Effects of subsidies on growth and welfare in a quality-ladder model with elastic labor*

Ruiyang Hu† University of Macau
Yibai Yang‡ University of Macau
Zhijie Zheng§ Beijing Normal University

January 16, 2023

Abstract

This paper develops a quality-ladder growth model with elastic labor supply and distortionary taxes to analyze the effects of different subsidy instruments: subsidies to the production of final goods, subsidies to the purchase of intermediate goods, and subsidies to research and development (R&D). Moreover, the model is calibrated to the US data to compare the growth and welfare implications of these subsidies. The main results are as follows. First, we analytically show that an optimal coordination of all instruments attains the first-best outcome. Second, in the calibrated economy, we numerically find that for the use of a single instrument, R&D subsidy is less growth-enhancing and welfare-improving than the other subsidies, whereas for the use of a mix of two instruments, subsidizing the production of final goods and the purchase of intermediate goods is most effective in promoting growth but least effective in raising welfare.

JEL classification: D61; E62; O31; O38
Keywords: Economic Growth; R&D; Quality Ladder; Subsidies

*We are grateful to Helmuth Cremer (the Editor), the Associate Editor, and two anonymous Referees for their helpful comments and generous suggestions. We also thank Angus Chu, Philipp Boeing, Cody Hsiao, Defu Li, Yu Pang, Dongying Sun, Xinyang Wei, and Yan Zhang and seminar/conference participants at Macau University of Science and Technology, the 2019 Lingnan Macroeconomics Meeting, and the 2021 Asia-Pacific Innovation Conference for the useful discussion and feedback. Hu gratefully acknowledges the financial support from the Start-Up Research Grant of University of Macau. Yang gratefully acknowledges financial support from the Research Grant of the Department of Science and Technology of Guangdong Province and the support by the Asia Pacific Academy of Economics and Management at University of Macau.

†Department of Economics, University of Macau, Taipa, Macao, China. *Email* address: ruiyang.hu.econ@gmail.com
‡Department of Economics, University of Macau, Taipa, Macao, China. *Email* address: yibai.yang@hotmail.com.
§Bay Area International Business School, Beijing Normal University, Zhuhai 519087, China. *Email* address: zhengzhijie1919@gmail.com.

1 Introduction

In this study, we explore the effects of subsidization on economic growth and social welfare in a Schumpeterian economy with elastic labor supply and distortionary taxes. In many industrialized economies where research activities for innovations are the major engine of growth, it is observed that research and development (R&D) activities are highly intervened by government policies. The narrative of government intervention on R&D activities has been justified by a spate of endogenous-growth-theory literature that highlight the presence of a positive R&D externality, since inventors of new products face knowledge spillovers and it is difficult for them to fully appropriate the benefits of innovations (e.g., Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992, and Jones and Williams 2000).[1] In subsequent R&D-based growth models, the granting of monopoly rights to innovators and subsidies to R&D are the two major forms of government policy instruments exploited to deal with such an R&D externality, both of which, unfortunately, have limitations.[2] Granting monopoly power to successful innovators in the form of patent protection, which incentivizes entrepreneurs to carry out R&D activities by allowing for monopolistic profits, would reduce the demand for production inputs to the level below the first-best allocation.[3] Similarly, subsidizing R&D investment also aims to effectively promote costly research activities to stimulate growth,[4] thereby potentially generating sizable effects on welfare. Nevertheless, the tool of R&D subsidies still cannot remove the distortion of monopoly pricing, since this policy instrument mainly affects the competitive R&D sector instead of the monopolistic production sector.

To internalize R&D externalities and remove the monopoly-pricing distortion, the existing studies exploiting the R&D-based growth framework have used a subsidization-policy regime, which includes subsidies to manufacturing and subsidies to R&D. In the presence of inelastic labor supply and lump-sum taxes through which subsidies are financed, the model of Barro and Sala-I-Martin (2003) with expanding varieties of new products shows that subsidies to manufacturing (through either final goods produced by competitive firms or the purchase of intermediate goods produced by monopolistic firms) are able to effectively restore the social optimum, which, however, cannot be achieved by subsidizing R&D activities. In contrast, the model of Acemoglu (2009) with improving quality of existing products implies that subsidizing both manufacturing and R&D is able to remove the distortions and replicate the socially optimal allocation.[5]

[1] In addition to this positive R&D externality, there can also be negative R&D externalities due to duplicative R&D (i.e., congestion externalities in Jones and Williams 1998, 2000) and business stealing (Bloom *et al.* 2013). However, the positive R&D externality tends to substantially outweigh the negative externalities (Grossmann *et al.* 2013), and this is consistent with the existing empirical evidence suggesting that the social return to R&D exceeds the private return by a wide margin. See Hall *et al.* (2010) for a complete review of the econometric literature on measuring the private and social returns to R&D.

[2] See Park (2008) for a strengthening of patent protection across countries since the signing of the agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) in 1994. In addition, see Impullitti (2010) for evidence on the widespread use of R&D subsidization in OECD countries since 1980s.

[3] The recent empirical evidence in Brown *et al.* (2017) finds that the protection for intellectual property has positive effects on R&D in OECD economies.

[4] See, for example, Minniti and Venturini (2017) who find that R&D tax credits have positive effects on productivity growth of the US economy.

[5] Yang (2018) shows that in a quality-ladder growth model with lump-sum taxes, mixing the subsidies to manufacturing and R&D can also achieve the social optimum even if labor supply is elastic.

In this study, we extend the R&D-based growth framework of Acemoglu (2009) with quality improvement, and examine the growth and welfare effects of three types of subsidy instruments: subsidies to the production of final goods, subsidies to the purchase of intermediate goods, and subsidies to R&D.[6] Specifically, this study revisits the implications of these subsidies and their combinations by means of growth maximization and welfare maximization. The major contribution of this paper is to provide novel findings, along with important policy implications, in the context of quality-ladder models taking into account elastic labor supply and distortionary taxes on wage income, which not only better capture households' labor supply decisions and labor market distortions, but also are shown in the literature to be important factors for welfare analysis in dynamic general equilibrium models.[7] Under the variety-expansion growth framework, Zeng and Zhang (2007) consider elastic labor supply with the financing option altered to distortionary taxes on wage income, and reveal that using a single subsidy alone or even their combination (with a consumption tax as well) cannot reach the social optimum when labor income tax is within the reasonable range. Moreover, extending Matsuyama (1999, 2001) by incorporating elastic labor and capital accumulation, Wan and Zhang (2021) suggest that the efficient allocation can be obtained by appropriating tax rules and subsidy regimes, and eliminating distortions requires policymakers to set opposite taxes on consumption and labor income. Since such an important practice under the quality-ladder framework is largely missing, our study also contributes to the literature by filling this gap.

Similar to the canonical R&D-based growth model, the inefficiencies present in this model originate from the distorted resource allocation in production caused by monopoly price and in innovation caused by R&D externalities. Nevertheless, by incorporating elastic labor supply and distortionary labor taxes, an additional inefficiency is introduced through the consumption-labor tradeoff in households' decision, which affects the resource allocation in production as well.[8] Therefore, the policymaker can adjust the equilibrium allocation to mitigate these distortions by properly implementing the subsidy tools.

The findings of this study are summarized as follows. First, it is shown that subsidies to manufacturing are more effective in eliminating the distortions in monopoly pricing and labor supply, whereas subsidies to research are more effective in eliminating the distortion in R&D externalities. Thus, an optimal mix of three policy instruments eliminates the three layers of inefficiencies in the decentralized equilibrium, restoring the social optimum. Second, in the calibrated economy, when only a single subsidy tool is used, subsidizing R&D investment is less effective than subsidizing manufacturing in terms of promoting growth and raising welfare. This is because the benefits of innovations and the removal of inefficiencies are both less sensitive to the decrease in research expenditures resulting from subsidies to R&D than to the increase in production volume induced by subsidies to manufacturing. In other words, most of

[6]Throughout this study, subsidies to the production of final goods and those to the purchase of intermediate goods are collectively called subsides to manufacturing.

[7]Bilbiie *et al.* (2019) investigate the welfare effect of distortions in a dynamic stochastic general equilibrium model featuring monopolistic competition and endogenous product creation. Comparing the market equilibrium with the socially efficient allocation, their quantitative analysis indicates that distortions due to elastic labor on its own account for a significant share (5%) of consumption. Their model suggests that achieving the first-best allocation requires subsidies on labor and physical capital investment.

[8]In Zeng and Zhang (2007), inefficiencies from monopoly pricing and labor supply are classified to the *static* distortion, whereas inefficiency from R&D externalities is classified to the *dynamic* distortion.

the inefficiencies in the decentralized equilibrium of this model stem from the distorted resource allocation caused by monopoly pricing and labor supply. Third, as for the use of a combination of any two instruments, subsidizing the production of final goods and the purchase of intermediate goods is most effective in promoting growth but least effective in raising welfare. The reason is that using two forms of subsidies to manufacturing together expands the dimension that enlarges the production size and hence is most growth-stimulating. Nevertheless, these two subsidy instruments remove the distortions stemming from only the manufacturing sector, so their combination generates less welfare as compared to the combinations of policy instruments involving R&D subsidies, which remove the distortions stemming also from the research sector. These results highlight the importance of the coordinated use of subsidies to R&D and subsidies to manufacturing in raising social welfare.

To thoroughly investigate the effects of elastic labor and distortionary taxes, we exploit the constant Frisch elasticity preference of households as in Trabandt and Uhlig (2011). It is shown analytically that the elasticity of labor supply plays an important role in determining the steady-state allocation of labor, consumption and R&D investment, and the steady-state level of welfare. In contrast, our quantitative analysis calibrated to the US economy suggests that intertemporal elasticity of substitution in consumption can remarkably affect the magnitude of the optimal subsidy rates. It is found that, under the first-best allocation, higher intertemporal elasticity of substitution is associated with a higher subsidy to final goods producers, a lower subsidy to intermediate goods producers, and a lower tax rate on R&D activities.

This study also contributes to the literature by comparing the welfare implications between models with and without elastic labor. While empirical evidence is mixed, studies based on microeconomic data typically report low values of the Frisch elasticity estimate, which potentially motivates researchers to consider an analytical framework with inelastic labor for simplification.[9] However, this study shows that it is of central importance to distinguish the implications under low elasticity of labor supply and under perfectly inelastic labor supply. Otherwise, considering low elasticity of labor supply as perfectly inelastic labor supply could result in misleading policy and welfare implications. In particular, when labor supply is perfectly inelastic, distortions induced by consumption-leisure tradeoff no longer exist, and hence, it would become impossible to uniquely pin down the optimal combination of three subsidy tools in our model. According to our quantitative analysis, the potential welfare loss associated with the misuse of subsidy instruments is increasing in the true value of the Frisch elasticity, and its magnitude can be significant in certain circumstances.

This study relates to the vast literature that explore the effects of R&D subsidies in innovation-driven growth models; see for example, Segerstrom (1998), Lin (2002), Dinopoulos and Syropoulos (2007), Şener (2008), Impullitti (2010), and Chu and Cozzi (2018), in which either variety expansion or quality improvement is considered as the process of innovation, in addition to Peretto (1998), Segerstrom (2000), Chu *et al.* (2016), and Chu and Wang (2022), in which the two dimensions of innovation are combined. While inspiring, the aforementioned studies mainly focus on financing subsidy costs with non-distortionary taxes, ruling out the distortionary effects of taxes on the aggregate equilibrium allocation. Our study differs from theirs by considering

[9] For recent surveys of literature inferring labor supply elasticity based on microeconomic data, please refer to Meghir and Phillips (2010) and Saez *et al.* (2012).

the impacts of R&D subsidies when subsidization is financed by distortionary labor income taxes that distort the consumption-labor decision.[10]

This study also relates to the literature on R&D-based growth models that consider the mixed use of subsidies to research and intermediate goods. Grossmann *et al.* (2013) show that in a semi-endogenous growth model put forward by Jones (1995), a combination of a time-varying subsidy to R&D and a constant subsidy to intermediate-good production can achieve the socially optimal growth path. Furthermore, Li and Zhang (2014) show that in the Matsuyama (1999) model of growth through cycles, using subsides to R&D and the purchase of intermediate goods, either individually or jointly, yields significant welfare gains. However, the analysis of these interesting studies focuses on dynamic general equilibrium frameworks with inelastic labor supply.[11] Therefore, the present paper complements their interesting studies by investigating the welfare implications when labor is supplied elastically. It turns out that elastic labor supply plays a crucial role in our model in attaining the social optimum.

Taking into account elastic labor supply, Annicchiarico *et al.* (2022) investigate optimal taxation in a scale-free quality-ladder model featuring distortionary labor and capital income taxes, levied by an exogenous public sector which consumes a fixed fraction of gross output. They find that, facing a larger share of capital income or a larger share of government expenditures in GDP, an increase in the ratio of capital to labor income tax leads to sizable welfare gains. Their analysis, however, focuses on the second-best allocation by assessing the welfare effect of the redistribution of the tax burden between labor and capital income taxes, rather than investigating the possibility of eliminating all distortions to achieve the social optimum. Our study complements their interesting study by exploring subsidy regimes that can potentially replicate the socially optimal allocation.

Finally, our study complements the recent study of Wan and Zhang (2021) who consider a variety-expansion model with elastic labor by comparing the roles of subsidies on welfare. Similar to their results, our numerical results find large potential gains in welfare from implementing optimal subsidies. In particular, our results continue to suggest an increase in manufacturing subsidies to increase labor and the demand for products. Nevertheless, in contrast to their study, our results suggest an increase in labor income taxes to raise the financing of subsidization. In addition, the results suggest a decrease in subsidies on research costs to eliminate the inefficiencies of (negative) R&D externalities. These welfare comparisons reveal the importance of the process of innovation (variety expansion versus quality improvement) regarding the design of optimal subsidy instruments in R&D-based growth models.

The rest of this paper is organized as follows. Section 2 presents the model setup. Section 3 characterizes the decentralized equilibrium and explores the growth effects of subsidies. Section 4 derives the first-best optimal outcome and analyzes the subsidy policy that restores the social

[10] Atkeson *et al.* (2019) also consider various innovation subsidies based on the framework of Garcia-Macia *et al.* (2019) and explore both the dynamic and steady-state properties of aggregate productivity. Our study complements this seminal research by studying the growth and welfare effects of production and R&D subsidies in addition to wage income taxes. More importantly, the analytical tractability of the model allows us to clearly disentangle the impacts of different policy tools.

[11] One exception is Nuño (2011), who finds that in the presence of elastic labor supply, the optimal mix of subsidies to research and intermediate-good production can replicate the first-best allocation in a Schumpeterian growth model with business cycles. Nevertheless, the financing of subsidies in his analysis relies on a lump-sum tax on households.

optimum. Section 5 performs a numerical analysis in a calibrated economy to evaluate the growth-maximizing and welfare-maximizing subsidy instrument(s). A model with inelastic labor supply is discussed in Section 6. Finally, Section 7 concludes the study.

2 The model

In this study, we extend the version of the quality-ladder growth model in Acemoglu (2009) (Chapter 14), which originates from Grossman and Helpman (1991), by incorporating (a) subsidies to the production of final goods, the purchase of intermediate goods, and expenditures on R&D, and (b) elastic labor supply. Moreover, this model introduces distortionary labor income taxes to finance the subsidies as in Zeng and Zhang (2007). This study analyzes the growth and welfare implications of subsidization by controlling one or a mix of these policy instruments.

2.1 Households

Suppose that the economy admits a unit continuum of identical households. Following Trabandt and Uhlig (2011) and Annicchiarico *et al.* (2022), we exploit a framework with constant Frisch elasticity and assume that the lifetime utility function of each household is given by

$$U_t = \int_{t=0}^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma}\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]^{\sigma} - 1}{1-\sigma} dt, \tag{1}$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution (IES) in consumption, $\eta > 0$ governs the Frisch elasticity of labor supply, ρ is the discount rate, C_t and L_t denote the household's consumption of final goods and labor hours supplied, respectively, and the parameter $\theta > 0$ captures the intensity of leisure preference relative to consumption.[12]

There is no population growth in this economy.[13] Each household chooses consumption C_t and labor supply L_t to maximize its lifetime utility (1) subject to the instantaneous budget constraint such that

$$\dot{A}_t = r_t A_t + W_t(1-\tau_t)L_t - C_t, \tag{2}$$

where A_t is the real value of financial assets owned by each household, W_t is the real wage rate, r_t is the real interest rate, and τ_t is the tax rate on labor wage.[14] The standard dynamic optimization

[12]Note that, when $\sigma = 1$, the utility function becomes

$$U_t = \int_0^{\infty} e^{-\rho t}\left(\ln C_t - \theta L_t^{1+\frac{1}{\eta}}\right) dt.$$

The corresponding equilibrium conditions can be obtained by setting σ in the generalized version of our model to unity. One exception lies in the steady-state welfare function, on which we provide detailed discussion in Appendix A.2.

[13]Our model results are robust to the setting of population growth in which the counterfactual scale effect is sterilized in a fully-endogenous approach. See the detailed discussion in Cozzi (2017a).

[14]Here we consider a wage income tax instead of an asset income tax, because if R&D subsidies were financed by a tax on asset income, then raising R&D subsidies would generate a counteracting effect on innovation, which would make the impact of R&D subsides unclear.

implies the consumption-labor decision given by

$$\frac{W_t(1-\tau_t)}{C_t} = \frac{\sigma\theta(1+\eta)L_t^{\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}, \tag{3}$$

and the usual Euler equation given by

$$\frac{\dot{C}_t}{C_t} + \frac{\theta(1-\sigma)(1+\eta)L_t^{\frac{1}{\eta}}\dot{L}_t}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} = \frac{r_t-\rho}{\sigma}. \tag{4}$$

Moreover, households own a balanced portfolio of all firms in the economy. The transversality condition is given by $\lim_{t\to\infty} e^{-\rho t}C_t^{-\sigma}A_t = 0$, which implies that the discounted present value of assets or debt should be zero when time goes to infinity.[15]

2.2 Final goods

Final goods Y_t are produced competitively by using labor and a continuum of intermediate goods according to the following production function:

$$Y_t = \frac{L_t^{1-\beta}}{\beta}\int_0^1 q_t(\nu)X_t(\nu)^\beta d\nu, \qquad \beta \in (0,1) \tag{5}$$

where L_t is the level of labor, $X_t(\nu)$ is the quantity of intermediate good in industry line $\nu \in [0,1]$ whose quality is $q_t(\nu)$, and β measures the importance of intermediate good ν relative to labor in producing final goods. In addition, the quality $q_t(\nu)$ evolves as follows:

$$q_t(\nu) = \lambda^{n_t(\nu)}q_0(\nu), \tag{6}$$

where $\lambda > 1$ represents the step size of each quality improvement, $n_t(\nu)$ is the number of innovations in line ν that have occurred between time 0 and time t. Then the profit function of the competitive final-good producers is given by

$$\hat{\pi}_t = (1+s_{y,t})\frac{L_t^{1-\beta}}{\beta}\int_0^1 q_t(\nu)X_t(\nu)^\beta d\nu - W_tL_t - (1-s_{x,t})\int_0^1 P_t(\nu)X_t(\nu)d\nu, \tag{7}$$

where $s_{y,t} \in (0,1)$ ($s_{y,t} \in (-1,0)$) is the subsidy rate (tax rate) to the production of final goods and $s_{x,t} \in (0,1)$ ($s_{x,t} \in (-1,0)$) is the subsidy rate (tax rate) to the purchase of intermediate goods.

With free entry and profit maximization, equation (5) yields the conditional demand functions

[15]In the steady state, as will be shown, the real interest rate r_t is constant. Therefore, we have $e^{-\rho t}C_t^{-\sigma} = e^{-rt}$, and the transversality condition also implies that neither assets nor debt should grow at the rate of return to assets (or higher) in the long run.

for inputs, namely, the demand for labor:

$$L_t = (1 + s_{y,t})(1-\beta)\frac{Y_t}{W_t}, \tag{8}$$

and the demand for the intermediate good ν:

$$X_t(\nu) = \left(\frac{1+s_{y,t}}{1-s_{x,t}}\right)^{\frac{1}{1-\beta}} \left(\frac{q_t(\nu)}{P_t(\nu)}\right)^{\frac{1}{1-\beta}} L_t, \tag{9}$$

where $P_t(\nu)$ is the price of the ν-th intermediate good relative to final goods.

2.3 Intermediate goods

In each industry line $\nu \in [0,1]$, intermediate goods are produced by a monopolistic leader who holds a patent on the latest innovation and replaced by the products of an entrant who has a new innovation due to the *Arrow replacement effect*. The marginal cost of producing a unit of intermediate good is $\psi q_t(\nu)$ units of final goods, where $\psi \in (0,1)$. Thus, the ν-th intermediate-good producer maximizes her profits $\pi_t(\nu) = [P_t(\nu) - \psi q(\nu)]x_t(\nu)$ subject to the intermediate-good demand in (9), which yields the profit-maximizing price such that $P_t(\nu) = (\psi/\beta)q_t(\nu)$. This implies that all monopolists charge a price equal to a constant markup $1/\beta$ over their marginal cost of production $\psi q_t(\nu)$. Then, without loss of generality, normalizing $\psi \equiv \beta$ yields

$$P_t(\nu) = q_t(\nu). \tag{10}$$

Substituting (10) into (9) generates the quantity of intermediate good ν such that

$$X_t(\nu) = \left(\frac{1+s_{y,t}}{1-s_{x,t}}\right)^{\frac{1}{1-\beta}} L_t. \tag{11}$$

Additionally, according to (11), the profit function of the monopolistic firm is given by

$$\pi_t(\nu) = (1-\beta)\left(\frac{1+s_{y,t}}{1-s_{x,t}}\right)^{\frac{1}{1-\beta}} q_t(\nu)L_t, \tag{12}$$

which shows that the monopolistic profit is increasing in product quality.

Technological progress in this model stems from the realization of quality improvements in $q_t(\nu)$ across all industry lines. Define the aggregate quality index Q_t by a combination of the total quality of intermediate goods such that

$$Q_t = \int_0^1 q_t(\nu)d\nu. \tag{13}$$

Substituting (11) and (13) into the final-good production function in (5) yields the total output in the final-good sector such that

$$Y_t = \frac{1}{\beta} \left(\frac{1+s_{y,t}}{1-s_{x,t}} \right)^{\frac{\beta}{1-\beta}} Q_t L_t, \tag{14}$$

which shows that the aggregate output is linearly increasing in the aggregate quality of intermediate goods. Next, using (11), the aggregate spending on intermediate goods is obtained by

$$X_t \equiv \int_0^1 P_t(\nu) X_t(\nu) d\nu = \beta \left(\frac{1+s_{y,t}}{1-s_{x,t}} \right)^{\frac{1}{1-\beta}} Q_t L_t. \tag{15}$$

Then the labor wage rate is given by

$$W_t = (1+s_{y,t}) \left(\frac{1-\beta}{\beta} \right) \left(\frac{1+s_{y,t}}{1-s_{x,t}} \right)^{\frac{\beta}{1-\beta}} Q_t. \tag{16}$$

Using (12) and (13) yields the aggregate profit that occurs in the intermediate-good sector, which is given by

$$\Pi_t \equiv \int_0^1 \pi_t(\nu) d\nu = (1-\beta) \left(\frac{1+s_{y,t}}{1-s_{x,t}} \right)^{\frac{1}{1-\beta}} Q_t L_t. \tag{17}$$

Observing (17) reveals that the total monopolistic profit Π_t created by all inventions is increasing in the policy instrument $s_{y,t}$ to the production of final goods and $s_{x,t}$ to the purchase of intermediate goods, respectively.

2.4 Innovations and R&D

Denote by $V_t(\nu)$ the real value of a firm who holds the most recent innovation in line ν. Accordingly, the Hamilton-Jacobi-Bellman (HJB) equation for $V_t(\nu)$ is given by

$$r_t V_t(\nu) = \pi_t(\nu) + \dot{V}_t(\nu) - p_t(\nu) V_t(\nu), \tag{18}$$

which is the no-arbitrage condition for the value of the asset (in the form of a patented innovation). Equation (18) implies that the return on this asset $r_t V_t(\nu)$ equals the sum of the profit flow $\pi_t(\nu)$, the capital gain $\dot{V}_t(\nu)$, and the potential losses $p_t(\nu)V_t(\nu)$ that occur due to creative destruction, where $p_t(\nu)$ denotes the Poisson arrival rate of the next successful innovation in each instant of time. Specifically, based on the lab-equipment assumption, the formulation of $p_t(\nu)$ is given by

$$p_t(\nu) = \frac{\zeta z_t(\nu)}{L_t q_t(\nu)}, \tag{19}$$

where $\zeta > 0$ is R&D productivity and $z_t(\nu)$ is the amount of final goods spent in R&D. Equation (19) means that the probability of the next successful innovation is increasing in R&D expenditures $z_t(\nu)$ whereas decreasing in quality $q_t(\nu)$; research on more advanced products becomes more difficult, so one unit of R&D spending is proportionately less effective when applied to a more sophisticated product.[16] Moreover, to eliminate the scale effect in this model, we use the *fully endogenous* solution by assuming that the arrival rate of innovation depends on R&D expenditures per unit of labor.[17]

New innovations in each line are invented by R&D firms, who have free entry into the research market and incur positive expenditures on R&D subject to policy interventions in the form of subsidization (taxation) at the rate of $s_{r,t} \in (0,1)$ ($s_{r,t} \in (-1,0)$). Hence, the expected profit of an R&D firm who spends $z_t(\nu|q\lambda^{-1})$ in R&D in line ν that has quality $q\lambda^{-1}$ at time t must be zero such that $p_t(\nu|q\lambda^{-1})V_t(\nu|q) - (1-s_{r,t})z_t(\nu|q\lambda^{-1}) = 0$, and it, together with (19) (in the form of $p_t(\nu|q\lambda^{-1}) = [\zeta z_t(\nu|q\lambda^{-1})]/[L_t q_t(\nu)\lambda^{-1}]$ if the initial quality is $q\lambda^{-1}$), implies the zero-expected-profit condition as follows:

$$V_t(\nu) = \frac{(1-s_{r,t})q_t(\nu)L_t}{\zeta\lambda}, \tag{20}$$

where we do not include the quality argument q in variables when it does not cause confusion.

Since A_t is the aggregate market value of all intermediate products, using (13) yields

$$A_t = \int_0^1 V_t(\nu)d\nu = \frac{(1-s_{r,t})Q_t L_t}{\zeta\lambda}, \tag{21}$$

implying that A_t is increasing in the aggregate quality of goods.

2.5 Government budget

Suppose that the policymaker can intervene the production of final goods, the purchase of intermediate goods, and the expenditures on R&D by choosing policy tools $s_{y,t}$, $s_{x,t}$, and $s_{r,t}$, respectively. These government interventions are financed by a distortionary tax levied on households' labor income. Then the government's budget constraint is given by

$$\tau_t W_t L_t = s_{y,t}Y_t + s_{x,t}X_t + s_{r,t}Z_t, \tag{22}$$

where $Z_t \equiv \int_0^1 z_t(\nu)d\nu$ is the total spending on R&D. In (22), the left-hand side is the tax revenues collected from households and the right-hand side is the expenditures for subsidization. Hence, in this model, the government can implement the subsidy (or tax) instruments to affect the input allocation and steer the market economy.

3 Decentralized equilibrium

An equilibrium consists of a sequence of allocation $[C_t, Y_t, L_t, X_t(\nu), z_t(\nu)]_{t=0,\nu\in[0,1]}^{\infty}$ and a sequence of prices $[r_t, W_t, P_t(\nu), q_t(\nu), V_t(\nu)]_{t=0,\nu\in[0,1]}^{\infty}$. In each instant of time,

[16] For the model to generate stationary balanced growth, we assume that the arrival rate of innovation in (19) is independent of R&D expenditures across intermediate industries as in Aghion and Howitt (1992).

[17] In the current literature, the fully endogenous solution proposed by Peretto (1998), Young (1998), and Howitt (1999) and the semi-endogenous solution proposed by Jones (1995), Kortum (1997), and Segerstrom (1998) are the two main approaches to remove scale effects. See Cozzi (2017a,b) for detailed discussions.

• households choose $[C_t, L_t]$ to maximize their utility taking $[r_t, W_t]$ as given;

• competitive final-good firms produce $[Y_t]$ and choose $[L_t, X_t(\nu)]$ to maximize profits taking $[W_t, P_t(\nu), q_t(\nu)]$ as given;

• monopolistic leaders for intermediate goods produce $[X_t(\nu)]$ and choose $[P_t(\nu)]$ to maximize profits;

• R&D firms choose $[z_t(\nu)]$ to maximize profits taking $[q_t(\nu), V_t(\nu)]$ as given;

• the goods market clears such that $Y_t = C_t + X_t + Z_t$;

• the financial market clears such that $A_t = \int_0^1 V_t(\nu)d\nu$.

3.1 Balanced growth path

In this subsection, we define the decentralized equilibrium and prove that the economy jumps immediately to a unique and stable balanced growth path (BGP). For an arbitrary path of subsidy rates $[s_{y,t}, s_{x,t}, s_{r,t}]_{t=0}^{\infty}$, we obtain the following result.

Proposition 1. *Holding constant s_y, s_x, and s_r, the economy jumps immediately to a unique and stable balanced growth path along which variables $\{W_t, Q_t, X_t, Z_t, Y_t, C_t\}$ grow at the same and constant rate g and labor supply $L_t = L$ is stationary.*

Proof. See Appendix A. □

From Proposition 1, given a stationary time path of the policy levers, we can derive the steady-state levels of variables in our interest along the BGP as follows. First, for a given level of quality $q(\nu)$ (which is constant over time until there is a new innovation in this line), the value of a firm in line ν (i.e., $V(\nu)$) does not change between time t and time $t + \Delta t$ (where Δt is an interval of time), namely $\dot{V}_t(\nu) = 0$. Thus, using (18) implies that $V(\nu)$ will be constant such that

$$V(\nu) = \frac{\pi(\nu)}{r + p(\nu)}, \tag{23}$$

where $\pi(\nu)$, r, and $p(\nu)$ are the steady-state levels of monopolistic profit, the interest rate, and the arrival rate of successful innovation in line ν, respectively. Combining (12), (20), and (23) yields

$$r + p(\nu) = \zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{\beta}}\left(\frac{1}{1-s_r}\right), \tag{24}$$

which implies that along the BGP, the arrival rate of the next successful innovation is independent of the line index ν, denoted by p. In addition, the aggregate expenditures on R&D can be expressed by

$$Z_t = \int_0^1 z(\nu)d\nu = \frac{p}{\zeta}Q_t L. \tag{25}$$

According to Proposition 1, substituting (24) into (4) yields the steady-state growth rate g of consumption such that

$$g = \frac{r-\rho}{\sigma} = \frac{1}{\sigma}\left[\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}\left(\frac{1}{1-s_r}\right) - p - \rho\right]. \tag{26}$$

Next, to pin down the growth rate of the aggregate quality index Q_t, we know that in an interval of time Δt, there are $p_t \Delta t$ sectors that experience one innovation, and this increases their productivity by λ. Hence, the dynamics of Q_t is given by

$$Q_{t+\Delta t} = p_t \Delta t \int_0^1 \lambda q_t(\nu) d\nu + (1 - p_t \Delta t) \int_0^1 q_t(\nu) d\nu = Q_t [1 + p_t \Delta t (\lambda - 1)] \tag{27}$$

Now subtracting Q_t from both sides in (27), dividing it by Δt, and taking the limit as $\Delta t \to 0$ yields the steady-state growth rate of aggregate quality (which is also g) such that

$$g = \frac{\dot{Q}_t}{Q_t} = p(\lambda - 1), \tag{28}$$

where $\dot{Q}_t = \lim_{\Delta t \to 0} (Q_{t+\Delta t} - Q_t)/\Delta t$. Then combining (26) and (28) yields the steady-state arrival rate of innovation such that

$$p = \frac{1}{1 + \sigma(\lambda - 1)} \left[\zeta \lambda (1 - \beta) \left(\frac{1 + s_y}{1 - s_x} \right)^{\frac{1}{1-\beta}} \left(\frac{1}{1 - s_r} \right) - \rho \right], \tag{29}$$

and the steady-state growth rate of aggregate quality is obtained by substituting (29) into (28) such that

$$g = \frac{\lambda - 1}{1 + \sigma(\lambda - 1)} \left[\zeta \lambda (1 - \beta) \left(\frac{1 + s_y}{1 - s_x} \right)^{\frac{1}{1-\beta}} \left(\frac{1}{1 - s_r} \right) - \rho \right]. \tag{30}$$

It can be seen that the steady-state growth rate of aggregate quality g in (30) is independent of labor supply L, implying that the scale effect is eliminated. Consequently, we have the following result.

Proposition 2. *The steady-state growth rate of aggregate quality g is increasing in the subsidy rate s_y to final-good production, the subsidy rate s_x to the purchase of intermediate goods, and the subsidy rate s_r to R&D.*

Proof. Equation (30) shows that g is increasing in s_y, s_x, and s_r. □

Intuitively, on the one hand, either a higher subsidy rate s_y to the production of final goods or a higher subsidy rate s_x to the purchase of intermediate goods can increase the demand for intermediate goods $X_t(\nu)$ in (9), which raises the profits of monopolistic firms in the intermediate-good sector brought by innovations. On the other hand, a higher subsidy rate s_r to R&D decreases the cost of research. The above policy changes increase the benefits of innovations and raise the incentives for R&D, so more resources are reallocated toward conducting research activities; namely R&D expenditures tend to rise. Hence, equation (19) implies that the economy exhibits a higher arrival rate of successful innovation, leading to a higher rate of economic growth g.[18] These comparative statics for the subsidy rates are consistent with those in Barro and Sala-I-Martin (2003) and Zeng and Zhang (2007).

[18] Recall that in Proposition 1, the steady-state growth rate of aggregate quality equals the counterparts of output and consumption.

Moreover, using (22), we can compute the steady-state rate of labor income tax such that

$$\tau = \frac{1}{1-\beta}\left(\frac{s_y}{1+s_y}\right) + \frac{\beta^2}{1-\beta}\left(\frac{s_x}{1-s_x}\right) + \frac{p}{\zeta}\left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+s_y}\right)^{\frac{1}{1-\beta}}(1-s_x)^{\frac{\beta}{1-\beta}}s_r, \tag{31}$$

which is a composite function of the subsidy rates s_y, s_x, and s_r, where the steady-state arrival rate of innovation p is given by (29). In addition, using (31), we can derive the steady-state level of labor supply such that

$$L(s_y,s_x,s_r) = \left\{\frac{1}{\frac{\sigma\theta(1+\eta)}{\eta(1-\tau)(1-\beta)(1+s_y)}\left[1-\beta^2\left(\frac{1+s_y}{1-s_x}\right) - \frac{\beta}{1+\sigma(\lambda-1)}\left(\frac{\lambda(1-\beta)}{(1-s_r)}\left(\frac{1+s_y}{1-s_x}\right) - \frac{\rho}{\zeta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{-\beta}{1-\beta}}\right)\right] - \theta(\sigma-1)}\right\}^{\frac{\eta}{1+\eta}}. \tag{32}$$

From the above analysis, it can be seen that in the case of a higher s_y, s_x, and s_r, τ would increase because heavier taxation is required to balance the government budget, and such a higher rate of labor income tax would decrease the supply of labor, which is captured by the negative relation between τ and L in (32). Nevertheless, a higher s_y, s_x, and s_r raises the growth rate g as shown in Proposition 2, and the enlarged production volume of output Y_t induces higher demand for labor L in equilibrium. Therefore, the overall effects of s_y, s_x, and s_r on L depend on the relative strength of these two opposing forces, and thus, seem analytically difficult to assess. We leave this discussion to the numerical analysis later on.

4 Optimal policy analysis

In this section, we study the socially optimal solution that maximizes the welfare of the model economy, followed by an analysis of the optimal policy regime showing how an appropriate joint choice on subsidy rates can be made so as to replicate the first-best allocation.

4.1 Socially optimal solution

As for the first-best outcome, the social planner chooses a time path of consumption C_t and labor supply L_t to maximize the households' lifetime utility given by (1), subject to the resource constraint $Y_t = C_t + X_t + Z_t$ and the technology constraint given by

$$\dot{Q}_t = \frac{\zeta(\lambda-1)Z_t}{L_t}, \tag{33}$$

which is obtained by combining (19) and (28). Moreover, in the intermediate-good sector, the socially optimal price $p_t(\nu)$ in line ν equals the marginal cost of production $\beta q_t(\nu)$. Therefore, the demand for intermediate goods in line ν is given by

$$X_t(\nu) = \beta^{\frac{1}{\beta-1}}L_t. \tag{34}$$

Aggregating these demand functions across all industry lines yields the total expenditures on the purchase of intermediate goods such that

$$X_t = \int_0^1 P_t(\nu)X_t(\nu)d\nu = \beta^{\frac{\beta}{\beta-1}}Q_tL_t. \tag{35}$$

Next, the total output in the social optimum can be found by substituting (34) into (5), which is given by

$$Y_t = \frac{(L_t)^{1-\beta}}{\beta}\int_0^1 q_t(\nu)X_t(\nu)^{\beta}d\nu = \beta^{\frac{1}{\beta-1}}Q_tL_t. \tag{36}$$

Therefore, using (34), (36), and the resource constraint in (33), we obtain the dynamics of aggregate quality such that

$$\dot{Q}_t = \frac{\zeta(\lambda-1)}{L_t}\left[\beta^{\frac{1}{\beta-1}}Q_tL_t - \beta^{\frac{\beta}{\beta-1}}Q_tL_t - C_t\right]. \tag{37}$$

Then the social planner's solution can be derived by setting up the following current-value Hamiltonian:

$$\hat{H}_t(C_t, L_t, Q_t, \hat{\mu}_t) = \frac{C_t^{1-\sigma}\left[1-\theta(1-\sigma)L_t^{\frac{1+\eta}{\eta}}\right]^{\sigma} - 1}{1-\sigma} + \hat{\mu}_t\left\{\frac{\zeta(\lambda-1)}{L_t}\left[\beta^{\frac{1}{\beta-1}}Q_tL_t - \beta^{\frac{\beta}{\beta-1}}Q_tL_t - C_t\right]\right\}, \tag{38}$$

where $\hat{\mu}_t$ is the costate variable associated with the constraint (37). Thus, the first-order conditions are respectively given by

$$\frac{\partial \hat{H}_t}{\partial C_t} = 0 \Rightarrow \left[1-\theta(1-\sigma)L^{\frac{1+\eta}{\eta}}\right]^{\sigma} C_t^{-\sigma} = \frac{\zeta(\lambda-1)}{L_t}\hat{\mu}_t; \tag{39}$$

$$\frac{\partial \hat{H}_t}{\partial L_t} = 0 \Rightarrow \zeta(\lambda-1)\hat{\mu}_t = \theta\sigma\left(\frac{1+\eta}{\eta}\right)\left[1-\theta(1-\sigma)L^{\frac{1+\eta}{\eta}}\right]^{\sigma-1} C_t^{-\sigma}L_t^{2+\frac{1}{\eta}}; \tag{40}$$

$$\frac{\partial \hat{H}_t}{\partial Q_t} = \hat{\mu}_t\zeta\beta^{\frac{1}{\beta-1}}(1-\beta)(\lambda-1) = \rho\hat{\mu}_t - \dot{\hat{\mu}}_t, \tag{41}$$

with the trasversality condition $\lim_{t\to 0} e^{-\rho t}\hat{\mu}_tQ_t = 0$. Multiplying (37) by $\hat{\mu}_t$ and multiplying (41) by Q_t, respectively, we can use (39) to obtain a differential equation for the control variable $\hat{\mu}_tQ_t$ such that $\dot{\hat{\mu}}_tQ_t + \hat{\mu}_t\dot{Q}_t = \rho\hat{\mu}_tQ_t - 1$, implying that $\hat{\mu}_tQ_t$ must jump immediately to its steady-state value given by $1/\rho$. This implies that the dynamical behavior of the model in the social optimum is also characterized by saddle-point stability.

Moreover, by inserting (39) into (40), we can see that the first-best level of labor supply L_t^* is stationary such that

$$L^* = \left[\frac{\eta}{\theta(\sigma+\eta)}\right]^{\frac{\eta}{1+\eta}}. \tag{42}$$

Accordingly, combining the saddle-point stability condition with (37), (39), (41) implies that in the first-best outcome, variables $-\hat{\mu}_t$, Q_t, and C_t all grow at the same rate given by

$$g^* = \frac{\zeta \beta^{\frac{1}{\beta-1}}(1-\beta)(\lambda-1)-\rho}{\sigma}, \tag{43}$$

which is again independent of the labor supply L^*.

Comparing the (steady-state) equilibrium rate of economic growth g in (30) and the first-best rate of economic growth g^* in (43) reveals that g can be higher or lower than g^*, depending on the values of parameters $\{\zeta, \lambda, \beta, \rho, \sigma\}$ in g^*. This implication, which is well known in the Schumpeterian growth model, is associated with various sources of R&D externalities. Specifically, a higher ζ or λ implies a worsening of the *surplus-appropriability problem* and a higher β implies a worsening of the *business-stealing effect*; both of these effects are a positive externality, making g^* exceed g. In contrast, a higher ρ or σ implies a strengthening of the *intertemporal-spillover effect*, which is a negative externality, making g^* lower than g.[19]

4.2 First-best policy instruments

We consider a combination of policy instruments (including the form of subsidies and/or taxes) that the policymaker can use to replicate the first-best optimal outcome. Given that both the decentralized equilibrium as shown in Subsection 3.1 and the optimal outcome as shown in Subsection 4.1 feature saddle-point stability, the optimal policy analysis in this subsection is based on steady-state comparisons and no transitional dynamics are considered. Specifically, comparing the decentralized equilibrium to the social optimum reveals that inefficiencies in the decentralized setting arise from three layers of distortions as follows.

4.2.1 Monopoly pricing

The first distortion is present in the ratio of intermediate-good expenditures and total outputs X_t/Y_t. This ratio equals the relative importance of intermediate goods in final-good production β in the social optimum where no policy interventions are involved, whereas it equals $\beta^2[(1+s_y)/(1-s_x)]$ in equilibrium where both subsidy rates for final-good production and the purchase of intermediate goods are involved. It can be seen that without these policy instruments (i.e., $s_y = s_x = 0$), the ratio X_t/Y_t in equilibrium is always lower than in the first-best outcome, producing an allocation inefficiency. As shown in Acemoglu (2009), this distortion stems from the monopoly rights protected by patents to preserve incentives for inventors to create higher quality products. Thus, if these policy tools are set to satisfy the following condition:

$$\frac{1+s_y}{1-s_x} = \frac{1}{\beta}, \tag{44}$$

[19]According to the conventional literature in quality-ladder growth models, overinvestment in R&D (and thereby a suboptimally high rate of equilibrium growth) is explained by the presence of the business-stealing effect that arises as the latest innovator destroys and/or appropriates previous incumbents' rents. Nevertheless, a recent study by Denicolò and Zanchettin (2014) argues that the possibility of excessive R&D in these models is attributed less to the business-stealing effect than other effects such as monopoly distortion effects and the congestion effect.

where the elasticity of product substitution $1/\beta$ measures the degree of monopoly pricing (i.e., the markup), then this layer of monopolistic distortion will be eliminated.

4.2.2 R&D externalities

The second distortion is present in the allocation of aggregate R&D expenditures Z_t (relative to total outputs), which determines the arrival rate of innovation p (i.e., equation (29)) and the growth rate of aggregate quality (i.e., equation (30)) in the steady state. Given that the setting $(1+s_y)/(1-s_x) = 1/\beta$ holds the optimal ratio of X_t and Y_t, the R&D subsidy rate s_r is the feasible policy lever that can adjust the equilibrium level of aggregate spending on R&D. Specifically, when the value of s_r induces $g(s_r)|_{(1+s_y)/(1-s_x)=1/\beta} > (<) g^*$, a too high (low) level of R&D expenditures is attained in the decentralized equilibrium, producing an allocation inefficiency. As shown in Subsection 4.1, this distortion stems from the presence of various types of R&D externalities in the canonical Schumpeterian growth model (i.e., the inclusion of the surplus-appropriability problem, the business-stealing effect, and the intertemporal-spillover effect), and the wedge between g and g^* is determined by parameter values that represent the overall impact of these R&D externalities. Therefore, if the choice of the R&D subsidy policy is designed to satisfy $g(s_r)|_{(1+s_y)/(1-s_x)=1/\beta} = g^*$, then this layer of distortion also will be eliminated. This yields the first-best design of the R&D subsidy rate given by

$$s_r^* = \frac{(1-\sigma)\zeta(\lambda-1)(1-\beta)\beta^{\frac{1}{\beta-1}} - \rho}{[1+\sigma(\lambda-1)]\zeta(\lambda-1)(1-\beta)\beta^{\frac{1}{\beta-1}} - \rho}. \tag{45}$$

Notice that the value of σ (the inverse of IES) plays an important role in determining the sign of s_r^* in (45). Specifically, s_r^* is negative (positive) for $\sigma > (<) 1-\rho/A$, where $A \equiv \zeta(\lambda-1)(1-\beta)\beta^{\frac{1}{\beta-1}}$, implying that the first-best R&D policy can either tax or subsidize the aggregate research spending.[20] For example, when $\sigma = 1 > 1-\rho/A$ (which is the benchmark case in our numerical analysis), we have $s_r^*|_{\sigma=1} = 1/(1-\lambda A/\rho)$, which is negative given that the optimal growth rate $g^*|_{\sigma=1} = A-\rho$ is positive. Intuitively, when $s_r|_{\sigma=1} = 0$, given that the optimal ratio of X_t and Y_t holds (i.e., $(1+s_y)/(1-s_x) = 1/\beta$), the steady-state growth rate $g|_{\sigma=1}$ becomes always higher than the socially optimal counterpart $g^*|_{\sigma=1}$, meaning that the negative R&D externalities dominates the positive R&D externalities in the model. To remove this inefficiency, imposing an R&D tax (i.e., $s_r^*|_{\sigma=1} < 0$) becomes necessary for increasing the cost of research, thus reducing the research incentives and the resulting R&D level to equate the steady-state growth rate and the first-best growth rate.[21]

[20]It is straightforward to show that the denominator of s_r^* in (45) (i.e., $[1+\sigma(\lambda-1)]A-\rho$) can only be positive given that $\lambda > 1$ and $A > \rho$ due to the positive rate of optimal growth g^*. Therefore, the sign of s_r^* is determined by the sign of its numerator (i.e., $(1-\sigma)A-\rho$), that is, $s_r^* < (>) 0$ when $\sigma > (<) 1-\rho/A$.

[21]As for the implications for optimal R&D policy in scale-invariant growth frameworks, in the model of Segerstrom (1998) with diminishing technological opportunities (DTO), either R&D taxes or subsidies are optimal for small-sized innovations, where R&D taxes are optimal for sufficiently large-sized innovations. However, in the model of Dinopoulos and Syropoulos (2007) with rent protection activities (RPA), R&D taxes are optimal for small- and large-sized innovations, and R&D subsidies are optimal only for medium-sized innovations. This result still applies to the model with both DTO and RPA, as shown in Şener (2008).

4.2.3 Consumption-labor tradeoff

The third distortion is present in the supply of labor L_t, which also determines the level of consumption (relative to total outputs, i.e., C/Y) in the steady state via the consumption-labor decision. To see this relation, combining (39) and (40) shows that the first-best level of labor supply L^* depends only on the structural parameters θ, σ and η such that

$$1 = \sigma\theta\left(1+\frac{1}{\eta}\right)\left[\frac{(L^*)^{\frac{1+\eta}{\eta}}}{1-\theta(1-\sigma)(L^*)^{\frac{1+\eta}{\eta}}}\right]. \tag{46}$$

In addition, using (3), (14) and (16) shows that the steady-state level of labor supply L depends on the subsidy rates s_y, s_x, s_r and the labor income tax rate τ, in addition to other parameters:

$$\frac{(1+s_y)(1-\beta)(1-\tau)}{C_t/Y_t} = \sigma\theta\left(1+\frac{1}{\eta}\right)\left[\frac{L^{\frac{1+\eta}{\eta}}}{1-\theta(1-\sigma)L^{\frac{1+\eta}{\eta}}}\right], \tag{47}$$

where L is given by (32), τ is given by (31), and using (14), (15), (25) and the resource constraint expresses the ratio of consumption and final good such that

$$\frac{C_t}{Y_t} = 1-\beta^2\left(\frac{1+s_y}{1-s_x}\right) - \frac{p\beta}{\zeta}\left(\frac{1+s_y}{1-s_x}\right)^{-\frac{\beta}{1-\beta}}. \tag{48}$$

Therefore, equations (46) and (47) imply that the presence of elastic labor introduces one additional layer of distortion, which comes from the use of labor income taxes and is determined by the consumption-labor decision. Given that setting $(1+s_y)/(1-s_x) = 1/\beta$ holds the optimal ratio of X_t and Y_t and that $s_r = s_r^*$ holds the optimal spending Z_t on R&D, there is one degree of freedom in the set of policy tools $\{s_y, s_x, s_r\}$ that can adjust the equilibrium level of labor supply L. Specifically, when the choice of $\{s_y, s_x, s_r\}$ induces $L(s_y, s_x, s_r) > (<) L^*$, too much (little) labor is supplied in the decentralized equilibrium as compared to the social optimum, which also produces an allocation inefficiency.[22] Thus, if the policy mix $\{s_y, s_x, s_r\}$ is chosen to satisfy the condition such that

$$L(s_y, s_x, s_r) = L^* = \left[\frac{\eta}{\theta(\sigma+\eta)}\right]^{\frac{\eta}{1+\eta}}, \tag{49}$$

then the last layer of distortion will be removed and the socially optimal outcome can be attained accordingly.

Further exploration of equation (49) yields the analytical solution to s_y^*, which is given by

$$s_y^* = \frac{\beta^{\frac{\beta}{1-\beta}}(s_r^*-1)p^*}{(\beta-1)\zeta} - 1, \tag{50}$$

[22] For example, when there is no fiscal policy (i.e., $s_y = s_x = s_r = 0$) in the market economy with $\sigma = 1$, the equilibrium labor supply is given by $L^{(1+\eta)/\eta} = \{\eta/[\theta(1+\eta)]\}/\{1+\beta\rho/[\zeta\lambda(1-\beta)]\}$. This produces an inefficient supply of labor, since the equilibrium labor is smaller than the optimal labor given by $(L^*)^{(1+\eta)/\eta} = \eta/[\theta(1+\eta)]$, giving rise to using subsidy tools to eliminate this distortion.

where p^* is obtained by setting $(1+s_y)/(1-s_x) = 1/\beta$ and $s_r = s_r^*$ in (29). For the signs of $\{s_y^*, s_x^*\}$, equation (50) implies that s_y^* will be positive if ζ is sufficiently small. Moreover, s_x^* can be positive at the same time if $s_y^* < (1-\beta)/\beta$. Our numerical exercise in Section 5 will show that the mix of a negative rate of s_r^* and positive rates of $\{s_y^*, s_x^*\}$ applies to a wide range of parametrization.

Observing that both s_r^* and p^* are unaffected by η and θ, we find that the optimal design of subsidy rates, namely the combination of s_y^*, s_x^* and s_r^*, is independent of the Frisch elasticity of labor supply and the leisure preference.[23] To see the intuition, let

$$\tilde{L}^*(\eta,\theta) = \left[\frac{(L^*)^{\frac{1+\eta}{\eta}}}{1-\theta(1-\sigma)(L^*)^{\frac{1+\eta}{\eta}}}\right],$$

$$\tilde{L}(\eta,\theta) = \left[\frac{L^{\frac{1+\eta}{\eta}}}{1-\theta(1-\sigma)L^{\frac{1+\eta}{\eta}}}\right].$$

Combining (46), (47) and (48) yields

$$\frac{\tilde{L}^*(\eta,\theta)}{\tilde{L}(\eta,\theta)} = \frac{1-\beta^2[(1+s_y)/(1-s_x)] - p(\beta/\zeta)[(1+s_y)/(1-s_x)]^{-\frac{\beta}{1-\beta}}}{(1+s_y)(1-\beta)(1-\tau)}. \tag{51}$$

It is straightforward to see that the right-hand side of (51) is independent of η and θ. Given other model parameters, varying these two parameters would affect the level of $\tilde{L}^*(\eta,\theta)$ and $\tilde{L}(\eta,\theta)$ by the same proportion, but not their *ratio*. In addition, eliminating the distortion in the consumption-leisure tradeoff simply requires choosing the policy instruments to ensure that the right-hand side of (51) is equal to unity.

Nevertheless, it is worth emphasizing that the presence of parameters η and θ regulates that labor needs to be supplied elastically, resulting in an additional source of distortions caused by the consumption-labor tradeoff. Hence, replication of the socially optimal allocation would require the joint use of all three subsidy instruments. Comparing to the case with inelastic labor, in Section 6, we show that the presence of elastic labor (in terms of the labor supply elasticity η) in the current model is crucial for determining the unique optimal design of three subsidy instruments, which helps to reduce welfare losses. Additionally, the magnitude of parameters η and θ plays an important role in determining the steady-state allocation of labor, consumption and R&D investment, and the steady-state level of social welfare. These results will be quantified in the numerical analysis.

Summarizing the above results yields the following proposition.

Proposition 3. *The economy can achieve the first-best outcome in equilibrium with an optimal mix of policy instruments $\{s_y, s_x, s_r\}$ determined by (44), (45), and (49).*

Proof. Proven in the text. □

[23] To be precise, the socially optimal combination of policy instruments is independent of the Frisch elasticity of labor supply and the leisure preference when these two parameters take strictly positive values. When $\eta = 0$, however, labor is supplied inelastically. As shown in Section 6, the design of optimal policy instruments under inelastic labor supply would be different.

This result is in a sharp contrast to that in Zeng and Zhang (2007). In their variety-expansion model with distortionary taxes and elastic labor supply, subsidies to the production of outputs and the purchase of intermediate goods are equivalent in terms of their growth and welfare effects. Therefore, the two subsidy rates are integrated to become an effective subsidy rate to production (i.e., s_f in their context), which reduces one degree of freedom in policy implementation. In their model, only two subsidy tools (i.e., the production subsidy and R&D subsidy) can be used to optimize the equilibrium allocation. In the presence of three distortions as described above, the optimal combination of policy tools $\{s_f, s_x\}$ only generates the second-best solution. Even when an additional policy lever, namely consumption tax τ_c, is included for use in their model, in addition to the optimized rates of manufacturing subsidy s_f and R&D subsidy s_r, the social optimum still would not be achievable with the income tax rate τ being smaller than 1.[24] Nevertheless, in our quality-ladder model, the subsidy rate to output production s_y and that to the user cost of intermediate goods s_x operate separately. Hence, optimizing a combination of subsidy tools $\{s_y, s_x, s_r\}$ suffices to eliminate all distortions in our model.[25] With the current framework, this result holds for a wide range of parametrization in which the income tax rate τ does not exceed unity, as shown in the next section.

5 Quantitative analysis

In this section, we calibrate the model to the US data to perform a quantitative analysis. First, we numerically evaluate the effects of subsidy instruments in terms of growth maximization. Second, we quantify the effects of subsidy instruments in terms of welfare maximization.[26] In each exercise, we consider (a) the case where a mix of all instruments is implemented, (b) the case where a single instrument is implemented, and (c) the case where a combination of two instruments is implemented, respectively.[27]

5.1 Calibration

To perform this numerical analysis, the strategy is to assign steady-state values to the following structural parameters $\{\sigma, \eta, \rho, \beta, \zeta, \lambda, \theta, s_y, s_x, s_r\}$. In the benchmark analysis, we follow the

[24] This setting in Zeng and Zhang (2007) implies that one policy instrument in their model can be feasibly set with negative values, as in our current model.

[25] In the blocking-patent model of Yang (2018) with subsidization, given elastic labor supply and a lump-sum tax, distortions from monopoly pricing and the consumption-labor tradeoff are consolidated to one layer of distortion from the (inverse) supply of labor in manufacturing terms. In addition to the distortion from R&D externalities, by fixing the patent-policy regime, an optimal mix of two subsidy rates (to production and R&D) will suffice to recover the first-best outcome.

[26] Note that the main focus of these quantitative exercises is not on the sizes of the growth-maximizing and welfare-maximizing subsidy/tax rates, but rather on the comparisons in the policy effectiveness for growth and welfare among the decentralized equilibrium and the outcomes with the optimal policy instrument(s).

[27] There are two main purposes of considering the use of a single instrument and a mix of two instruments in this section. First, this analysis aims to highlight how quantitatively efficient each subsidy instrument is in terms of promoting growth and raising welfare. Second, the results obtained in this analysis serve as counterparts to those in Zeng and Zhang (2007) who perform a similar numerical analysis.

literature (e.g., Guvenen 2006) to set the IES in consumption σ to unity.[28] As for the magnitude of the Frisch elasticity, this study follows Trabandt and Uhlig (2011) and Annicchiarico *et al.* (2022) to choose $\eta = 1$, which is consistent with the survey evidence in Kimball and Shapiro (2008).[29] We choose a standard value of 0.05 for the discount rate ρ. As for the production parameter, we calibrate the value of β by setting the markup ratio $P_t(\nu)/[\psi q_t(\nu)] = 1/\beta$ to 1.5, which is in line with the recently estimated markup values of the US economy considered in Barkai (2020) and Loecker *et al.* (2020). As for the R&D productivity, we calibrate the value of ζ by following Zeng and Zhang (2007) to choose the average growth rate of GDP (i.e., g in (30)) in the US for the last 30 years, which has been roughly 3%, according to the Conference Board Total Economy Database. As for the step size of quality improvement, we calibrate the value of λ by setting the time between arrivals of innovation $1/p$ to about 3 years, as in Acemoglu and Akcigit (2012). As for the leisure preference parameter, we calibrate the value of θ by matching the standard moment of labor supply L to $1/3$. Finally, given that the US has used an R&D subsidy but not the manufacturing subsidy (including subsides to the production of final goods and to the purchase of intermediate goods) in the past three decades, we choose the market values of $s_y = s_x = 0$ and follow Grossmann *et al.* (2013) to calibrate the value of s_r by targeting the current US R&D subsidy rate, which is approximately 6.6% (OECD 2009, 2013). Table 1 summarizes the values of parameters and variables in this quantitative exercise.

Table 1: Calibration

Targeted Moments								g 0.030	p 0.333	L 0.333
Parameters	σ 1.0	η 1.0	ρ 0.050	β 0.667	ζ 1.063	λ 1.090	θ 4.151	s_y 0.000	s_x 0.000	s_r 0.066

5.2 Numerical results

Before proceeding to the cases in which policy instruments are employed, this analysis starts from comparisons in the growth rate and welfare level between the decentralized equilibrium in which realistic values are calibrated (i.e., the benchmark case) and an extreme scenario in which no policy tools are introduced (i.e., the no-policy case). The purpose of this exercise is to quantify the differences in growth and welfare of the equilibrium level in our model as compared to in the original quality-ladder model. The growth rates and welfare levels of these two cases are shown in Table 2. It can be seen that as compared to our benchmark case, the growth rate declines by 0.230% (percentage point) and the welfare level declines by 0.08% (percent change)

[28] The benchmark value of σ implies that the absolute value of optimal R&D subsidy s_r^* does not exceed 100%. Indeed, empirical evidence on the magnitude of IES in the existing literature is mixed (e.g., in a detailed survey by Thimme 2017). Therefore, as will be shown, we consider other permissible values for σ as robustness checks.

[29] This value of the Frisch elasticity η is also close to the mean value of estimates in Smets and Wouters (2007) and in Hall (2009) (i.e., 0.52 and 1.9, respectively).

when all policy interventions are dismantled.[30] [31] Notice that in this comparison, the equilibrium subsidy rates to final-good production and the purchase of intermediate goods are identical in the benchmark case and in the no-policy case (i.e., $s_y = s_x = 0$). Therefore, the growth and welfare differences between the two cases are only driven by the presence of subsidies to R&D, which effectively stimulates the arrival rate of innovation in equilibrium, as shown in (29). This result tends to justify the use of R&D subsidies in promoting growth and raising welfare with the current US policy in the absence of the use of any manufacturing subsidies.

Moreover, from the no-policy case to the benchmark case, the labor income tax rate τ rises from 0 to 4.14% to finance the use of R&D subsidies s_r, and the labor supply L rises slightly from 0.3330 to 0.3333 in response. Corresponding to the discussion in Subsection 3.1, this result implies that the effect of s_r through the growth channel that stimulates the demand for production labor dominates the counterpart through the taxation channel that stifles the supply of labor.

Table 2: Growth and welfare under the benchmark case and the no-policy case

Benchmark	g	U	L	C_0	τ
($s_y = s_x = 0$, $s_r = 0.066$)	0.0300	-32.2895	0.3333	0.1732	0.0414
No-policy	g	U	L	C_0	τ
($s_y = s_x = s_r = 0$)	0.0277	-32.3054	0.3330	0.1809	0.0000

5.2.1 Growth-maximizing subsidization

According to Proposition 2, a higher rate of subsidies to the production of final goods (i.e., s_y), the purchase of intermediate goods (i.e., s_x), and R&D (i.e., s_r) leads to a quantitatively identical effect on the steady-state rate of economic growth (namely, a higher g). Therefore, this subsection quantifies and compares the size of the effect of each subsidy instrument in terms of growth maximization.

First, we consider the case in which the three policy instruments are used. As shown in Table 3, the maximized rate of economic growth g, conditional on the baseline calibration, is 10.74% and the growth-maximizing rates of subsidy are given by $s_y = 0.445$, $s_x = 0.045$, and $s_r = 0.010$, respectively. It is found that the significant increase by 7.74% (percentage point) in g is mainly driven by the use of subsidies in manufacturing s_y and s_x. It can be seen that among these growth-maximizing rates of subsidy, s_y is the largest whereas s_r is the smallest in terms of magnitude, implying that s_r tends to be less effective in enhancing economic growth than s_y and s_x. In other words, in this model, the benefit of innovations is much more sensitive to the increase in monopolistic profits (due to more production sales in final goods) rather than the reduction in research costs. As compared to the benchmark case, the large growth effect significantly stimulates the demand for labor L in manufacturing, and the labor income tax rate τ also rises dramatically to finance the higher level of subsidy expenditure. In addition, Table

[30]See Appendix A.2 for the derivation of the steady-state welfare function. The welfare difference is expressed as the usual equivalent variation in consumption flow such that $\exp(\rho\Delta U) - 1$, where ΔU denotes the difference in the steady-state welfare.

[31]In our welfare analysis, the initial aggregate quality index Q_0 in (A.20) is set to unity. Accordingly, the term C_0 is interpreted as the consumption level in the initial period ($t = 0$) normalized by the aggregate quality index.

3 reveals that using all subsidy tools generates excess growth compared to the growth rate of 5.76% in the social planner's solution (which will be estimated in Subsection 5.2.2). It is also worth noting that the model implication on the growth-maximizing policy instruments is largely robust to setting σ to 2, an equally commonly considered value in the literature. With a lower IES, however, the maximized growth rate is potentially higher (11.49%), whereas the equilibrium labor becomes lower (0.7896).[32]

Table 3: Growth maximization under a combination of all instruments

	g	L	τ
Benchmark calibration ($s_y = 0.445$, $s_x = 0.045$, $s_r = 0.010$)	0.1074	0.9170	0.9935
$\sigma = 2$ ($s_y = 0.365$, $s_x = 0.110$, $s_r = 0.040$)	0.1149	0.7896	0.9949

Second, we consider the case in which only a single subsidy instrument is used. The result is shown in Table 4, which presents the situations of growth maximization for each subsidy tool.[33] Specifically, under the growth-maximizing rate of s_y, the growth rate is 10.32%, whereas under the growth-maximizing rate of s_x, the growth rate is 10.28%. Nevertheless, under the growth-maximizing rate of s_r, the growth rate is 7.8%. This result implies that when only a single subsidy instrument is implemented to stimulate growth, subsidizing R&D (the purchase of final goods) is the least (most) effective; this policy implication differs from the comparison in growth effectiveness of subsidy instruments in Zeng and Zhang (2007), in which R&D subsidy is more growth-enhancing than the other subsidies. Additionally, the above analysis justifies the fact in Table 3 that the growth-maximizing rate of s_y (s_r) is the largest (smallest) among the three subsidy tools if the choice of all tools becomes available.

Table 4: Growth maximization under a single instrument

	g	L	τ
Subsidies to production of final goods ($s_y = 0.499$, $s_x = 0$, $s_r = 0$)	0.1032	0.0772	0.9987
Subsidies to purchase of intermediate goods ($s_y = 0$, $s_x = 0.332$, $s_r = 0$)	0.1028	0.9554	0.6627
R&D subsidies ($s_y = 0$, $s_x = 0$, $s_r = 0.612$)	0.0780	0.0670	0.9987

Finally, it is interesting to see how the growth effect changes when a mix of any two subsidy instruments is used. Table 5 displays the growth-maximization solutions for three different combinations of subsidy rates accordingly. It can be seen that the three strategies of policy com-

[32] We do not conduct a sensitivity analysis by varying the Frisch elasticity and leisure preference, because equation (30) shows that the BGP growth rate is independent of η and θ, and the change in the equilibrium labor, while keeping growth-maximizing subsidies the same, is only of the second-order importance.

[33] In this numerical analysis, to ensure that the consumption level is positive and that the labor supply is bounded between 0 and 1, we restrict the range of s_y to $[-0.99, 0.499]$, of s_x to $[-0.99, 0.332]$, and of s_r to $[-0.99, 0.612]$, respectively.

binations produce similar rates of growth; they are all higher than the growth rates by using a single subsidy tool but lower than the growth rate by using the three tools together. Notice that the policy combinations with subsidization to R&D generate lower growth rates than the one without it, which confirms the previous finding that s_r is the least effective to enhance growth. In particular, subsidizing the mix of intermediate goods production and R&D yields the lowest growth-maximizing rate, whereas subsidizing the manufacturing factors yields the highest growth-maximizing rate.

Table 5: Growth maximization under a combination of two instruments

Subsidies to manufacturing	g	L	τ
($s_y = 0.410$, $s_x = 0.070$, $s_r = 0$)	0.1070	0.9349	0.9727
Subsidies to production of final goods and R&D	g	L	τ
($s_y = 0.475$, $s_x = 0$, $s_r = 0.050$)	0.1035	0.0270	0.9998
Subsidies to purchase of intermediate goods and R&D	g	L	τ
($s_y = 0$, $s_x = 0.300$, $s_r = 0.135$)	0.1033	0.9706	0.7144

5.2.2 Welfare-maximizing subsidization

Optimizing a mix of all instruments. The analysis now quantifies the effects of welfare-maximizing subsidies when the three policy instruments $\{s_y, s_x, s_r\}$ are implemented jointly, as displayed in Table 6. Using our benchmark calibration, the optimal mix of all subsidy instruments is given by $s_y^* = 0.400$, $s_x^* = 0.067$, and $s_r^* = -0.743$. Recalling Proposition 3, the use of this optimal mix of subsidy rates induces the decentralized equilibrium to achieve the first-best solution. Intuitively, the first-best outcome in this model is restored by adjusting three policy levers to remedy three distortions occurring in the decentralized equilibrium. First, given $s_y = s_x = 0$ in equilibrium, the fraction $(1+s_y)/(1-s_x) = 1$ implies that the subsidy rate to the production of final goods is less compatible with the subsidy rate to the purchase of intermediate goods in the sense that $(1+s_y)/(1-s_x)$ is smaller than its optimal value $1/\beta$. This is the first inefficiency stemming from the distorted ratio of intermediate-good expenditure and total outputs X_t/Y_t in equilibrium, in which subsidies to manufacturing are absent. Second, there is a layer of inefficiency stemming from the allocation on the aggregate R&D spending, because with the suboptimal subsidy rate to R&D (i.e., $s_r = 0.066$), the equilibrium growth rate of $g = 0.030$ differs from the socially optimal growth rate of $g^* = 0.0576$. Third, there is another layer of inefficiency stemming from the supply of labor, since in the presence of distortionary labor income tax τ, the suboptimal subsidy rates $\{s_y, s_x, s_r\}$ yield a level of labor supply at $L = 0.333$ in equilibrium, which is smaller than the socially optimal level at $L^* = 0.3471$. Thus, when the subsidy rates to manufacturing and the subsidy rate to R&D are adjusted simultaneously to their first-best levels $\{s_y^*, s_x^*, s_r^*\}$, the above layers of distortions are eliminated by reallocating the resources in the use of final goods and in the supply of labor. As a result of correcting inefficiencies, the increment in welfare from the decentralized equilibrium to the social optimum is considerable (approximately by 35%). The result on an increase in subsidies to manufacturing and labor supply is consistent with the counterpart in Wan and Zhang (2021).

Table 6: Welfare maximization under a combination of all instruments

All subsidies	g^*	U^*	L^*	C_0^*	τ^*
($s_y^* = 0.400$, $s_x^* = 0.067$, $s_r^* = -0.743$)	0.0576	-21.1008	0.3471	0.1815	0.6679

Two interesting points around the above results are worthwhile discussing. First, the first-best rate of R&D subsidy s_r^* becomes negative (i.e., an R&D tax) under the benchmark calibration. This is because when the subsidy rates to manufacturing are fixed at their first-best levels (i.e., $s_y^* = 0.400$ and $s_x^* = 0.067$), the equilibrium outcome with no R&D subsidies (i.e., $s_r = 0$) will generate a higher growth rate to the economy as compared to the social optimum (i.e., $g = 0.1036 > g^* = 0.0575$). Hence, to depress the equilibrium R&D, the level of s_r has to be lowered to be smaller than 0. The mechanism for the first-best rate of R&D subsidy to be negative in this model is similar to the mechanism for the first-best rule of profit division to be positive in Yang (2018), who considers the effects of blocking patents in a quality-ladder model; a negative R&D subsidy rate and a positive profit-division rule play the same role in mitigating the R&D level and the growth rate. In other words, achieving the social optimum in quality-ladder models would require a policy instrument that can be growth-depressing, since the class of quality-ladder models features the possibility of overinvestment in R&D that could lead to suboptimally excess growth. This is in a sharp contrast to the policy analysis of subsidization in variety-expansion models.[34]

Additionally, the majority of welfare improvements moving from the decentralized equilibrium to the social optimum stems from remedying the distortions in production. This can be seen in the following policy experiment that decomposes the distortions present in the current framework. Consider an intermediate case where the subsidy rates are given by $s_y = 0$, $s_x = 0.333$, and $s_r = -0.743$, and the resulting level of welfare is $U = -21.3739$. In this case, the markup ratio and the equilibrium labor supply L attain their first-best levels of $1/\beta$ and L^*, respectively. Therefore, given that the ratio $(1 + s_y)/(1 - s_x)$ in the intermediate case is identical to its counterpart in the first-best case (which is 1/1.5), the welfare difference between the first-best case and the intermediate case stems from the distortion in R&D externalities, denoted by $\xi_1 = 1.37\%$. In addition, it is straightforward to see that the welfare difference between the intermediate case and the benchmark case stems from the distortions in monopoly pricing and labor supply, denoted by $\xi_2 = 74.97\%$. Accordingly, it is obvious that the magnitude of ξ_2 is much more significant than that of ξ_1.[35]

Sensitivity. To examine the sensitivity of the above numerical analysis, we consider several exercises with respect to the structural parameters $\{\beta, \rho, \lambda, \zeta, \theta, \sigma, \eta\}$. First, we vary the value of β

34For example, in Barro and Sala-I-Martin (2003) with inelastic labor and lump-sum taxes, using a subsidy to the production of final goods or to the purchase of intermediate goods alone, either of which is growth-enhancing, can induce the decentralized equilibrium to achieve the social optimum, given that the equilibrium growth rate in their setting is lower than the socially optimal growth rate. In Zeng and Zhang (2007) who consider the model of Barro and Sala-I-Martin (2003) with elastic labor and distortionary taxes, optimizing a combination of analogous subsidy tools in addition to a consumption tax, with the labor income tax being smaller than 1, cannot eliminate all distortions to achieve the first-best outcome.

35This finding is consistent with Nuño (2011) and Yang (2018), both of whom argue that most of the welfare losses in the decentralized equilibrium of R&D-based growth models are attributed to the presence of suboptimal choices of policy tools that affect the resource allocation in the monopolistic intermediate-good sector.

to 0.6849, so that the implied markup ratio $1/\beta = 1.46$ is consistent with the average level of the empirical estimates in Loecker *et al.* (2020) for US firms during 1996–2005. Second, the values of $\{\rho, \lambda, \zeta, \sigma\}$ are changed, so that the resulting optimal growth rate g^* is maintained at the rate of 0.0501, which is the value generated by setting β alone to 0.6849 in the first sensitivity exercise.[36] Additionally, we recalibrate the model by using the leisure preference parameter $\theta = 3$ and the Frisch elasticity $\eta = 3$, respectively. Table 7 presents the welfare level U^*, the growth rate g^*, the labor supply L^*, the consumption level C_0^*, and the tax rate τ^*, respectively, under the alternative sets of structural parameters.

It can be seen that, the qualitative pattern and the quantitative magnitude of the main results are quite robust. First, the optimal subsidy rate to output production s_y^* continues to be the largest among the policy instruments, whereas the optimal subsidy rate to R&D s_r^* is still negative. This result continues to imply that s_y^* and s_x^* are the two subsidy rates that eliminate most of the inefficiencies. Second, under the new parameter settings of ζ, λ and β, the welfare level U^* declines, as compared to the counterpart in the benchmark first-best case as shown in Table 6. Although a higher σ tends to raise welfare by increasing C^* and decreasing L^*, it amplifies the negative welfare effect brought by a lower g^*, so the level of U^* still declines. Nevertheless, under a higher value of ρ, the considerable rise in C_0^* generates a sufficiently positive welfare effect, which dominates the negative welfare effect from the decrease in g^*. Hence, the resulting level of welfare U^* increases in this case. As for the parameter θ, a lower value implies less disutility from each unit of labor supply, leading to higher L^* and C_0^*, which raises welfare. In the meantime, the optimal combination of subsidies, along with the growth rate of first-best allocation, is independent of θ. Hence, a lower θ unambiguously improves the social welfare. Similarly, the optimal subsidy rates and the growth rate are unaffected by varying η. However, a higher Frisch elasticity, keeping θ at the same level as in the benchmark case, drastically reduces L^* and C_0^*, resulting in a significant decline in U^*.

It is worth pointing out that the above welfare analysis aims to numerically explore the channels through which the three subsidies, when combined as a unified policy tool, are translated into welfare gains in a canonical quality-ladder framework. The magnitude of the optimal subsidy rates reported in the baseline case, however, may not yield direct policy implications on the desirable level of the real-world subsidy and tax rates, since they indeed hinge critically upon the calibrated model parameters. In particular, the optimal tax rate on R&D activities under the baseline analysis seems unusually high, which might lead to a misinterpretation that our study suggests a dramatic tax reform that is seemingly impossible to implement. In fact, we perform an additional analysis and find that the optimal subsidy rates are largely affected by the parameters λ and β. As shown in Table 8, once the value of λ is raised to 1.2 (implying that the arrival rate of successful innovation equals 0.15), this alternative calibration suggests that the optimal R&D tax declines to 52.6%. Further increasing the value of λ to 1.4 sharply reduces the magnitude of the optimal R&D tax rate to 34.4%. However, it is worth emphasizing that the key model implications remain consistent with the baseline analysis. Regarding the parameter β, this study considers the upper and lower bound of the markup value (i.e. 1.2 and 1.6, respectively) in Barkai (2020).

[36]Notice that from (43), the parameter values of $\{\beta, \rho, \lambda, \zeta, \sigma\}$ determine the socially optimal growth g^*. Given that g^* is nonlinear in β, it is technically convenient to firstly set the alternative value of β, yielding a new value of g^*. Then the alternative values of other values are set to match this implied first-best growth rate in order to compare the welfare effects of these parameters in removing distortions.

Table 7: Sensitivity checks: varying the values of structural parameters

Parameters affecting growth rate					
$\beta = 1/1.46$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.349$, $s_x^* = 0.076$, $s_r^* = -0.845$)	-24.0751	0.0501	0.3471	0.1815	0.6299
$\zeta = 0.9891$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.386$, $s_x^* = 0.076$, $s_r^* = -0.845$)	-22.6426	0.0501	0.3471	0.1949	0.6397
$\lambda = 1.0838$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.393$, $s_x^* = 0.072$, $s_r^* = -0.854$)	-22.6426	0.0501	0.3471	0.1949	0.6415
$\rho = 0.0574$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.370$, $s_x^* = 0.086$, $s_r^* = -0.960$)	-20.8049	0.0501	0.3471	0.2085	0.6104
$\sigma = 1.1483$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.370$, $s_x^* = 0.086$, $s_r^* = -0.960$)	-25.9256	0.0501	0.3349	0.2011	0.6104
Parameters NOT affecting growth rate					
$\theta = 3$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.400$, $s_x^* = 0.067$, $s_r^* = -0.743$)	-17.8539	0.0576	0.4082	0.2135	0.6679
$\eta = 3$	U^*	g^*	L^*	C_0^*	τ^*
($s_y^* = 0.400$, $s_x^* = 0.067$, $s_r^* = -0.743$)	-30.6010	0.0576	0.2771	0.1449	0.6679

In addition to the observation that a higher markup ratio reduces the optimal R&D tax rate, it is also found that promoting social welfare relies more heavily on subsidizing final (intermediate) goods when the markup ratio $1/\beta$ is relatively large (small). In particular, when $\lambda = 1.4$ and $\beta = 1/1.6$, the model-implied s_r^* is merely -32.7%, and the optimal subsidy rate to final goods s_y^* is almost three times as large as s_x^*.[37]

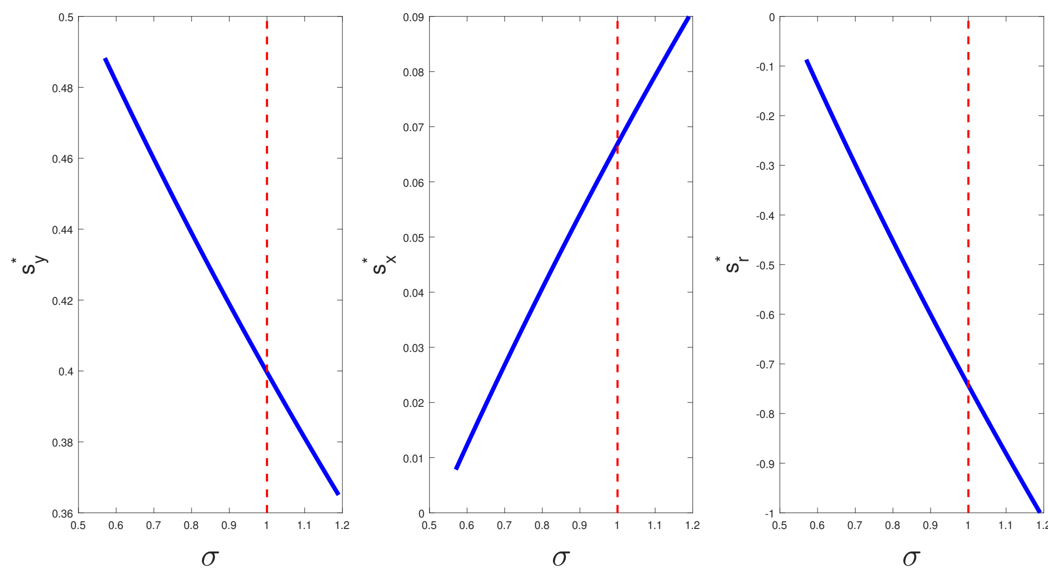
Finally, we quantify the effects of two important parameters σ and η in the utility function (1). In Figure 1, we plot the relation between the welfare-maximizing subsidy rates and σ, the inverse of the IES parameter. It is shown that a higher value of σ implies that it is socially optimal to tax R&D activities more heavily. This is consistent with the implication in Section 4.1: the R&D externality caused by a higher σ makes the optimal growth rate g^* lower than the equilibrium growth rate g, so a higher optimal R&D subsidy rate s_r^* is required for removing this inefficiency. In particular, when $\sigma = 0.57$, the model-implied optimal R&D tax rate is merely 10%. It is also found that a larger value of σ is associated with a higher (lower) optimal subsidy rate to the purchase of intermediate goods s_x^* (final-good production s_y^*). In addition, since our model suggests that both the optimal subsidy rates and the economic growth rate are unaffected by the Frisch elasticity, Figure 2 plots welfare, labor and consumption in the first-best allocation against the structural parameter η. It is seen that both L^* and C_0^* are increasing (at a diminishing rate) in η. However, the disutility from working seems to outweigh the gain in utility from additional consumption, which leads to a negative relation between U^* and η.[38]

[37] All quantitative exercises in Table 8 recalibrate θ when alternative values of λ and β are considered, which causes the equilibrium labor L^* to vary.

[38] The quantitative practice reported in Figures 1 and 2 recalibrate θ and ζ for each alternative value of σ and η

Table 8: Sensitivity checks: effects of λ and β on optimal subsidies.

	U^*	g^*	L^*	C_0^*	τ^*
$\lambda = 1.2$ $(s_y^* = 0.342,\ s_x^* = 0.105,\ s_r^* = -0.526)$	-17.5286	0.0708	0.3575	0.1664	0.6917
$\lambda = 1.4$ $(s_y^* = 0.294,\ s_x^* = 0.138,\ s_r^* = -0.344)$	-12.3058	0.0896	0.3685	0.1485	0.7232
$\lambda = 1.2;\ \beta = 1/1.2$ $(s_y^* = 0.047,\ s_x^* = 0.128,\ s_r^* = -0.639)$	-36.6257	0.0569	0.3636	0.0846	0.5532
$\lambda = 1.2;\ \beta = 1/1.6$ $(s_y^* = 0.441,\ s_x^* = 0.100,\ s_r^* = -0.498)$	-13.4387	0.0754	0.3560	0.1864	0.7232
$\lambda = 1.4;\ \beta = 1/1.2$ $(s_y^* = 0.005,\ s_x^* = 0.163,\ s_r^* = -0.407)$	-32.1426	0.0735	0.3771	0.0760	0.5971
$\lambda = 1.4;\ \beta = 1/1.6$ $(s_y^* = 0.391,\ s_x^* = 0.131,\ s_r^* = -0.327)$	-7.9662	0.0949	0.3663	0.1661	0.7518

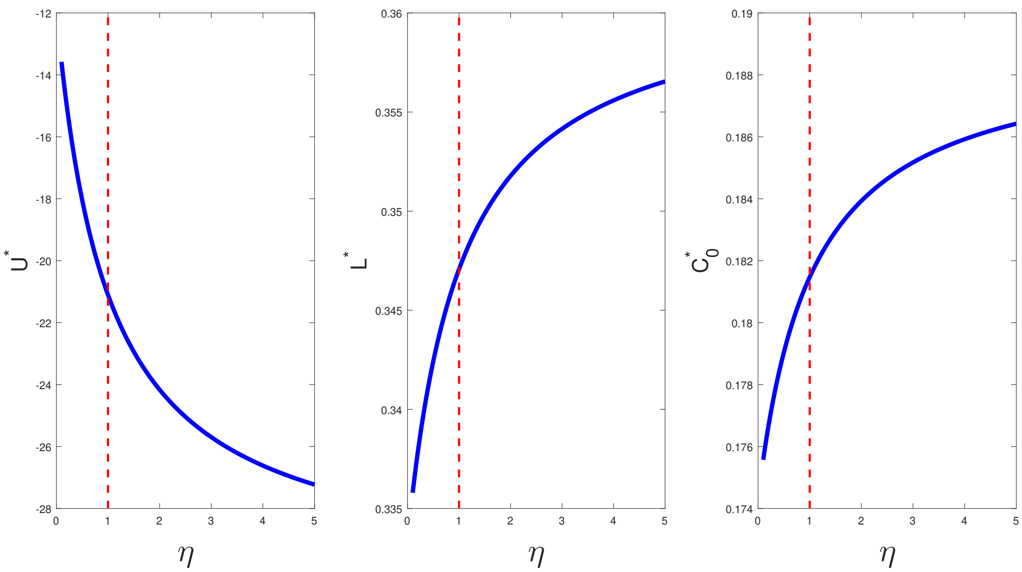


Notes: The range in consideration for σ is $[0.57, 1.19]$ to ensure studied variables fall into the permissible interval.

Fig. 1. Effects of σ on welfare-maximizing subsidy rates

Optimizing a single instrument. Next, we consider the cases in which only one single instrument is exploited. Given the benchmark calibration, Table 9 presents the details on welfare maximization by optimizing each subsidy tool. This result of welfare maximization implies that when only a single subsidy instrument is adopted to promote welfare, subsidizing R&D (the production of final goods) is least (most) effective. The reason is as follows. Most of inefficiencies in this model are realized by correcting the distortions in monopoly pricing and labor supply,

considered, which is different from the sensitivity analysis in Table 7.



Notes: The range in consideration for η is $[0.1, 5]$.

Fig. 2. Effects of η on welfare, labor and consumption

mainly through the use of s_y and s_x.[39] Therefore, when the subsidy rate is implemented alone, s_y and s_x tend to be more welfare-effective than s_r, given that the latter is used to mainly correct the distortion brought by R&D externalities.[40] This policy implication again differs from the comparison in welfare effectiveness of subsidy instruments in Zeng and Zhang (2007), in which R&D subsidy can be more welfare-improving than the other subsidies.

Table 9: Welfare maximization under a single instrument

Subsidies to production of final goods	g	U	L	τ
($s_y = 0.308$, $s_x = 0$, $s_r = 0$)	0.0672	-24.1517	0.3263	0.7064
Subsidies to purchase of intermediate goods	g	U	L	τ
($s_y = 0$, $s_x = 0.224$, $s_r = 0$)	0.0641	-24.5182	0.3951	0.3849
R&D subsidies	g	U	L	τ
($s_y = 0$, $s_x = 0$, $s_r = 0.060$)	0.0298	-32.2893	0.3333	0.1740

Optimizing a mix of two instruments. We also consider a policy experiment in which a mix of two instruments is optimized. Table 10 reports the quantitative implications of the optimal policy combinations under the benchmark parametrization. Specifically, subsidies to manufac-

[39] Similar to the analysis of growth maximization, this welfare-maximization analysis justifies our findings in Table 6 such that the welfare-maximizing rate of s_y (s_r) is the largest (smallest) among the three subsidy tools if the choice of all tools becomes available.

[40] The welfare improvements from the decentralized equilibrium to the outcomes by optimizing the subsidy to output production and to the purchase of intermediate goods are significantly large (which are 50.21% and 47.49% of consumption, respectively), whereas the counterpart by optimizing R&D subsidy alone is marginally small (which is roughly 0.001% of consumption).

turing (namely the combination of s_y and s_x) yield a lower level of welfare than those under the combination of s_x and s_r and under the combination of s_y and s_r.

It is obvious that an optimal mix of two subsidy rates is substantially welfare-improving than the decentralized equilibrium and the outcomes optimizing a single subsidy. For example, the welfare level under the combination of s_y and s_x, which is the lowest one among the three combinations of any two subsidies, is higher than the welfare level under the decentralized equilibrium by 50.76% of consumption. However, the welfare level under this combination is higher, merely by 0.37% of consumption, than the welfare level under optimizing s_y alone, which is the largest one among the outcomes using a single subsidy. Moreover, the welfare level under an optimal mix of two subsidy rates is considerably lower than the counterpart under the optimal combination of three subsidy rates (i.e., the social optimum).[41]

Table 10: Welfare maximization under a combination of two instruments

	g	U	L	τ
Subsidies to manufacturing ($s_y = 0.230$, $s_x = 0.060$, $s_r = 0$)	0.0673	-24.0785	0.3478	0.6461
Subsidies to production of final goods and R&D ($s_y = 0.495$, $s_x = 0$, $s_r = -0.730$)	0.0574	-21.2145	0.3214	0.7309
Subsidies to purchase of intermediate goods and R&D ($s_y = 0$, $s_x = 0.310$, $s_r = -0.695$)	0.0531	-22.0221	0.4145	0.2314

Notice that among the three combinations of two policy instruments, the combinations with subsidies to R&D (namely either subsidizing output production s_y and R&D s_r or subsidizing the purchase of intermediate goods s_x and R&D s_r) are more welfare-enhancing than subsidies to manufacturing only (i.e., subsidizing output production s_y and the purchase of intermediate goods s_x). This finding is in line with the argument in Zeng and Zhang (2007): the joint use of subsidies is to take advantage of their relative strength in correcting different types of distortions. Specifically, subsidies to manufacturing tend to be more effective in eliminating the distortions from monopoly pricing and the consumption-labor tradeoff, both of which are considered as the efficiency losses related to production (i.e., the static distortion), whereas subsidies to R&D tend to be more effective in eliminating the distortion from R&D externalities, which is considered as the efficiency losses related to innovation (i.e., the dynamic distortion). As a result, mixing subsidies that remedy different types of inefficiencies (i.e., inefficiencies from both production and innovation) does better than mixing subsidies that remedy the same type of inefficiencies (i.e., inefficiencies from only production).

6 Discussion on optimal subsidies with inelastic labor supply

The analysis above suggests that the economic growth rate and the optimal subsidy rates are independent of the Frisch elasticity of labor supply η. In this section, however, we show that the

[41] The welfare level under the combination of s_y and s_x, the combination of s_x and s_r, and the combination of s_y and s_r, is lower than the welfare level under the socially optimal outcome by 13.83%, 0.57%, and 4.50% of consumption, respectively.

setting of elastic labor supply is of central importance to the design of the welfare-maximizing subsidy instruments.

In the literature, the Frisch elasticity may take various values depending on the measurement approach. Empirical studies using micro-level data typically report low estimates of the Frisch elasticity (i.e. below 0.5), whereas the estimates in macroeconomic analysis vary in a wider range.[42] Empirical evidence favoring low values of the Frisch elasticity might motivate theorists to simplify their analytical framework by assuming inelastic labor supply. However, our analysis below shows that this simplifying assumption tends to create bias at least in the design of the optimal subsidy rates under our theoretical model. In particular, we find that it is important to distinguish the case with low elasticity of labor supply (i.e. small value of η) from the case with completely inelastic labor supply (i.e. $\eta = 0$). If labor is supplied with low elasticity but not completely inelastically, a variant of our model featuring inelastic labor supply would not uniquely pin down the optimal combination of s_y^*, s_x^* and s_r^*. Consequently, policymakers would have to make arbitrary choices on certain subsidy rates, and welfare losses are likely to occur.

When labor supply is inelastic, the utility maximization problem facing the households is given by:

$$U_t = \int_{t=0}^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma} dt, \tag{52}$$

subject to the budget constraint

$$\dot{A}_t = r_t A_t + W_t L - C_t + \Gamma_t, \tag{53}$$

where Γ_t denotes lump-sum tax (or transfer) and labor L is inelastically supplied. Other sectors in the modeled economy remain unchanged. It is straightforward to show that replicating the first-best allocation requires elimination of distortions stemming from monopoly pricing and R&D externalities, and the conditions to be satisfied are exactly the same as those in (44) and (45). Due to inelastic labor supply, however, distortions induced by the consumption-leisure tradeoff no longer exist. In addition to an explicit derivation of the inelastic-labor model, one can also see this implication from the analytical expressions for steady-state labor in the decentralized equilibrium and its first-best counterpart in the baseline model. Letting $\eta = 0$ in (32) and (42), we see that the values of L and L^* are both equal to unity (i.e., $0^0 = 1$), independent of the policy instruments and the structural parameters.

Hence, under inelastic labor supply, we need to pin down three policy instruments based on two equations. We see that the optimal subsidy rate to R&D is still uniquely determined by (45), which is identical to that in the elastic-labor model. For subsidies to the manufacturing industries, however, the inelastic-labor model implies that any combination (s_y, s_x) satisfying $(1 + s_y)/(1 - s_x) = 1/\beta$ is socially optimal. In sharp contrast, the elastic-labor model suggests that s_y^* and s_x^* are uniquely determined, given that they are important policy instruments affecting the consumption-labor tradeoff. Therefore, it is clear that policymakers might misuse the policy instruments if they consider low elasticity of labor supply as perfectly inelastic labor supply.

Using the benchmark calibration, we assess the potential welfare losses conditional on 5 different values of the Frisch elasticity, and present our findings in Figure 3 accordingly. In this

[42] See Chetty *et al.* (2011) and Chetty (2012) for a detailed discussion.

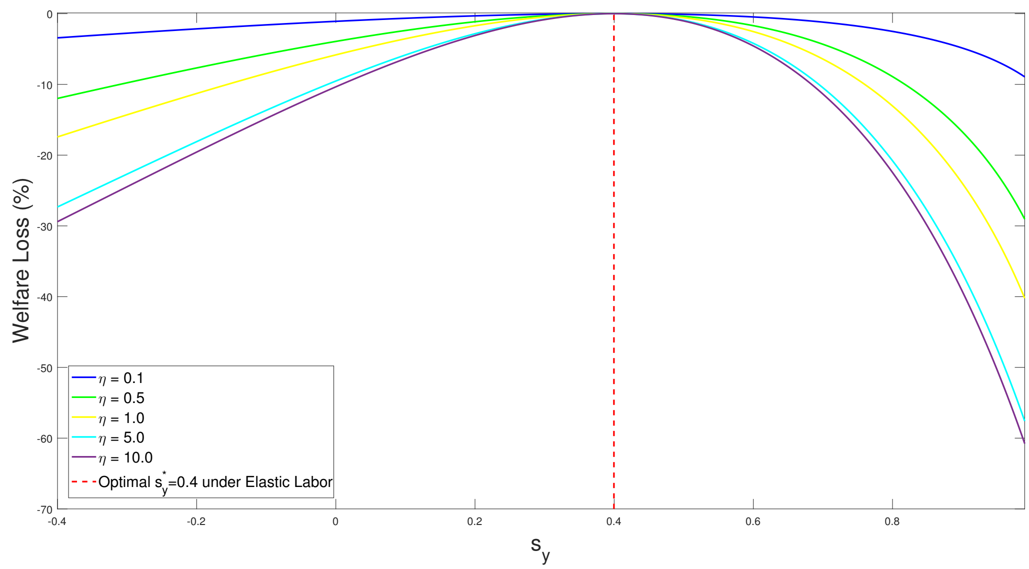


Fig. 3. Potential Welfare Loss due to Inelastic Labor

practice, the welfare under inelastic labor supply is computed by choosing the value of s_y from [-0.4 0.9], and setting the values of s_x and s_r such that equations (44) and (45) hold. The utility for all combinations of s_y^*, s_x^* and s_r^*, conditional on the calibrated value of η, is then compared with the first-best outcome under elastic labor as reported in Table 6. In Figure 3, we find that the welfare losses tend to get intensified when the wedge between the optimal final-good subsidy rate under inelastic labor supply (s_y^{IL*}) and the counterpart under elastic labor supply (s_y^{EL*}) becomes larger. In addition, given any s_y^{IL*} such that $s_y^{IL*} \neq s_y^{EL*}$, the welfare losses, which increase with the labor elasticity η, can be potentially substantial. The intuition for these results are straightforward. When the elasticity of labor supply is small, assuming inelastic labor would not leave out the response of labor to wage too much, and hence, the welfare differences tend to be small. To the contrary, when the elasticity of labor supply is large, setting it to zero in the theoretical model would ignore the response of labor supply to fluctuations in wage, leading to considerable welfare costs.

In practice, policymakers exploiting the model with inelastic supply might consider to set s_y^{IL*} or s_x^{IL*} to 0 for convenience. According to our calculation, when $\eta = 0.1$ and $s_y^{IL*} = 0$ (which implies $s_x^{IL*} = 1 - \beta = 1/3$), the potential welfare loss is approximately 1%. When $\eta = 0.5$ (namely the upper bound of Frisch elasticity estimates in most microeconomic studies), the potential welfare loss surges to 3.92%. If η takes the value of unity as calibrated in Trabandt and Uhlig (2011) and Annicchiarico *et al.* (2022), the welfare loss further increases to 5.83%. Hence, the above numerical analysis highlights the importance of elastic labor supply to welfare implications.

7 Conclusion

In this study, we explore the growth and welfare implications of a subsidization-policy regime in a quality-ladder model with elastic labor supply, where subsidies are financed by distortionary labor income taxes. This subsidization regime includes three policy instruments: subsidies to the production of final goods, subsidies to the purchase of intermediate goods, and subsidies to R&D. In this model, the equilibrium allocation is subject to three layers of distortions, namely, the distortions on monopoly pricing, labor supply, and R&D externalities. Therefore, the policymaker can adjust the equilibrium allocation to mitigate these distortions by properly implementing the subsidy tools.

The result in the current study differs substantially from those in the literature. In the presence of lump-sum taxes, Barro and Sala-I-Martin (2003) show that in a variety-expansion model with inelastic labor supply, the social optimum can be attained by subsidizing manufacturing (through either the production of final output or the purchase of intermediate products), whereas the analysis of Acemoglu (2009) (and Yang 2018) implies that in a quality-ladder model with inelastic (elastic) labor supply, the social optimum can be achieved by subsidizing manufacturing and research together. With elastic labor supply and distortionary taxes, Zeng and Zhang (2007) show that in a variety-expansion model, the social optimum could not be restored by using a single type of subsidies or their combination within a reasonable range in taxes. Nevertheless, under a similar setting of labor and taxation as in Zeng and Zhang (2007), our analysis shows that in a quality-ladder model, the mix of subsidies to the production of final goods, the purchase of intermediate goods, and research is able to replicate the first-best optimal outcome by correcting all distortions occurring in the decentralized equilibrium. Specifically, subsidies to manufacturing tend to remove the distortions on monopoly pricing and the consumption-labor tradeoff, whereas subsidies to R&D tend to remove the distortion on R&D externalities. Therefore, the process of innovation is crucial in determining the possibility of which the social optimum is attained in a decentralized equilibrium with the aid of subsidies.

To quantify the effectiveness of subsidy tools in promoting economic growth and raising social welfare, this model is calibrated to the US data to perform a numerical analysis on growth maximization and welfare maximization. First, the use of more types of subsidies ameliorates the effects on maximizing growth and welfare. Second, as for the use of a single instrument, we find that R&D subsidy is less growth-enhancing and welfare-improving than the other types of subsidies. Finally, as for the use of a mix of two instruments, subsidizing final-good production and the purchase of intermediate goods is most effective in promoting growth but least effective in raising welfare. These quantitative results differ significantly from some existing studies showing that R&D subsidy is more growth-enhancing and welfare-improving than the other subsidies. Although subsidizing R&D investment is the common practice as observed in many industrialized countries, the present study provides an important policy implication on extending the dimensionality of the fiscal-policy (or industrial-policy) system by increasing the number of subsidy/tax tools, given that the mechanism of these dimensions works differently in allocating resources and eliminating inefficiencies.

Appendix A

A.1 Proof of Proposition 1

In this proof, we examine the stability of this model given a stationary path of $s_{y,t}$, $s_{x,t}$, and $s_{r,t}$. First, define the transformed variable $\Phi_t \equiv C_t/Y_t$. Taking the log of Φ_t and differentiating it with respect to time yields

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t}. \tag{A.1}$$

Substituting (4) into (A.1) yields

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{r_t - \rho}{\sigma} - \frac{\theta(1-\sigma)(1+\eta)L_t^{\frac{1}{\eta}}\dot{L}_t}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} - \frac{\dot{Y}_t}{Y_t}. \tag{A.2}$$

We now construct a relation between r_t and L_t. Consider that for the monopolist in line ν, quality q is given. Hence, from (18), we have $\dot{V}_t(\nu)/V_t(\nu) = \dot{L}_t/L_t$. Then using (12) for $\pi_t(\nu)$ and (20), we can rewrite the no-arbitrage condition (18) as

$$r_t = \frac{\dot{L}_t}{L_t} + \frac{\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}}{1-s_r} - p_t(\nu). \tag{A.3}$$

This implies that $p_t(\nu) = p_t$ is identical across industries. Moreover, from (14), we derive the growth rate of output such that

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{Q}_t}{Q_t} + \frac{\dot{L}_t}{L_t} = (\lambda-1)p_t + \frac{\dot{L}_t}{L_t}, \tag{A.4}$$

where we have applied the definition of the growth rate of aggregate technology $\dot{Q}_t/Q_t = p_t(\lambda - 1)$. By inserting (A.3) and (A.4) back into (A.2), we obtain

$$\begin{aligned}
\frac{\dot{\Phi}_t}{\Phi_t} &= \frac{1}{\sigma}\left\{\frac{\dot{L}_t}{L_t} + \frac{\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}}{1-s_r} - p_t - \rho\right\} - \frac{\theta(1-\sigma)(1+\eta)L_t^{\frac{1}{\eta}}\dot{L}_t}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} - (\lambda-1)p_t - \frac{\dot{L}_t}{L_t} \\
&= \left\{\frac{1-\sigma}{\sigma} - \frac{\theta(1-\sigma)(1+\eta)L_t^{1+\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}\right\}\frac{\dot{L}_t}{L_t} - \left(\lambda - 1 + \frac{1}{\sigma}\right)p_t + \frac{1}{\sigma}\left[\frac{\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}}{1-s_r} - \rho\right].
\end{aligned} \tag{A.5}$$

So far, we have three endogenous variables $\{p_t, L_t, \Phi_t\}$ in (A.5) and next derive their relations. To do so, we first use the government budget constraint (22) to express τ_t as a function of p_t. By plugging (14), (15), (16) and (25) into (22),[43] we obtain

$$
\begin{aligned}
&\tau_t W_t L_t = s_y Y_t + s_x X_t + s_r Z_t \\
\Leftrightarrow &\tau_t (1+s_y)\left(\frac{1-\beta}{\beta}\right)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t L_t = \frac{s_y}{\beta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t L_t + s_x\beta\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}} Q_t L_t + \frac{s_r p_t}{\zeta} Q_t L_t \\
\Leftrightarrow &\tau_t (1+s_y)\left(\frac{1-\beta}{\beta}\right) = \frac{s_y}{\beta} + s_x\beta\left(\frac{1+s_y}{1-s_x}\right) + \frac{s_r p_t}{\zeta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{-\beta}{1-\beta}} \\
\Leftrightarrow &\tau_t = \underbrace{\left[\frac{s_r\beta}{\zeta(1-\beta)(1+s_y)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}}\right]}_{\chi_1>0} p_t + \underbrace{\left[\frac{s_y}{(1-\beta)(1+s_y)} + \frac{\beta^2 s_r}{(1-\beta)(1-s_x)}\right]}_{\chi_2>0}.
\end{aligned}
\tag{A.6}
$$

Relating p_t with Φ_t and L_t by substituting (16) and (A.6) into (3) yields

$$
\begin{aligned}
&\frac{W_t(1-\tau_t)}{C_t} = \frac{\sigma\theta(1+\eta)L_t^{\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} \\
\Leftrightarrow &\frac{(1+s_y)\left(\frac{1-\beta}{\beta}\right)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t(1-\chi_1 p_t-\chi_2)}{(C_t/Y_t)Y_t} = \frac{\sigma\theta(1+\eta)L_t^{\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} \\
\Leftrightarrow &\frac{(1+s_y)\left(\frac{1-\beta}{\beta}\right)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t(1-\chi_1 p_t-\chi_2)}{\Phi_t\frac{1}{\beta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t L_t} = \frac{\sigma\theta(1+\eta)L_t^{\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} \\
\Leftrightarrow &\frac{(1+s_y)(1-\beta)(1-\chi_1 p_t-\chi_2)}{\Phi_t} = \frac{\sigma\theta(1+\eta)L_t^{1+\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} \\
\Leftrightarrow &p_t = \frac{1-\chi_2}{\chi_1} - \frac{\sigma\theta(1+\eta)\Phi_t L_t^{1+\frac{1}{\eta}}}{\eta\chi_1(1+s_y)(1-\beta)\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]},
\end{aligned}
\tag{A.7}
$$

where χ_1 and χ_2 are constants. Moreover, substituting (14), (15) and (25) into the final-good

[43]Note that equation (25) holds since we have proven above that $p_t(\nu) = p_t$, which does not rely on any BGP condition.

market-clearing condition shows that

$$
\begin{aligned}
&Y_t = C_t + X_t + Z_t \Leftrightarrow 1 = \Phi_t + \frac{X_t}{Y_t} + \frac{Z_t}{Y_t} \\
\Leftrightarrow &1 = \Phi_t + \frac{\beta\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}} Q_t L_t}{\frac{1}{\beta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t L_t} + \frac{p_t Q_t L_t/\zeta}{\frac{1}{\beta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t L_t} \\
\Leftrightarrow &1 = \Phi_t + \beta^2\left(\frac{1+s_y}{1-s_x}\right) + \frac{\beta p_t}{\zeta\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}} \\
\Leftrightarrow &1 = \Phi_t + \beta^2\left(\frac{1+s_y}{1-s_x}\right) + \frac{\frac{\beta(1-\chi_2)}{\chi_1} - \frac{\beta\sigma\theta(1+\eta)\Phi_t L_t^{1+\frac{1}{\eta}}}{\eta\chi_1(1+s_y)(1-\beta)\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}}{\zeta\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}} \\
\Leftrightarrow &1 - \beta^2\left(\frac{1+s_y}{1-s_x}\right) = \Phi_t + \underbrace{\frac{\beta(1-\chi_2)}{\zeta\chi_1\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}}}_{\chi_3} - \underbrace{\frac{\beta\sigma\theta(1+\eta)}{\eta\zeta\chi_1(1+s_y)(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}}}_{\chi_4} \cdot \frac{\Phi_t L_t^{1+\frac{1}{\eta}}}{\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}
\end{aligned}
\tag{A.8}
$$

where

$$
\chi_3 = \frac{(1-\beta)(1+s_y) - s_y - \left(\frac{1+s_y}{1-s_x}\right)\beta^2 s_r}{s_r}
$$

and

$$
\chi_4 = \frac{\sigma\theta(1+\eta)}{\eta s_r}
$$

are constants. Therefore, (A.8) is eventually reduced to

$$
\begin{aligned}
&1 - \beta^2\left(\frac{1+s_y}{1-s_x}\right) = \Phi_t + \frac{(1-\beta)(1+s_y) - s_y}{s_r} - \beta^2\left(\frac{1+s_y}{1-s_x}\right) - \frac{\sigma\theta(1+\eta)}{\eta s_r} \cdot \frac{\Phi_t L_t^{1+\frac{1}{\eta}}}{\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]} \\
\Leftrightarrow &\underbrace{\frac{\beta(1+s_y) - 1 + s_r}{s_r}}_{\chi_5} = \Phi_t\left\{1 - \frac{\chi_4 L_t^{1+\frac{1}{\eta}}}{\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}\right\} \\
\Leftrightarrow &\Phi_t = \frac{\chi_5\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}{1 - \left[\chi_4 + \theta(1-\sigma)\right]L_t^{1+\frac{1}{\eta}}}.
\end{aligned}
\tag{A.9}
$$

Since $\sigma > 0$ and the calibrated value of θ exceeds 1, the condition $\sigma > (1-\theta)/\theta$ holds, ensuring that the term $1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}$ must be positive. Therefore, equation (A.9) implies that

$$\frac{\chi_5}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}>0$$

must hold. Moreover, differentiating both sides of (A.9) with respect to time t yields

$$\dot{\Phi}_t=\frac{\chi_4\chi_5\left(1+\frac{1}{\eta}\right)L_t^{\frac{1}{\eta}}\dot{L}_t}{\left\{1-[\chi_4+\theta(1-\sigma)]\,L_t^{1+\frac{1}{\eta}}\right\}^2}. \tag{A.10}$$

Combining (A.9) and (A.10) yields

$$\frac{\dot{\Phi}_t}{\Phi_t}=\frac{\chi_4\left(1+\frac{1}{\eta}\right)L_t^{1+\frac{1}{\eta}}}{\left\{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\right\}\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}\frac{\dot{L}_t}{L_t}. \tag{A.11}$$

Finally, by substituting (A.11) into (A.5), we can obtain the one dimensional differential equation of L_t:

$$\begin{aligned}&\frac{\dot{L}_t}{L_t}\left\{\frac{\chi_4\left(1+\frac{1}{\eta}\right)L_t^{1+\frac{1}{\eta}}}{\left\{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\right\}\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}+\frac{\sigma-1}{\sigma}+\frac{\theta(1-\sigma)(1+\eta)L_t^{1+\frac{1}{\eta}}}{\eta\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}\right\}\\&=\frac{1}{\sigma}\left[\frac{\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}}{1-s_r}-\rho\right]-\left(\lambda-1+\frac{1}{\sigma}\right)p_t.\end{aligned} \tag{A.12}$$

The right-hand side of (A.12) can be rewritten by using (A.7) and (A.9):

$$
\frac{1}{\sigma}\left[\frac{\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}}{1-s_r}-\rho\right]-\left(\lambda-1+\frac{1}{\sigma}\right)\left\{\frac{1-\chi_2}{\chi_1}-\frac{\sigma\theta(1+\eta)L_t^{1+\frac{1}{\eta}}\frac{\chi_5\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}}{\eta\chi_1(1+s_y)(1-\beta)\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]}\right\}
$$

$$
=\underbrace{\frac{\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}}{\sigma(1-s_r)}-\frac{\rho}{\sigma}-\frac{(1-\chi_2)\left(\lambda-1+\frac{1}{\sigma}\right)}{\chi_1}}_{\chi_6}
$$

$$
+\underbrace{\frac{\zeta\sigma\theta(1+\eta)\chi_5\left(\lambda-1+\frac{1}{\sigma}\right)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}}{\eta\beta s_r}}_{\chi_7}\cdot\frac{L_t^{1+\frac{1}{\eta}}}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}
$$

Because there is no population growth, i.e., $\dot{L}_t/L_t=0$, it must be true that

$$
\chi_6+\underbrace{\frac{\zeta\sigma\theta(1+\eta)\left(\lambda-1+\frac{1}{\sigma}\right)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}}L_t^{1+\frac{1}{\eta}}}{\eta\beta s_r}}_{>0}\cdot\frac{\chi_5}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}=0,
$$

which implies that the condition

$$
\chi_6<0
$$

must hold, because $\chi_5/\{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\}>0$ according to (A.9). In addition, the terms associated with $\dot{L}_t/L_t$ on the left-hand side of (A.12) can be rewritten as

$$
\frac{\chi_4(1+1/\eta)L_t^{1+\frac{1}{\eta}}+\theta(1-\sigma)(1+1/\eta)L_t^{1+\frac{1}{\eta}}\left\{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\right\}}{\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]\left\{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\right\}}+\frac{\sigma-1}{\sigma}
$$

$$
=\frac{(1+1/\eta)L_t^{1+\frac{1}{\eta}}\left\{\chi_4+\theta(1-\sigma)-\theta(1-\sigma)[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\right\}}{\left[1-\theta(1-\sigma)L_t^{1+\frac{1}{\eta}}\right]\left\{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}\right\}}+\frac{\sigma-1}{\sigma}
$$

$$
=\frac{(1+1/\eta)[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}+\frac{\sigma-1}{\sigma}.
$$

The one-dimensional differential equation for L_t in (A.12) can be further reduced to

$$\frac{\dot{L}_t}{L_t} = \frac{\chi_6 + \frac{\chi_7 L_t^{1+\frac{1}{\eta}}}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}}{\frac{\sigma-1}{\sigma} + \frac{(1+1/\eta)[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}{1-[\chi_4+\theta(1-\sigma)]L_t^{1+\frac{1}{\eta}}}} = \frac{\chi_6 + \frac{\chi_7}{L_t^{-(1+1/\eta)}-[\chi_4+\theta(1-\sigma)]}}{\frac{\sigma-1}{\sigma} + \frac{(1+1/\eta)[\chi_4+\theta(1-\sigma)]}{L_t^{-(1+1/\eta)}-[\chi_4+\theta(1-\sigma)]}}. \tag{A.13}$$

We next show that $\partial(\dot{L}_t/L_t)/\partial L_t > 0$. First, along a BGP, by imposing $\dot{L}_t/L_t = 0$ in (A.13), we can obtain the steady-state value of L such that

$$L = \left(\frac{\chi_6}{\chi_6[\chi_4+\theta(1-\sigma)] - \chi_7}\right)^{\frac{\eta}{1+\eta}}. \tag{A.14}$$

Given the condition $\chi_6 < 0$, the parameter space is restricted such that

$$\chi_7 - \chi_6[\chi_4+\theta(1-\sigma)] > 0. \tag{A.15}$$

to ensure $0 < L < 1$. Next, define $F(L_t) = \{L_t^{-(1+1/\eta)} - [\chi_4+\theta(1-\sigma)]\}^{-1}$, we have $F'(L_t) > 0$. Therefore, from (A.13), we can calculate

$$\begin{aligned}
&\frac{\partial(\dot{L}_t/L_t)}{\partial L_t} \gtreqless 0 \\
\Leftrightarrow &\chi_7 F'(L_t)\left\{\frac{\sigma-1}{\sigma} + \left(1+\frac{1}{\eta}\right)[\chi_4+\theta(1-\sigma)]F(L_t)\right\} \\
&- [\chi_6+\chi_7 F(L_t)]\left(1+\frac{1}{\eta}\right)[\chi_4+\theta(1-\sigma)]F'(L_t) \gtreqless 0 \\
\Leftrightarrow &\chi_7\left(\frac{\sigma-1}{\sigma}\right) - \chi_6\left(1+\frac{1}{\eta}\right)[\chi_4+\theta(1-\sigma)] \gtreqless 0.
\end{aligned} \tag{A.16}$$

Recall that $\chi_4 = \sigma\theta(1+\eta)/(\eta s_r)$. Then we have

$$\chi_4 + \theta(1-\sigma) = \theta\left[\frac{\sigma(1+\eta)}{\eta s_r} - \sigma + 1\right] = \theta\left[\frac{\sigma}{s_r} - \sigma + \frac{\sigma}{\eta s_r} + 1\right] > 0, \tag{A.17}$$

as $s_r < 1$. Therefore, the condition $\chi_6 < 0$ and (A.17) together guarantee $\chi_7(1-1/\sigma) - \chi_6(1 + 1/\eta)[\chi_4+\theta(1-\sigma)] > 0$ in (A.16). Given that L_t is a control variable and $\partial(\dot{L}_t/L_t)/\partial L_t > 0$, the dynamics of L_t is characterized by saddle-point stability such that L_t jumps immediately to its steady-state value. Equations (A.9) and (A.11) then imply that $\Phi_t = \Phi$ is also stationary and $\dot{\Phi}_t/\Phi_t = 0$. Moreover, (A.6) and (A.7) immediately follow that $\tau_t = \tau$ and $p_t = p$ are also time invariant, and then X_t/Y_t and Z_t/Y_t from (A.8) are both stationary, implying that variables $\{Y_t, C_t, X_t, Z_t\}$ have an identical growth rate g. Finally, since $L_t = L$, from (16) we have

$$g = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{W}_t}{W_t} = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{X}_t}{X_t} = \frac{\dot{Z}_t}{Z_t}. \tag{A.18}$$

A.2 Derivation of the steady-state welfare function

The steady-state welfare function is obtained by imposing the BGP in the utility function (1). When $\sigma > 0$ and $\sigma \neq 1$, integrating it yields

$$U_0 = \frac{1}{1-\sigma}\left\{\frac{\left[1-\theta(1-\sigma)L^{1+\frac{1}{\eta}}\right]^{\sigma} C_0^{1-\sigma}}{\rho - g(1-\sigma)} - \frac{1}{\rho}\right\}, \tag{A.19}$$

where C_0 is the initial level of consumption. Using (3) and (16), the term C_0 can be re-expressed as follows:

$$C_0 = \left(\frac{1-\tau}{\sigma\theta}\right)\left(\frac{\eta}{1+\eta}\right)\left(\frac{1-\beta}{\beta}\right)(1+s_y)^{\frac{1}{1-\beta}}(1-s_x)^{\frac{\beta}{\beta-1}}\frac{\left[1-\theta(1-\sigma)L^{1+\frac{1}{\eta}}\right]}{L^{\frac{1}{\eta}}}Q_0, \tag{A.20}$$

where Q_0 is the initial level of aggregate quality, and τ and L are given by (31) and (32), respectively. Hence, given that τ and L are functions of policy instruments $\{s_y, s_x, s_r\}$, the welfare level U_0 is also a function of policy instruments $\{s_y, s_x, s_r\}$.

When $\sigma = 1$, the utility function becomes

$$U_t = \int_0^{\infty} e^{-\rho t}\left(\ln C_t - \theta L_t^{1+\frac{1}{\eta}}\right) dt.$$

Integrating it yields

$$U_0 = \frac{1}{\rho}\left(\ln C_0 + \frac{g}{\rho} - \theta L^{1+\frac{1}{\eta}}\right), \tag{A.21}$$

and the expression for C_0 can be attained by setting $\sigma = 1$ in (A.20).

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton, NJ: Princeton University Press.

— and Akcigit, U. (2012). Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association*, **10** (1), 1–42.

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, **60** (2), 323–51.

Annicchiarico, B., Antonaroli, V. and Pelloni, A. (2022). Optimal factor taxation in a scale free model of vertical innovation. *Economic Inquiry*, **60** (2), 794–830.

Atkeson, A., Burstein, A. T. and Chatzikonstantinou, M. (2019). Transitional dynamics in aggregate models of innovative investment. *Annual Review of Economics*, **11**, 273–301.

Barkai, S. (2020). Declining labor and capital shares. *Journal of Finance*, **75** (5), 2421–2463.

Barro, R. J. and Sala-I-Martin, X. (2003). *Economic Growth*. Cambridge, MA: The MIT Press.

Bilbiie, F. O., Ghironi, F. and Melitz, M. J. (2019). Monopoly power and endogenous product variety: Distortions and remedies. *American Economic Journal: Macroeconomics*, **11** (4), 140–74.

Bloom, N., Schankerman, M. and Reenen, J. V. (2013). Identifying technology spillovers and product market rivalry. *Econometrica*, **81** (4), 1347–1393.

Brown, J. R., Martinsson, G. and Petersen, B. C. (2017). What promotes r&d? comparative evidence from around the world. *Research Policy*, **46** (2), 447–462.

Chetty, R. (2012). Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply. *Econometrica*, **80** (3), 969–1018.

—, Guren, A., Manoli, D. and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, **101** (3), 471–75.

Chu, A. C. and Cozzi, G. (2018). Effects of patents versus r&d subsidies on income inequality. *Review of Economic Dynamics*, **29**, 68–84.

—, Furukawa, Y. and Ji, L. (2016). Patents, r&d subsidies, and endogenous market structure in a schumpeterian economy. *Southern Economic Journal*, **82** (3), 809–825.

— and Wang, X. (2022). Effects of r&d subsidies in a hybrid model of endogenous growth and semi-endogenous growth. *Macroeconomic Dynamics*, **26** (3), 813–832.

Cozzi, G. (2017a). Combining semi-endogenous and fully endogenous growth: A generalization. *Economics Letters*, **155** (C), 89–91.

— (2017b). Endogenous growth, semi-endogenous growth... or both? a simple hybrid model. *Economics Letters*, **154** (C), 28–30.

Denicolò, V. and Zanchettin, P. (2014). What causes over-investment in r&d in endogenous growth models? *Economic Journal*, **124** (581), 1192–1212.

Dinopoulos, E. and Syropoulos, C. (2007). Rent protection as a barrier to innovation and growth. *Economic Theory*, **32** (2), 309–332.

Garcia-Macia, D., Hsieh, C.-T. and Klenow, P. J. (2019). How destructive is innovation? *Econometrica*, **87** (5), 1507–1541.

Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of growth. *Review of Economic Studies*, **58** (1), 43–61.

Grossmann, V., Steger, T. and Trimborn, T. (2013). Dynamically optimal r&d subsidization. *Journal of Economic Dynamics and Control*, **37** (3), 516–534.

Guvenen, F. (2006). Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. *Journal of Monetary Economics*, **53** (7), 1451–1472.

Hall, B. H., Mairesse, J. and Mohnen, P. (2010). *Measuring the Returns to R&D*, Elsevier, *Handbook of the Economics of Innovation*, vol. 2, pp. 1033–1082.

Hall, R. E. (2009). Reconciling cyclical movements in the marginal value of time and the marginal product of labor. *Journal of Political Economy*, **117** (2), 281–323.

Howitt, P. (1999). Steady endogenous growth with population and r&d inputs growing. *Journal of Political Economy*, **107** (4), 715–730.

Impullitti, G. (2010). International competition and u.s. r&d subsidies: A quantitative welfare analysis. *International Economic Review*, **51** (4), 1127–1158.

Jones, C. I. (1995). R&d-based models of economic growth. *Journal of Political Economy*, **103** (4), 759–84.

— and Williams, J. C. (1998). Measuring the social return to r&d. *Quarterly Journal of Economics*, **113** (4), 1119–1135.

— and — (2000). Too much of a good thing? the economics of investment in r&d. *Journal of Economic Growth*, **5** (1), 65–85.

Kimball, M. S. and Shapiro, M. D. (2008). *Labor supply: Are the income and substitution effects both large or both small?* Tech. rep., National Bureau of Economic Research.

Kortum, S. S. (1997). Research, patenting, and technological change. *Econometrica*, **65** (6), 1389–1420.

Li, B. and Zhang, J. (2014). Subsidies in an economy with endogenous cycles over investment and innovation regimes. *Macroeconomic Dynamics*, **18** (06), 1351–1382.

Lin, H. C. (2002). Shall the northern optimal r&d subsidy rate inversely respond to southern intellectual property protection? *Southern Economic Journal*, **69** (2), 381–397.

Loecker, J. D., Eeckhout, J. and Unger, G. (2020). The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics*, **135** (2), 561–644.

Matsuyama, K. (1999). Growing through cycles. *Econometrica*, **67** (2), 335–348.

— (2001). Growing through cycles in an infinitely lived agent economy. *Journal of Economic Theory*, **100** (2), 220–234.

Meghir, C. and Phillips, D. (2010). Labour supply and taxes. *Dimensions of tax design: The Mirrlees review*, pp. 202–74.

Minniti, A. and Venturini, F. (2017). The long-run growth effects of r&d policy. *Research Policy*, **46** (1), 316–326.

Nuño, G. (2011). Optimal research and development and the cost of business cycles. *Journal of Economic Growth*, **16** (3), 257–283.

OECD (2009). Oecd science, technology and industry scoreboard 2009. *OECD Publishing*, **Paris**.

— (2013). Oecd science, technology and industry scoreboard 2013: Innovation for growth. *OECD Publishing*, **Paris**.

Park, W. G. (2008). International patent protection: 1960-2005. *Research Policy*, **37** (4), 761–766.

Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, **3** (4), 283–311.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, **98** (5), S71–102.

Saez, E., Slemrod, J. and Giertz, S. H. (2012). The elasticity of taxable income with respect to marginal tax rates: A critical review. *Journal of Economic Literature*, **50** (1), 3–50.

Segerstrom, P. S. (1998). Endogenous growth without scale effects. *American Economic Review*, **88** (5), 1290–1310.

— (2000). The long-run growth effects of r&d subsidies. *Journal of Economic Growth*, **5** (3), 277–305.

Şener, F. (2008). R&d policies, endogenous growth and scale effects. *Journal of Economic Dynamics and Control*, **32** (12), 3895–3916.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, **97** (3), 586–606.

Thimme, J. (2017). Intertemporal substitution in consumption: A literature review. *Journal of Economic Surveys*, **31** (1), 226–257.

Trabandt, M. and Uhlig, H. (2011). The laffer curve revisited. *Journal of Monetary Economics*, **58** (4), 305–327.

Wan, J. and Zhang, J. (2021). Optimal growth through innovation, investment, and labor. *European Economic Review*, **132**, 103644.

Yang, Y. (2018). On the optimality of ipr protection with blocking patents. *Review of Economic Dynamics*, **27**, 205–230.

Young, A. (1998). Growth without scale effects. *Journal of Political Economy*, **106** (1), 41–63.

Zeng, J. and Zhang, J. (2007). Subsidies in an r&d growth model with elastic labor. *Journal of Economic Dynamics and Control*, **31** (3), 861–886.