friedman rule to be suboptimal, conditional on a higher \( \mu \).\(^{14}\) Moreover, we can infer that if \( \partial U/\partial i \rvert_{i=0} < 0 \), then R&D must be underinvested in equilibrium. It implies that R&D underinvestment is the necessary condition for the Friedman rule to be optimal.

In the case in which the CIA constraint is only present on downstream R&D, the analytical solution for the optimal interest rate \( i^* \) is found to exist. Using the first-order condition of \( U \) with respect to \( i \), we derive \( i^* \) for \( \xi_s = 0 \) given by

\[
i^* = \max \left[ \frac{\ln \rho}{\rho} \left( 1 + \frac{\varphi}{\varphi_x} + \frac{\rho}{\varphi_y} \right) \left( \varphi_y - \varphi_x \left( \frac{\mu - 1}{\mu} \right) \right) - 1, 0 \right],
\]

and the value of \( i^* \) is chosen based on the sign of \( \partial U/\partial i \rvert_{i=0} \) in (38). The above results are summarized in Proposition 3.

**Proposition 3.** Suppose that only the CIA constraint on downstream R&D is present. Then the optimal nominal interest rate \( i^* \) is given by (40). Furthermore, when \( \varphi_y < \varphi_x \), R&D overinvestment in the zero-nominal-interest-rate equilibrium is sufficient for the Friedman rule to be suboptimal.

*Proof.* Proven in the text.

As for the comparative statics of \( i^* \) when it is positive, the analysis is similar to that in Chu and Cozzi (2014), and the intuition is given as follows. First, a higher discount rate \( \rho \) implies a worsening of the intertemporal spillover effect, which is a negative externality that leads R&D overinvestment to be more likely to occur. Thus, \( i^* \) is increasing in \( \rho \). This model features a (positive) negative surplus-propriability effect on the productivity in the (downstream) upstream R&D, in the sense that a higher \( \varphi_x/\varphi_y \) is more likely to cause R&D overinvestment. Thus, \( i^* \) is increasing in \( \varphi_x/\varphi_y \). Additionally, a larger patent breadth \( \mu \) strictly increases downstream R&D and thus R&D overinvestment is more likely to occur. At the same time, a larger \( \mu \) reinforces the monopoly-pricing distortion faced by the upstream sector, and thus enlarges the welfare gain of raising \( i \) through the increase in the upstream-R&D labor \( l_x^* \) even if R&D is underinvested. Thus, \( i^* \) is increasing in \( \mu \). Finally, a smaller step size \( z \) of innovation implies a negative externality that makes R&D overinvestment more possible, so that \( i^* \) is decreasing in \( z \).

### 4.2 CIA Constraint on Upstream R&D

We now turn to consider the optimal monetary policy in a case where the CIA constraint only on upstream R&D is present. For simplicity, we also normalize the CIA constraint on upstream

\(^{14}\) Specifically, a relatively large markup, i.e., \( \mu > \bar{\mu} = -\ln z(1 + \rho/\varphi_x + \rho/\varphi_y)(\varphi_x - \varphi_y)/(2\rho) + \sqrt{\ln z(1 + \rho/\varphi_x + \rho/\varphi_y)(\varphi_x - \varphi_y)/(2\rho) + \varphi_x \ln z(1 + \rho/\varphi_x + \rho/\varphi_y)/(\rho/\varphi_y)} \), would enable R&D underinvestment to yield a positive welfare-maximizing nominal interest rate. This implies that the suboptimality of the Friedman rule can be supported by R&D underinvestment if the welfare gain from reallocating labor through the increase in \( l_x \) and \( l_x^* \) dominates the welfare cost of a deterioration in R&D underinvestment through the decrease in \( l_x^* \).
R&D to unity in this case. Consequently, imposing $\xi_y = 0$ and $\xi_x = 1$ in (27)-(29) yields the equilibrium labor allocations such that

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{1 + \mu(\mu - 1) + \frac{\mu - 1}{1 + \rho}},$$

(41)

$$l^y_x = \frac{\mu(\mu - 1)\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \mu(\mu - 1) + \frac{\mu - 1}{1 + \rho} \varphi_y} - \frac{\rho}{\varphi_y},$$

(42)

$$l^x_x = \frac{\mu^{-1}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \mu(\mu - 1) + \frac{\mu - 1}{1 + \rho} \varphi_x} - \frac{\rho}{\varphi_x}.$$  

(43)

Equations (41)-(43) show that under $\xi_y = 0$, a higher rate of nominal interest $i$ continues to increase the manufacturing labor $l_x$ and decrease (increase) the labor in the constrained (unconstrained) R&D sector, i.e., the upstream R&D labor $l^y_x$ (the downstream R&D labor $l^x_x$). In addition, substituting (41)-(43) into (31) and evaluating $\partial U/\partial i$ at $i = 0$ yields

$$\text{sign} \left( \frac{\partial U}{\partial i} |_{i=0} \right) = \text{sign} \left\{ 1 - \frac{\ln z}{\mu^2 \rho} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) \left[ \varphi_x + \mu(\mu - 1)(\varphi_x - \varphi_y) \right] \right\},$$

(44)

which again can be either positive or negative. Given a high relative productivity of upstream R&D (i.e., $\varphi_y < \varphi_x$), comparing (41) and (32) at $i = 0$ shows that if R&D underinvestment arises in the equilibrium where $i = 0$ (i.e., $l^x_x |_{i=0} + l^y_x |_{i=0} < l^x_x + l^y_x$ or $1 - (\varphi_y \ln z) (1 + \rho/\varphi_x + \rho/\varphi_y)/(\mu^2 \rho) < 0$), the right-hand side of (44) will be given by

$$\frac{\partial U}{\partial i} |_{i=0} = 1 - \frac{\varphi_y \ln z}{\mu^2 \rho} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) - \frac{\ln z}{\mu^2 \rho} \left( 1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y} \right) (\mu - 1)(\varphi_y - \varphi_x) < 0,$$

(45)

which implies that R&D underinvestment is sufficient (but not necessary) for the Friedman rule to be optimal. Intuitively, by raising $i$ to be positive, on the one hand, the manufacturing-R&D reallocation effect increases the excess consumption (i.e., moving $l_x$ away from $l^x_x$) and at the same time decreases technology growth further from its suboptimal level (i.e., reducing $l^x_x$ and $l^y_x$ away from $l^x_x + l^y_x$); on the other hand, the cross-R&D-sector effect reduces technology growth through shifting R&D labor from the more productive sector to the less productive one (i.e., from $l^x_x$ to $l^y_x$). Therefore, the overall welfare worsens unambiguously.

Moreover, if $\partial U/\partial i |_{i=0} > 0$, then R&D must be overinvested (i.e., $1 - \varphi_y \ln z (1 + \rho/\varphi_x + \rho/\varphi_y) > -\ln z (1 + \rho/\varphi_x + \rho/\varphi_y) (\varphi_y - \varphi_x) (\mu - 1)/(\mu^2 \rho) > 0$), implying that R&D overinvestment is the necessary condition for the Friedman rule to be suboptimal; in particular, this can be guaranteed under a larger markup $\mu$.\footnote{It can be shown that the degree of R&D overinvestment has to be sufficiently high in order to make a positive rate of nominal interest optimal, and this can be achieved under a large markup. Specifically, if the degree of R&D overinvestment $l^x_x - l^y_x |_{i=0}$ is greater than a threshold value, then the LHS of (44) will suffice to be positive. Suppose $1 - l^y_x |_{i=0} = 1 - l^x_x + \chi \Leftrightarrow l^y_x |_{i=0} = l^x_x + \chi$, where $\chi > 0$. There thus exists a threshold value $\bar{\chi}$ such that if and only if $\chi > \bar{\chi}$, then $\partial U/\partial i |_{i=0} > 0$. Furthermore, $\bar{\chi}$ is given by $\rho/(\varphi_x \ln z) - (1 + \rho/\varphi_x + \rho/\varphi_y)/\mu^2$, which is increasing in $\mu$. Hence, the degree of R&D overinvestment is more likely to dominate the threshold value under a large markup.}
Then, differentiating $U$ with respect to $i$ yields the analytical solution for the optimal nominal interest rate $i^*$ for $\xi_y = 0$ given by

$$i^* = \max \left[ \frac{\ln \rho}{1 + \frac{\rho}{\phi} + \frac{\rho}{\phi_y}} \left( \frac{1}{\mu + \frac{1}{\mu-r}} \right) - \left( \mu + \frac{1}{\mu-r} \right) - 1, 0 \right]. \quad (46)$$

We summarize the above results in Proposition 4.

**Proposition 4.** Suppose that only the CIA constraint on upstream R&D is present. Then the optimal nominal interest rate $i^*$ is given by (46). Furthermore, when $\phi_y < \phi_x$, R&D underinvestment in the zero-nominal-interest-rate equilibrium is sufficient for the Friedman rule to be optimal.

*Proof.* Proven in the text. \qed

Again, investigating (46) shows that $i^*$, when it is positive, is increasing in $\rho$ and $\phi_y/\phi_x$, but decreasing in $z$. The intuition for these results is similar to that in Section 4.1. Nevertheless, in the case of (46), $i^*$ can be increasing or decreasing in $\mu$. The reason that $i^*$ increases in response to a larger $\mu$ also follows from the counterpart in Section 4.1. The difference in the current section is that the CIA constraint is now imposed on the upstream R&D sector. When the level of $\mu$ becomes relatively high, the reduction in the demand for intermediate goods (captured by $1/\mu$ in (12)) can make the monopoly-pricing distortion in the upstream sector less severe, which mitigates the welfare effect of increasing $i$. If this welfare effect is significantly attenuated, it is possible for $i^*$ to decrease in response to a larger $\mu$.

### 4.3 CIA Constraints on Two Sectors

In this section, we consider the case in which both downstream and upstream R&D sectors are subject to a CIA constraint. The strengths of CIA constraints on the two sectors are equal, namely, $\xi_x = \xi_y = \xi$, where we normalize $\xi = 1$ for analytical simplicity in this case as in the previous two subsections.\footnote{In the numerical exercise, we consider a case in which the strengths of the CIA constraints on the two sectors are allowed to differ. See Subsection 5.3.3.}

Under this setting, the equilibrium labor allocations are given by

$$l_x = \frac{1 + \frac{\rho}{\phi} + \frac{\rho}{\phi_y}}{1 + \frac{\mu - \rho}{\mu - \rho + \phi_x}}, \quad (47)$$

$$l_y^u = \frac{\mu(\mu - 1)}{1 + \mu - \rho + \phi_y} - \frac{\rho}{\phi_y}, \quad (48)$$

$$l_x^u = \frac{\mu(\mu - 1)}{1 + \mu - \rho + \phi_y} - \frac{\rho}{\phi_x}. \quad (49)$$

In contrast to the previous cases for the imposition of CIA parameters, under $\xi_x = \xi_y = 1$, a higher rate of nominal interest $i$ increases the manufacturing labor $l_x$ by decreasing the R&D
labor $l^Y_l$ and $l^{\xi}_l$ in both the upstream and downstream sectors; only the manufacturing-R&D reallocation effect is in play, whereas the cross-R&D-sector effect becomes absent.

Substituting (47)-(49) into (31) and evaluating the result at $i = 0$ yields

$$
\text{sign} \left( \frac{\partial U}{\partial i} |_{i=0} \right) = \text{sign} \left\{ 1 - \frac{1}{\mu^2} - \ln\left( \frac{\mu}{R} \right) \left( 1 + \frac{\rho}{\phi_t} + \frac{\rho}{\phi_x} \right) \left[ \mu \phi_t + \phi_y \right] \right\}.
$$

(50)

Given $\phi_y < \phi_x$, we compare $l^*_\xi$ in (32) and $l_{\xi}$ in (47) at $i = 0$. If R&D overinvestment in the equilibrium occurs, (i.e., $l^\ast_{\xi}|_{i=0} + l^\ast_{y}|_{i=0} > l^\ast_{\xi} + l^\ast_{y}$ or $1 - \phi_x \ln\left( 1 + \rho / \phi_x + \rho / \phi_y \right) / (\mu^2 \rho) > 0$), then the right-hand side of (50) becomes

$$
\frac{\partial U}{\partial i} |_{i=0} = \frac{\mu(\mu - 1)}{\mu^2} \left\{ 1 - \frac{\phi_y}{\phi_x} + \left[ 1 - \frac{\phi_x \ln\left( 1 + \frac{\rho}{\phi_x} + \frac{\rho}{\phi_y} \right)}{\mu^2 \rho} \right] \left( \frac{\phi_y}{\phi_x} + \frac{1}{\mu} \right) \right\} > 0.
$$

(51)

This shows that R&D overinvestment is sufficient (but not necessary) for the Friedman rule to be suboptimal. In the absence of the cross-R&D-sector effect, the manufacturing-R&D reallocation effect, when $i$ is raised, depresses the excess technology growth to compensate for a sufficient increase in consumption to approach their optimal levels. Accordingly, the overall welfare un-ambiguously improves.\(^{17}\)

Notice that if the analysis of a low relative productivity of upstream R&D (i.e., $\phi_x < \phi_y$) is also considered, one can show that R&D overinvestment is sufficient for the Friedman rule to be suboptimal irrespective of the relative productivity between the R&D sectors.\(^{18}\) This is because without the effect of the nominal interest rate on the labor reallocation across the R&D sectors, the relative magnitude of R&D productivity becomes invariant to the changes in welfare.

Additionally, we derive the optimal nominal interest rate $i^\ast$ for $\xi = \xi_y = 1$ by differentiating $U$ with respect to $i$, which is given by

$$
i^\ast = \max \left\{ \frac{\ln\left( 1 + \frac{\rho}{\phi_x} + \frac{\rho}{\phi_y} \right)}{1 + \mu} - 1, 0 \right\}.
$$

(52)

Then, we summarize the above results in Proposition 5.

**Proposition 5.** Suppose that the CIA constraints on both R&D sectors are equally present. Then the optimal nominal interest rate $i^\ast$ is given by (52). Furthermore, R&D overinvestment in the zero-nominal-interest-rate equilibrium is sufficient for the Friedman rule to be suboptimal (regardless of the relative R&D productivity).

*Proof.* Proven in the text. \(\square\)

Notice that the comparative statics of $i^\ast$ in (52) on $\rho$ and $\phi$ is similar to that in the previous sections. Moreover, $i^\ast$ is increasing in $\mu$. This is because both R&D sectors are now subject to an equal CIA constraint, and an increase in $\mu$ will increase $l^\ast_{\xi}$ and $l^\ast_{y}$ simultaneously. Therefore, $i^\ast$

\(^{17}\)In this case, R&D underinvestment is necessary for the optimality of the Friedman rule. Nevertheless, R&D underinvestment could still lead the Friedman rule to be suboptimal conditional on a large value of $\mu$.

\(^{18}\)The detailed analysis for this case is available upon request.
must increase unambiguously to reduce the levels of R&D labor in the two sectors in response. What is different here is that a higher \( q_y / q_x \) can either increase or decrease \( i^\ast \), and it depends on the level of \( \mu \). Specifically, if \( \mu \) is large (small), then raising \( q_y / q_x \) will decrease (increase) \( i^\ast \); the monopoly-pricing distortion faced by the upstream sector will intensify (attenuate) the effect of a relatively low level of \( q_x \) on the upstream-R&D labor \( l^\ast \), so R&D underinvestment (overinvestment) is more likely to occur and \( i^\ast \) would respond by declining (rising).

5 Numerical Analysis

In this section, we calibrate this two-R&D-sector model to the US economy to quantitatively analyze the growth and welfare effects of monetary policy.

5.1 Calibration

To perform this numerical analysis, the strategy is to assign steady-state values to the following structural parameters \( \{\rho, \mu, \xi_x, \xi_y, z, \rho_x, \rho_y\} \). We follow Acemoglu and Akcigit (2012) to set the discount rate \( \rho \) to 0.05. As for the level of patent breadth, we set \( \mu = 1.1 \) as the market value to capture the empirical findings from Laitner and Stolyarov (2004). As for the strength of CIA constraints on upstream and downstream R&D activities, we consider the presence of a CIA constraint on both R&D sectors in Section 4.3 as our benchmark. We thereafter consider the pairs of \( \{\xi_x = 0, \xi_y = 1\} \) and \( \{\xi_x = 1, \xi_y = 0\} \), to corresponds to the analysis in Section 4.1 and 4.2, respectively.

To calibrate the quality step size parameter \( z \), we use the arrival rate of innovation and the equilibrium growth rate. The arrival rate of innovation is set to 6.5\%, which is a reasonable value consistent with empirical estimates.\(^{19}\) According to the Conference Board Total Economy Database, the growth rate of total factor productivity (TFP) for the US economy from 1990-2016 is approximately 0.5\%, so we consider \( g = 0.005 \) as the benchmark value. Then, the quality step size is calibrated to \( z = 1.08 \), which is close to the estimated value in Acemoglu and Akcigit (2012) (i.e., 1.05) and Akcigit and Kerr (2018).\(^{20}\) Moreover, to pin down the remaining R&D productivity parameters \( \rho_x \) and \( \rho_y \), in addition to the benchmark growth rate, we use the ratio of scientists and engineers engaged in R&D over the manufacturing labor force.\(^{21}\) The average ratio is around 5.7\%, indicating \( (1 - l_x)/l_x = 0.057 \). Therefore, the aggregate R&D labor ratio \( l^\ast_y + l^\ast_x = 1 - l_x \) is around 0.05.\(^{22}\)

For the benchmark value of nominal interest rate, we adopt the following moments. First, the population growth rate is set to \( n = 0.01 \) to correspond to the data during 1990-2016 according

---

\(^{19}\)Existing studies have considered different values for the arrival rate of innovations. For example, Caballero and Jaffe (2002) estimate a mean rate of creative destruction of roughly 4\%. Lanjouw (1998) estimates the probability of obsolescence to be in the range of 7\% − 12\%, and Laitner and Stolyarov (2013) find a rate of 3.5\% per year of the rate of creative destruction. In this study, we consider an intermediate value of 6.5\% within the above estimates.

\(^{20}\)The quality step size is calibrated according to \( z = \exp(0.005/0.065) \approx 1.08 \), which is also consistent with the estimate from Akcigit and Kerr (2018) who find a radical innovation step size of 1.112 and an incremental one of 1.051.

\(^{21}\)The number of scientists and engineers engaged in R&D during 1990-2016 is obtained from Science and Engineering Indicators 2000 (Appendix Tables 3-25) published by the National Science Foundation. The data on manufacturing employees are obtained from the Bureau of Labor Statistics.

\(^{22}\)When using the data for an even longer period such as 1980-1997, the average ratio remains almost unchanged, although a slightly increasing trend in the R&D labor ratio can be observed.
to the Conference Board Total Economy Database. Second, the market-level nominal interest rate $i$ is calibrated by targeting at $\pi = 2.5\%$, which is the average annual inflation rate of the US economy within this period according to Bureau of Labor Statistics. Hence, the benchmark value of nominal interest rate is given by $i = r + \pi = g + \rho + n + \pi = 0.09$. Accordingly, Table 1 summarizes the calibrated values of parameters and variables in this quantitative exercise.\(^{23}\)

<table>
<thead>
<tr>
<th>Targets</th>
<th>$r$</th>
<th>$l^y_r + l^x_r$</th>
<th>$g$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>$\xi_y$</td>
<td>$\xi_x$</td>
<td>$z$</td>
<td>$\mu$</td>
</tr>
<tr>
<td></td>
<td>0.065</td>
<td>0.05</td>
<td>0.005</td>
<td>0.025</td>
</tr>
</tbody>
</table>

### 5.2 Growth and Welfare Implications of Monetary Policy

This subsection evaluates the growth and welfare effects of inflation in the case of an equal CIA constraint on both R&D sectors. Fig.1a indicates that for $\xi_x = \xi_y = 1$, the steady-state growth rate of technology $g$ is monotonically decreasing in the inflation rate. According to (48) and (49), a higher nominal interest rate $i$ raises the cost of both downstream and upstream R&D due to CIA constraints in these sectors. As discussed in Subsection 3.2, with the same degree of CIA constraints on both sectors, only the manufacturing-R&D reallocation effect is present. This induces labor to shift from the R&D sector to the manufacturing sector, thereby leading to a monotonic decrease in the rate of economic growth.

Moreover, the level of steady-state welfare is also decreasing in the inflation rate for $\xi_x = \xi_y = 1$, as shown in Fig.1b. It is known that a higher $i$ leads to a rise in $l^r_x$ but a decline in $l^y_r$ and $\lambda^x_r$, which, according to (31), increases the level of current consumption $c_0$ (i.e., a positive welfare effect) but decreases the growth rate $g$ (i.e., a negative welfare effect). In this benchmark case, the negative welfare effect always dominates the positive one, resulting in a negative relationship.

---

\(^{23}\)When solving for the values of $\varphi_x$ and $\varphi_y$, we restrict our analysis to positive values to ensure that the R&D labor allocations of $l^y_r$ and $l^x_r$ are positive.
between inflation and welfare. In addition, we define the change in steady-state welfare by the usual equivalent variation in consumption flow such that \( \kappa \equiv \exp(\rho \Delta U) - 1 \). Then, within the range of \( i \), we find that the welfare gain can be as large as approximately 1% of consumption per annum by moving the equilibrium from \( i = 0.2140 (\bar{\pi} = 0.15, \text{the upper bound of inflation rate}) \) to the welfare-maximizing outcome (i.e., the equilibrium at \( i = 0 \)).

Furthermore, we compare the socially optimal manufacturing labor \( l_x^* \) and the market equilibrium manufacturing labor \( l_x|_{i=0} \) in the zero-nominal-interest-rate equilibrium. Given a higher relative productivity of upstream R&D in our benchmark case where \( \phi_y = 0.5265 < \phi_x = 1.3136 \), we use (32)-(34) to find that the socially optimal labor allocations are given by \( l_x^* = 0.4946, l_y^r = 0, l_x^r = 0.5054 \), respectively. Evaluating (47) at \( i = 0 \) yields the equilibrium manufacturing labor of \( l_x = 0.9364 \), which implies \( l_x|_{i=0} > l_x^* \) and \( l_y^r|_{i=0} + l_x^r|_{i=0} < l_y^r + l_x^r \). This shows that relative to the first-best allocation, much less R&D labor is assigned in equilibrium (i.e., R&D underinvestment). According to (52), the optimal nominal interest rate is given by \( i^* = 0 \), and hence the Friedman rule is optimal.

### 5.3 Sensitivity Analysis

In this subsection, we perform several sensitivity checks to test the robustness of our quantitative results. Specifically, we first examine the cases where a CIA constraint is present in only one R&D sector to correspond to our theoretical analysis in Subsection 4.1 and Subsection 4.2, respectively.\(^25\) Moreover, we conduct a sensitivity exercise on the markup. Finally, we show that the inverted-U growth effect of inflation, as indicated in Proposition 2, is quantitatively available under an even larger markup.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \xi_y )</th>
<th>( \xi_x )</th>
<th>( \mu )</th>
<th>( \phi_y )</th>
<th>( \phi_x )</th>
<th>( l_y^r + l_x^r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1.1</td>
<td>0.7268</td>
<td>1.0404</td>
<td>0.07†</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1.1</td>
<td>0.5265</td>
<td>1.3136</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.2</td>
<td>0.5265</td>
<td>1.3136</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.26</td>
<td>0.5265</td>
<td>1.3136</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>1.888</td>
<td>0.5265</td>
<td>1.3136</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

Note: † represents the sensitivity check in which the parameters \( \phi_y \) and \( \phi_x \) are re-calibrated to match the R&D intensity as indicated. Additionally, in this case, there are two groups of productivity parameters generated, and we choose the group of \( \phi_y < \phi_x \) to be consistent with the benchmark case.

---

\(^24\)Throughout the quantitative analysis, we focus on an empirically realistic case of the inflation rate where \( \pi \leq 0.15 \). According to the Bureau of Labor Statistics, the maximum of quarterly and annual inflation rates for the US are 0.136 and 0.124, respectively, from 1958 (when the indicator is available) to 2016. Thus, to cover these empirical moments, we consider a slightly large value of 0.15 as the upper bound of inflation rate. Correspondingly, the upper bound of nominal interest rate is around 0.2140 in the benchmark case by solving the function that \( i - g(i) - \rho - n = 0.15 \).

\(^25\)We have also considered the case where a CIA constraint is exclusively present on consumption. Not surprisingly, when labor is elastically supplied, the presence of a CIA constraint on consumption leads a higher nominal interest rate to reduce the rate of economic growth and the level of social welfare, through a conventional effect of decreasing the aggregate labor supply and labor employment in both R&D sectors.
5.3.1 CIA parameters

First, in the case of a CIA constraint on the downstream R&D sector only (i.e., $\xi_x = 0$, $\xi_y = 1$), the benchmark value of aggregate R&D labor ratio is adjusted to a slightly larger one, such as 7%. The two sectoral productivity parameters are then re-calibrated to be $\varphi_y = 0.7268 < 1.0404 = \varphi_x$. Fig. 2a displays the growth effects of inflation for $\varphi_y < \varphi_x$. When the CIA constraint is only present in the downstream R&D sector, raising the nominal interest rate increases the R&D cost for only the downstream sector and hence shifts its R&D labor to both the upstream and manufacturing counterparts (by both cross-R&D and manufacturing-R&D reallocations). Moreover, in this case, the growth-retarding effect induced by the R&D labor flowing out from the downstream R&D sector (i.e., a decrease in $l_y^r$) is partially offset by the growth-enhancing effect caused by the R&D labor flowing into the upstream R&D sector (i.e., an increase in $l_x^r$). Thus, in spite of a relatively higher productivity in the upstream sector, the growth rate of technology continues to be decreasing in the inflation rate as in our benchmark case.

As for the welfare analysis, the optimal inflation rate turns to become positive, as displayed in Fig. 2b. In contrast to the benchmark case, this result implies the suboptimality of the Friedman rule. Recall that the welfare effect of inflation relies on its impacts on $l_x$ and $g$ according to (31). Raising $i$ from a lower level gives rise to a stronger effect of manufacturing-R&D allocation (which increases consumption and welfare) than that of cross-R&D allocation (which decreases growth and welfare), leading to a net welfare improvement. However, as the nominal interest rate rises and exceeds the threshold value $i = 0.1516$ (namely, the inflation rate is higher than the threshold value of $\pi = 8.68\%$), the effect of manufacturing-R&D allocation becomes increasingly weak, and it is eventually dominated by the effect of cross-R&D allocation. In this case, within the range of $i$, the largest welfare change is obtained by raising the nominal interest rate from $i = 0$ to the welfare-maximizing level ($i = 0.1516$), yielding a marginal gain of $\kappa = 0.0072\%$.

To examine the implication of Proposition 3, we show that the first-best labor allocations are $l_x^* = 0.6245$, $l_y^r = 0$, and $l_x^r = 0.3755$, given that the downstream R&D sector is less productive herein (i.e., $\varphi_y = 0.7268 < 1.0404 = \varphi_x$). The equilibrium manufacturing labor at $i = 0$ is $l_x = 0.9230$ according to (35), implying that $l_x|_{i=0} > l_x^*$ and $l_y^r|_{i=0} + l_x^r|_{i=0} < l_y^r + l_x^r$. Therefore, again, R&D underinvestment occurs in the zero-nominal-interest-rate equilibrium.

---

26 Because in the case of $\xi_x = 0$ and $\xi_y = 1$, using the benchmark values would lead the downstream R&D labor $l_y^r$ to be negative when $\pi \geq 0.028$, we recalibrate the productivity parameters by using an alternative R&D labor ratio.
Next, we perform the sensitivity check when only the upstream R&D activities are subject to a CIA constraint (i.e., $\xi_x = 1$, $\xi_y = 0$ and other parameters are maintained at their benchmark values). Fig. 3a illustrates the growth effect of inflation accordingly. In this case, given a low relative sectoral productivity in the downstream sector, raising the nominal interest rate tends to trigger off a significant cross-R&D labor reallocation from the more constrained upstream R&D sector (i.e., $l_x^r$ decreases) to the less constrained downstream R&D sector (i.e., $l_y^r$ increases), which causes the nominal interest rate to strictly decrease in the growth rate of technology.

![Fig. 3. (a) Inflation and economic growth; (b) Inflation and social welfare ($\xi_x > \xi_y$)](image)

In addition, Fig. 3b illustrates the welfare effect of inflation for the case with $\xi_x = 1$ and $\xi_y = 0$. It shows that an increase in the inflation rate causes welfare to decrease, implying the optimality of the Friedman rule. In this case, within the range of $i$, the welfare difference between the equilibrium at $i = 0.2145^{27}$ and the welfare-maximizing outcome (the equilibrium at $i = 0$) yields the largest welfare differential (i.e., $\kappa = 1.531\%$), indicating a more severe welfare loss as compared to the benchmark case. Furthermore, we obtain $l_x|_{i=0} > l_x^* + l_y^r|_{i=0} + l_x^r|_{i=0} < l_x^{r*}$, which, again, implies R&D underinvestment in the zero-nominal-interest-rate equilibrium.

### 5.3.2 Markup

To assess the role of a markup in the growth and welfare effects of inflation, we consider a larger value of the markup, i.e., $\mu = 1.2$, holding other parameters unchanged as in the benchmark case. The growth-retarding and welfare-decreasing effects of inflation are displayed in Fig. 4a and Fig. 4b, respectively. A comparison with Fig. 1a reveals that raising the markup increases the level of growth rate. Additionally, comparing Fig. 1b and Fig. 4b shows that a higher markup increases the welfare level through raising the level of growth rates, whereas it does not alter the monotonically decreasing relationship between inflation and social welfare. Thus, the welfare-maximizing equilibrium is at $i = 0$ in which the Friedman rule still holds.

Interestingly, when continuing to raise the markup to an even larger value of $\mu = 1.26$, we find that although the growth rate of aggregate technology is still monotonically decreasing in the inflation rate, social welfare becomes non-monotonic to inflation, with a welfare-maximizing equilibrium at $i = 0.2145$.

---

27The upper bound of inflation rate of 0.15 corresponds to $i = 0.2145$ in this case. It is obtained by solving the function indicated in Footnote 24.

28The markup values of 1.2 and 1.26 are in line with the intermediate values of the empirical estimates reported in Jones and Williams (2000) (i.e., 1.05-1.4).
inflation rate $\pi = 3.94\% \ (i = 0.1180)$. This is consistent with our analytical results shown in 4.3; the welfare-maximizing nominal interest rate $i^\star$ is increasing in $\mu$, and R&D underinvestment could also lead to the suboptimality of Friedman rule with the support of a higher markup.\footnote{In this case, R&D is underinvested such that $l_y^\star|_i=0 + l_x^\star|_i=0 = 0.2863 < l_y^0 + l_x^0 = 0.5054$.} 

These quantitative results are reported in Fig. 5a and Fig. 5b accordingly.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{(a) Inflation and economic growth; (b) Inflation and social welfare ($\mu = 1.2$)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{(a) Inflation and economic growth; (b) Inflation and social welfare ($\mu = 1.26$)}
\end{figure}

### 5.3.3 Inverted-U effect of inflation on growth

The last exercise is performed to examine the possibility of an inverted-U relationship between the inflation rate and economic growth rate; such a connection has been observed in recent empirical studies such as Bick (2010) and Kremer et al. (2013). We find that when raising the markup to $\mu = 1.888$,\footnote{Comin (2004) shows that the markups charged by innovators should be well above the generally estimated average markup in the economy. De Loecker et al. (2018) documents that the average markup has increased steadily in the U.S. since 1980. Thus, it is not unrealistic to consider a higher markup of $\mu = 1.888$.} and simultaneously adjusting the two CIA constraint parameters to $\zeta_x = 0.1$ and $\zeta_y = 1$,\footnote{It is useful to note that according to Proposition 2, either setting any one CIA constraint to zero or choosing any parameter groups with equal CIA constraints will lead to a monotonic effect of $i$ on $g$. Moreover, given the low relative productivity of downstream R&D, a positive growth effect of inflation is present when the upstream R&D sector is less cash constrained than its downstream counterpart. Intuitively, the growth-enhancing effect of inflation is in play when the labor reallocation operates from the less productive sector to the more productive one.} the growth rate of aggregate technology is an inverted-U function...
of inflation, as shown in Fig. 6a. This is consistent with the analytical result in Proposition 2. In addition, the growth-maximizing inflation rate is found to be around 2.73%, which is in line with the empirical estimates of Ghosh and Phillips (1998) (i.e., 2.5%) and López-Villavicencio and Mignon (2011) (i.e., 2.7%). Finally, Fig. 6b shows that the level of welfare is increasing in inflation, indicating that the Friedman rule is suboptimal in this case.

![Graph](image)

**Fig. 6.** (a) Inflation and economic growth (b) Inflation and social welfare.

## 6 Conclusion

In this study, we analyze the growth and welfare effects of monetary policy in a Schumpeterian growth model in which both vertical sectors engage in R&D activities and CIA constraints are present in R&D investment. We find that a higher nominal interest rate reallocates resources from the more cash-constrained R&D sector to the less constrained one. In addition to the usual growth-retarding effect of CIA constraints due to an increase in R&D costs, the cross-sector reallocation of R&D labor can generate a growth-enhancing effect, which leads economic growth to have an inverted-U shape in relation to the nominal interest rate. Moreover, we examine the necessary and sufficient conditions for the (sub)optimality of the Friedman rule by relating the underinvestment and overinvestment of R&D in the decentralized equilibrium. We find that this relationship is crucially affected by the presence of CIA constraints, the relative productivity between upstream R&D and downstream R&D, and the strength of markup; these factors determine the interaction between the welfare effects brought about by the reallocation of different types of labor, and thereby determine the optimal design of monetary policy. Finally, by calibrating this two-R&D-sector model to the US data, our quantitative results suggest that the growth-maximizing and welfare-maximizing nominal interest rates are generally zero, i.e., they are given by the Friedman rule. In addition, it is found that the welfare effect of monetary policy can be quantitatively significant by altering the nominal interest rate from the steady-state value to the optimal value. Therefore, our analysis serves to provide an analytical and quantitative justification that reveals the importance of the inclusion of R&D investment in an economy with vertical industries, when considering the role of monetary policy in economic growth and social welfare.

By incorporating a dilution effect on R&D productivity as in Chu and Cozzi (2014), this study removes the scale-effect problem present in the first-generation endogenous growth model such
as in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Alternatively, one can remove the scale effects in the Schumpeterian growth model by considering the semi-endogenous-growth approach as in Kortum (1997) and Segerstrom (1998) or the second-generation approach as in Peretto (1998, 2007). Due to its complexity, we leave this potentially interesting extension to future research.

Appendix A

Proof of Proposition 1

Suppose that a time path of \([i_t]_{t=0}^\infty\) is stationary. First, define a transformed variable \(\Psi_{y,t} \equiv N_t c_t / v_{y,t}\), and its law of motion is given by

\[
\dot{\Psi}_{y,t} = n + \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_{y,t}}{v_{y,t}}. \tag{A.1}
\]

Similarly, define another transformed variable \(\Psi_{x,t} \equiv N_t c_t / v_{x,t}\), and its law of motion is given by

\[
\dot{\Psi}_{x,t} = n + \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_{x,t}}{v_{x,t}}. \tag{A.2}
\]

To derive the law of motion for \(v_{y,t}\), substituting (12) into (19) yields

\[
\frac{\dot{v}_{y,t}}{v_{y,t}} = r_t + \phi_y l_{y,t} - \left(\frac{\mu - 1}{\mu}\right) \Psi_{y,t}. \tag{A.3}
\]

Likewise, combining (16) and (20) yields the law of motion for \(v_{x,t}\) such that

\[
\frac{\dot{v}_{x,t}}{v_{x,t}} = r_t + \phi_x l_{x,t} - \left(\frac{\mu - 1}{\mu^2}\right) \Psi_{x,t}. \tag{A.4}
\]

Plugging (A.3) and (A.4) into (A.1) and (A.2) respectively, along with the Euler equation (5), yields

\[
\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \left(\frac{\mu - 1}{\mu}\right) \Psi_{y,t} - \phi_y l_{y,t} - \rho, \tag{A.5}
\]

and

\[
\frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}} = \left(\frac{\mu - 1}{\mu^2}\right) \Psi_{x,t} - \phi_x l_{x,t} - \rho. \tag{A.6}
\]

Moreover, using the zero-expected-profit condition of upstream R&D (21) yields an expression of \(l_{x,t}\) such that

\[
l_{x,t} = \left(\frac{1 + \xi y}{\mu^2 \phi_y}\right) \Psi_{y,t}, \tag{A.7}
\]

See Chu et al. (2015) and Huang et al. (2015), respectively, for the use of the above approaches to eliminate scale effects in a monetary Schumpeterian growth model.
and using the zero-expected-profit condition of downstream R&D (21) to relate $l_{x,t}$ to $\Psi_{x,t}$ yields

$$l_{x,t} = \left( \frac{1 + \xi_x i}{\mu^2 \varphi_x} \right) \Psi_{x,t}. \tag{A.8}$$

Therefore, $\Psi_{x,t}$ can be expressed as a function of $\Psi_{y,t}$ such that

$$\Psi_{x,t} = \left[ \frac{(1 + \xi_y i) \varphi_x}{(1 + \xi_x i) \varphi_y} \right] \Psi_{y,t}. \tag{A.9}$$

Then, it is obvious that

$$\frac{\Psi_{y,t}}{\Psi_{y,t}} = \frac{\Psi_{x,t}}{\Psi_{x,t}}. \tag{A.10}$$

Using (A.10) together with (A.5), (A.6) and (A.9), we derive a relationship between $l_{r,t}^y$ and $l_{r,t}^x$ such that

$$l_{r,t}^y = \left( \frac{\varphi_y}{\varphi_x} \right) l_{r,t}^y + \left[ \frac{(\mu - 1)(1 + \xi_y i) \varphi_x}{\mu^2 \varphi_y (1 + \xi_x i)} - \frac{\mu - 1}{\mu} \right] \frac{\Psi_{y,t}}{\varphi_x}. \tag{A.11}$$

Finally, to derive the relationship between $\Psi_{y,t}$ and $l_{r,t}^y$, using the labor-market-clearing condition $l_{x,t} + l_{r,t}^y + l_{r,t}^x = 1$ and substituting (A.7) and (A.11) into it yields

$$\left( 1 + \frac{\varphi_y}{\varphi_x} \right) l_{r,t}^y = 1 - \left[ \frac{1 + \xi_y i}{\varphi_y} + \frac{(\mu - 1)(1 + \xi_y i)}{\varphi_y \mu^2 (1 + \xi_x i)} - \frac{\mu - 1}{\mu \varphi_x} \right] \Psi_{y,t}. \tag{A.12}$$

Substituting (A.12) into (A.6) yields an autonomous dynamical equation for $\Psi_{y,t}$ such that

$$\frac{\Psi_{y,t}}{\Psi_{y,t}} = \frac{\varphi_y}{\varphi_y + \varphi_x} \left[ \frac{1 + \xi_y i}{\mu^2} \left( 1 + \frac{\mu - 1}{1 + \xi_x i} \right) + \frac{\mu - 1}{\mu} \right] \Psi_{y,t} - \left( \frac{\varphi_x \varphi_y}{\varphi_y + \varphi_x} + \rho \right). \tag{A.13}$$

Given that $\Psi_{y,t}$ is a control variable and the coefficient on $\Psi_{y,t}$ is positive in (A.13), the dynamics of $\Psi_{y,t}$ is characterized by saddle-point stability in this model such that $\Psi_{y,t}$ jumps immediately to its interior steady-state value given by

$$\Psi_y = \frac{\mu^2 [\rho (1 + \varphi_x / \varphi_x) + \varphi_y]}{(1 + \xi_y i) [1 + (\mu - 1)/(1 + \xi_x i)] + \mu (\mu - 1)}. \tag{A.14}$$

Equations (A.7) and (A.12) imply that when $\Psi_{y,t}$ is stationary, $l_{x,t}$ and $l_{r,t}^y$ must be stationary, which in turn implies that $l_{r,t}^x$ is stationary as well according to (A.11).

References


