Supplementary Material for "Blocking Patents, Rent Protection and Economic Growth"

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S.1 Calibration details For Figure 5

In Figure 5, we explore how the initial pace of innovation influences the impact of forward protection through its effect on the use of RPAs. Specifically, we display the change in economic growth associated with strengthening forward protection from $s_0 = 0.15$ to $s = 0.30$ in three cases where patent lifespan equals 6, 11, and 16 years in the initial equilibrium. We repeat this exercise across values of $\eta_x \in [0, 1]$ in increments of 0.05. For each of the 21 values of η_x, we set δ accordingly then calibrate γ to match the innovation rate associated with each of the three patent lifespans. The resulting calibrated parameter values are listed in Table S.1. All other parameters remain as reported in Table 1.

Table S.1: Calibration for Figure 5

η_x value	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
δ	0.00	0.875	0.937	0.959	0.971	0.978	0.983	0.986	0.989	0.991	0.993
γ $(2/I = 6)$	0.438	3.23	5.88	8.39	10.8	13.0	15.2	17.2	19.2	21.1	22.9
γ $(2/I = 11)$	0.327	2.41	4.38	6.23	7.98	9.64	11.2	12.7	14.1	15.5	16.8
γ $(2/I = 16)$	0.196	1.44	2.60	3.69	4.72	5.69	6.61	7.48	8.31	9.11	9.87
η_x value	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	–
δ	0.994	0.995	0.996	0.997	0.998	0.998	0.999	0.999	0.999	1.00	–
γ $(2/I = 6)$	24.6	26.3	27.9	29.5	31.0	32.5	33.9	35.3	36.6	37.9	–
γ $(2/I = 11)$	18.1	19.3	20.5	21.6	22.7	23.7	24.8	25.8	26.7	27.7	–
γ $(2/I = 16)$	10.6	11.3	11.9	12.6	13.2	13.8	14.4	15.0	15.5	16.1	–

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S.2 Discussion of Optimal Patent Policy

In Section 4 of the main text, we show that the model features a single dynamic distortion arising from the allocation of non-specialized labor between the production of final goods and R&D. We then argue that the model tends toward an equilibrium in which the proportion of labor employed in R&D is inefficiently low. In this Section, we numerically examine the implications of this tendency for optimal patent policy.

We begin by illustrating the relationship between social welfare and the innovation rate across a (θ, s) policy grid using the calibrated model as described in Section 5. We focus on two representative cases: the $\eta_x = 0$ case, where RPAs have no impact on innovation difficulty, and the $\eta_x = 0.848$ case, where RPAs are a highly effective innovation deterrent. We examine the entire feasible range of patent policy, $s \in [0, 0.5]$ and $\theta \in [1, \lambda]$. All other parameters remain as reported in Section 5. For illustrative purposes, we express backward patent protection in terms of its corresponding markup over marginal cost relative to the markup with full backward protection, $(\theta - 1)/(\lambda - 1) \in [0, 1]$. Given the calibrated value of $\lambda = 1.1163$, the baseline value of $\theta = 1.08$ used in Section 5 corresponds to $(\theta - 1)/(\lambda - 1) = 0.688$.

As is apparent from Figure S.1, we find that changes to patent policy improve welfare if and only if they increase the innovation rate. As a result, the growth-maximizing patent policy is socially optimal in both cases. Intuitively, this is because the market equilibrium innovation rate remains below the socially optimal innovation rate across the entire patent policy grid. Optimal patent policy reduces the magnitude of the equilibrium misallocation of labor resources to the extent possible, but cannot eliminate it fully. Since strengthening backward protection always increases innovation, this implies that full backward protection is optimal in both cases. This is equivalent to preventing all imitation in our model. The associated optimal forward protection is case-specific and depends on whether stronger forward protection increases or decreases innovation. Since strengthening forward patent protection always reduces innovation when $\eta_x = 0$, optimal patent policy in this case is $(\theta^\star = \lambda, s^\star = 0)$. When $\eta_x = 0.848$, the equilibrium innovation rate associated with $\theta^\star = \lambda$ is sufficiently high such that forward protection increases innovation. Thus, optimal patent policy is $(\theta^\star = \lambda, s^\star = 0.5)$ in the case of $\eta_x = 0.848$.

These findings echo a large Schumpeterian growth literature that typically either assumes perfect backward patent protection, or otherwise rules out all imitation, and still finds substantial equilibrium underinvestment in R&D. Indeed, in influential analyses of this issue, Jones and Williams (2000) and Denicolo and Zanchettin (2014) conclude that R&D underinvestment persists in this class of endogenous growth models unless the monopoly-distortion effect discussed in Section 4 is exceptionally large or R&D is subject to significant congestion externalities that generate aggregate diminishing returns to R&D. Since we assume constant returns to scale in R&D and do not consider alternative policy remedies for this equilibrium inefficiency, namely R&D subsidies, we find that using patent policy to promote economic growth improves welfare.[1] Of course, the stark nature of our finding that it is socially optimal to implement the growth-maximizing patent policy should be interpreted cautiously and with these aspects of our

[1] Although it is *possible* to generate inefficiently high equilibrium R&D investment in our model within the (θ, s) policy grid that we consider, doing so requires implausible parameter values. In particular, it requires a very high value of λ so that the corresponding monopoly-distortion effect is sufficiently large. An analysis of this case is available from the authors upon request.

Figure S.1: Illustration of optimal patent policy

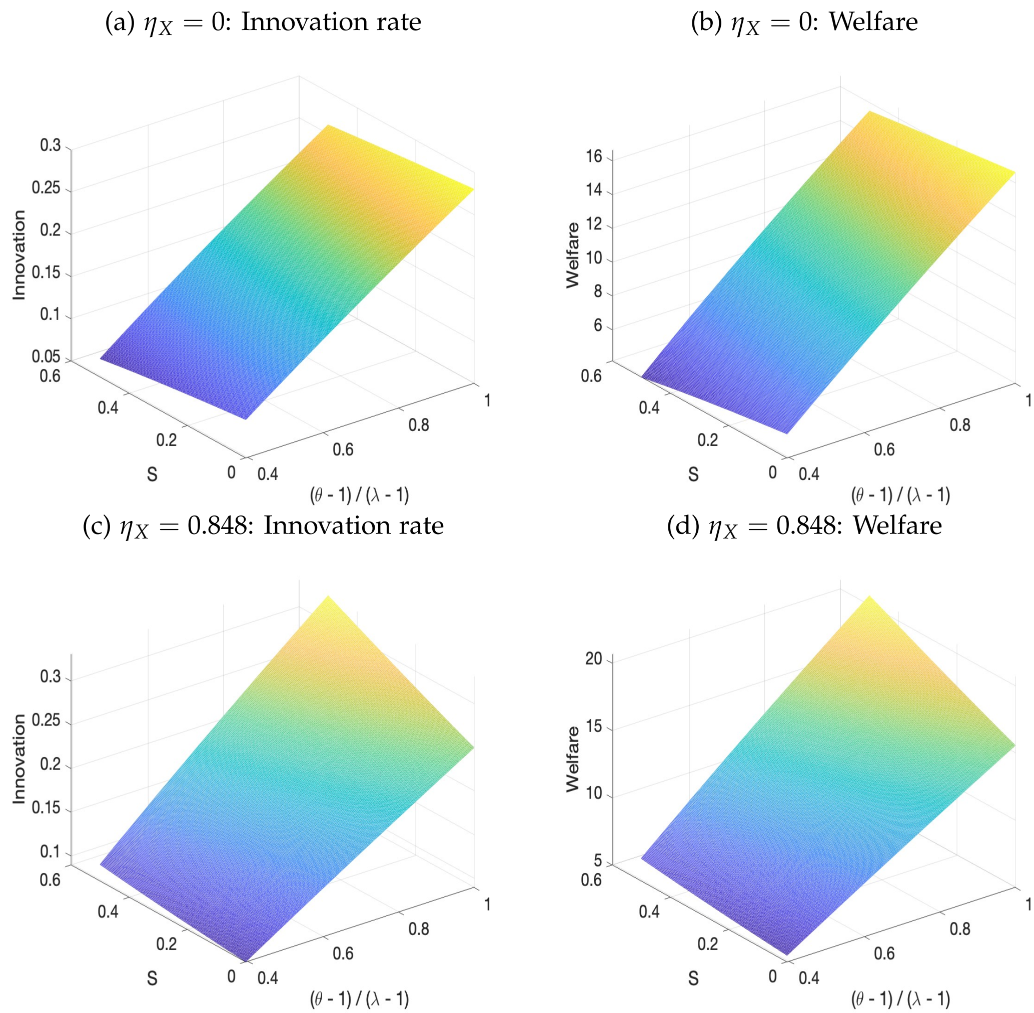

Figure S.1 displays the equilibrium innovation rate and social welfare associated with different patent policy combinations in two cases where $\eta_x = 0$ and $\eta_x = 0.848$. Since the innovation rate goes to zero when backward protection is sufficiently weak, we display results for $(\theta - 1)/(\lambda - 1) \geq 0.4$. Other than (θ, s), parameters remain as reported in Section 5.

modeling framework in mind. Nevertheless, our analysis demonstrates that forward patent protection can stimulate innovation through its interaction with patent holders' endogenous RPAs incentives, and thus, can play a role in growth-enhancing patent policy.

S.3 A Lab-Equipment Setting

In this section, we briefly demonstrate that the fully mobile labor version of the model examined in Section 6.1 is equivalent to a lab-equipment setting in which the input for both RPAs and R&D is a final consumption good. As in Section 6.1, we focus on the simple case where innovation difficulty $D(t)$ depends only on RPAs ($\delta = \eta_x = 1$). Thus, the innovation rate is given by $I(t) = R(t)/(\gamma X(t))$, where $R(t)$ and $X(t)$ denote the quantity of the final good devoted to R&D and RPAs respectively.

The lifetime utility of each household is given by

$$U \equiv \int_0^\infty e^{-(\rho-n)t} \ln c(t) dt, \tag{S.1}$$

where $c(t)$ denotes the per capita consumption. The dynamic optimization problem is to choose $c(t)$ to maximize (S.1) subject to the intertemporal budget constraint of $\dot{a}(t) = (r(t) - n)a(t) + w(t) - p(t)c(t)$, where $p(t)$ is the price of final good. This yields the familiar Euler equation,

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{p}(t)}{p(t)} = r(t) - \rho. \tag{S.2}$$

The final good $y(t)$ is competitively produced by a unit continuum of intermediate goods $m(t,i)$ for $i \in [0,1]$ according to the Cobb-Douglas production function:

$$y(t) = \exp\left(\int_0^1 \ln m(t,i) di\right). \tag{S.3}$$

Profit maximization yields the conditional demand for $m(t,i)$ such that $m(t,i) = (p(t)y(t))/p_m(t,i)$, where $p_m(t,i)$ is the price of $m(t,i)$.

Each intermediate good i is manufactured by a typical monopolistic firm employing labor according to the following production function:

$$m(t,i) = \lambda^{q(t,i)} L(t,i), \tag{S.4}$$

where λ measures the step size of each quality improvement, $q(t,i)$ is the number of innovations between time 0 and time t, and $L(t,i)$ is the production labor in industry i. The marginal cost of producing an intermediate good is $w(t)/\lambda^{q(t,i)}$. The profit-maximizing price is a constant markup over the marginal cost such that $p_m(t,i) = \theta(w(t)/\lambda^{q(t,i)})$, where the wage rate $w(t)$ is normalized to unity and the markup θ is the policy parameter for backward protection. The profit of this typical monopolistic firm is

$$\pi(t) = \left(\frac{\theta - 1}{\theta}\right) p_m(t,i)m(t,i) = \left(\frac{\theta - 1}{\theta}\right) p(t)y(t).$$

Free entry into research implies the first-order condition such that $I(t)V_1(t) = p(t)R(t)$, and substituting this expression into the innovation rate $I(t)$ yields

$$V_1(t) = \gamma p(t)X(t). \tag{S.5}$$

Omitting the dt notation, the no-arbitrage condition for the most recent innovator is

$$r(t)V_1(t) = (1 - s)\pi(t) - p(t)X(t) - I(t)[V_1(t) - V_2(t)] + \dot{V}_1(t), \tag{S.6}$$

which implies that optimal RPAs expenditure is given by

$$I(t)[V_1(t) - V_2(t)] = p(t)X(t). \tag{S.7}$$

Exactly as in (31), the ratio of the free-entry condition (S.5) and (S.7) yields the following,

$$\underbrace{I(t)[V_1(t) - V_2(t)]}_{RPAs} = \underbrace{\frac{V_1(t)}{\gamma}}_{R\&D}. \tag{S.8}$$

Since there is no change to the no-arbitrage condition for the second most recent innovator, we can use (S.6) and (S.7), to derive the following equilibrium expressions for $V_1(t)$ and $V_2(t)$,

$$V_1(t) = \frac{\pi(t)\{(\rho - n + I) + s[I - (\rho - n)]\}}{(\rho - n + 2I)(\rho - n + I)}, \qquad V_2(t) = \frac{s\pi(t)}{\rho - n + I}. \tag{S.9}$$

Substituting these expressions into (S.8) allows us to implicitly define I as a function of parameters exactly as in (32),

$$\gamma I\{(\rho - n + I) - s[I + 2(\rho - n)]\} = (\rho - n + I) + s[I - (\rho - n)]. \tag{S.10}$$

Equation (S.10) immediately implies that the innovation rate I is unaffected by backward protection θ and increases with forward protection s. Fundamentally, this is because the innovation rate is determined by relative R&D and RPAs investment. In both the lab-equipment and fully mobile labor versions of the model, these two activities share a common input, and thus marginal cost. This implies that the innovation rate is completely determined by the relative return to R&D and RPAs according to equation (S.8). Since these returns are both proportional to $\pi(t)$, neither backward patent protection nor the overall scale of the economy have any impact on the innovation rate.

Finally, aggregate technology $Z(t)$ is defined as

$$Z(t) \equiv \exp\left(\int_0^1 q(t,i)di \ln\lambda\right) = \exp\left(\int_0^t I(\omega)d\omega \ln\lambda\right), \tag{S.11}$$

where the last equality uses the law of large numbers. Since $L(t,i) = L(t)$ for all $i \in [0,1]$, substituting (S.4) into (S.3) yields $y(t) = Z(t)L(t)$, where $L(t) = N(t)$ is the total labor supply.[2] Differentiating the log of $Z(t)$ in (S.11) with respect to time yields the growth rate of technology given by

$$\frac{\dot{Z}(t)}{Z(t)} = I(t)\ln\lambda.$$

Therefore, the growth rate of final good is given by

$$\frac{\dot{y}(t)}{y(t)} = \frac{\dot{Z}(t)}{Z(t)} + \frac{\dot{L}(t)}{L(t)} = I(t)\ln\lambda + n.$$

[2]The main result in this setting continues to hold in the presence of elastic labor supply. Derivations are available upon request.

Consequently, the above expressions imply that the growth effects of $\{\theta, s\}$ in the lab-equipment setting are identical to those in Subsection 6.1 with mobile labor. We summarize these results by the following proposition.

Proposition S.1. *In the lab-equipment model with $\delta = \eta_x = 1$,*

- *Strengthening backward protection has no impact on economic growth.*
- *Strengthening forward protection always increases economic growth.*

S.4 Extension: Endogenous Patent Protection

The baseline model assumes that each patent holder's ability to prevent imitation and extract licensing fees from competitors is exogenous and defined by the patent policy parameters θ and s. In reality however, exercising the legal rights granted by a patent requires substantial resources. In this subsection, we examine how our results may change when incumbents also use RPAs to defend their existing patent against the threat that it will be invalidated in court. If an incumbent's patent is invalidated, the technology becomes freely available to imitators and subsequent innovators, resulting in the loss of both the incumbent's flow profits and claim over future profits through licensing revenue. In this way, even though θ and s remain exogenous, *effective* patent protection is endogenous since each incumbent's ability to maintain its patent rights depends on RPAs investment.

Let $m(t)$ denote the instantaneous probability that a patent will be invalidated. Together with the innovation rate, we define $m(t)$ according to

$$I(t) = \frac{L_I(t)}{D(t)}, \qquad m(t) \equiv \frac{\mu N(t)}{D(t)}, \tag{S.12}$$

where $\mu > 0$ is a parameter that captures the tendency of courts to invalidate standing patents. Note that the probability of patent invalidation increases with the size of the population. This reflects the idea that patent holders in a larger market likely face more challenges to the validity of their patents and must devote more resources to defend them. Since only incumbents with a valid patent have an incentive to conduct RPAs, we define the common difficulty term $D(t)$ as a stock variable that grows with the use of RPAs and depreciates at a constant rate $0 < \kappa < 1$. Let $\dot{D}(t) \equiv \gamma[\delta n_v(t) L_x(t) + (1-\delta)N(t)] - \kappa D(t)$, where $n_v(t)$ denotes the proportion of industries with a valid patent at time t.[3] A constant steady-state innovation and patent invalidation rate requires that $D(t)$, $L_x(t)$ and $L_I(t)$ all grow at the rate of population growth n. This implies that $\dot{D}(t)/D(t) = n$ and that $D(t) = \gamma[\delta n_v(t) L_x(t) + (1-\delta)N(t)]/(n+\kappa)$.

To maintain a symmetric equilibrium structure, we assume that R&D firms make their investment decisions without knowing if there will be a valid patent in the industry at the time they successfully innovate. The expected profit flow of a successful innovation is given by $\mathbb{E}\pi(t) = (1 - n_v(t))\pi(t) + (1-s)n_v(t)\pi(t)$. Omitting the dt notation, the no-arbitrage condition associated with the expected value of a successful innovation is $r(t)V_1(t) = \mathbb{E}\pi(t) - w_x(t)L_x(t) -$

[3] The stock variable formulation allows for the existence of a steady-state equilibrium in which all incumbents choose the same $L_x(t)$ while they have a valid patent and all industries share the same rate of innovation $I(t)$.

$I(t)[V_1(t) - V_2(t)] - m(t)V_1(t) + \dot{V}_1(t)$. Note that patent invalidation results in the loss of the incumbent innovator's total value $V_1(t)$. If innovation occurs while the incumbent's patent is active, the incumbent is entitled to a share of the new innovator's profit $s\pi(t)$ until either the new innovator has her patent invalidated or creative destruction occurs through further innovation. Thus, the no-arbitrage condition for $V_2(t)$ is given by $r(t)V_2(t) = s\pi(t) - (I(t) + m(t))V_2(t) + \dot{V}_2(t)$. Incumbent firms invest in RPAs to maximize their expected value $V_1(t)$. This results in the following demand for RPAs for each firm with an active patent,[4]

$$L_x(t) = \frac{\eta_x(t)\{I(t)[V_1(t) - V_2(t)] + m(t)V_1(t)\}}{w_x(t)}, \tag{S.13}$$

where $\eta_x(t)$ is the elasticity of innovation with respect to RPAs as before and the $m(t)V_1(t)$ term represents the component of RPAs demand motivated by patent defense.

We focus on a symmetric equilibrium in which $I(t)$, $m(t)$ and $n_v(t)$ are constant. Thus, the equilibrium flow into the valid patent pool $(1 - n_v)I$ must equal the flow out the pool $n_v m$. We obtain $n_v = I/(I + m)$. Since incumbents have a positive incentive to invest in RPAs only while their patent is valid, the RPAs labor market clearing condition is $\alpha N(t) = n_v L_x(t)$. Imposing this condition into the definition of $D(t)$, allows us to write the constant equilibrium m and η_x in terms of the parameters of the model such that $m = \mu(n + \kappa)/(\gamma[\delta\alpha + 1 - \delta])$ and $\eta_x = \delta\alpha(n + \kappa)/(\delta\alpha + 1 - \delta)$. Finally, we once again use the free-entry and non-specialized labor market clearing conditions to derive two equations in I and c. Rewriting the free-entry condition of $V_1(t) = D(t)$ gives,

$$\underbrace{\frac{\gamma[\delta\alpha + 1 - \delta]}{n + \kappa}}_{D(t)/N(t)} = \underbrace{\frac{c}{\theta}\left[\frac{\theta - 1}{(\rho - n + (1 + \eta_x)(I + m)}\right]\left\{\frac{s[\eta_x I - n_v(\rho - n)]}{(\rho - n + I + m)} + 1\right\}}_{V_1(t)/N(t)}. \tag{S.14}$$

The labor market clearing condition for non-specialized labor is given by $(1 - \alpha)N(t) = L_z(t) + L_I(t) = [(1 - n_v) + n_v/\theta]cN(t) + L_I(t)$. Note that industries without an active patent produce under competitive conditions with a price equal to firms' marginal cost of one. Using $D(t)$, we have

$$1 - \alpha = \frac{c}{\theta}[\theta(1 - n_v) + n_v] + \frac{\gamma(\delta\alpha + 1 - \delta)}{n + \kappa}I, \tag{S.15}$$

Repeating the analysis of Section 3, gives the following result

Proposition S.2. *In the model with patent invalidation,*

- *Strengthening backward protection always increases economic growth.*
- *Strengthening forward protection increases (decreases) the equilibrium growth rate if the innovation rate is sufficiently high (low), namely* $(\rho - n)/\eta_x - m < (>)I$.

[4]Technically, incumbent firms are differentiated into two groups based on whether the previous incumbent in their industry holds a patent claim to an s share of their flow profits. These groups would choose distinct levels of RPAs investment in order to optimally defend their different profit flows. However, it is straightforward to show that our approach, where all incumbents choose the same $L_x(t)$ based on average profit flows $\mathbb{E}\pi(t)$ while their patents are active, produces an identical equilibrium.

Thus, we continue to find that the growth impact of strengthened forward protection is controlled by a threshold innovation rate that is determined by the magnitude of the growth-reducing backloading effect and the growth-enhancing cost-saving effect. However, the presence of patent invalidation does influence the relative importance of these two effects. On the one hand, strengthening forward protection is less effective at reducing total demand for RPAs because an incumbent's incentives to use RPAs to defend her existing patent is relatively insensitive to changes in s. This tends to imply a smaller cost-saving effect than in the baseline model, particularly when the patent defense component of RPAs demand, $mV_1(t)$, is large. However, since optimal RPAs expenditure still decreases in $I[V_1(t) - V_2(t)]$, this effect still appears as a positive $s\eta_x I$ term in (S.14). On the other hand, new innovators need only pay licensing fees if the previous incumbent holds a valid patent. This reduces the expected profit impact of increasing s and implies a smaller backloading effect. Thus, the backloading effect appears in (S.14) as a negative $sn_v(\rho - n)$ term. Since $n_v = I/(I+m)$, the backloading effect is decreasing in the rate of patent invalidation, with the baseline model corresponding to $m = 0$. All together, we find that forward protection is more likely to be growth enhancing when patent holders also use RPAs to defend against patent invalidation.

S.5 Characterizing the Equilibrium with Incumbent R&D

We begin by deriving the no-arbitrage conditions associated with each type of incumbent firm. A firm that holds patent rights over the second most recent innovation in a typical industry receives a flow of licensing payments $s\pi(t)$ until the next innovation arrives, either from a new entrant or from the current quality leader. Omitting the dt notation, this implies the following no-arbitrage condition, $r(t)V_2(t) = s\pi(t) - [I(\lambda, t) + I(\lambda^2, t)]V_2(t) + \dot{V}_2(t)$. In equilibrium, this yields

$$V_2(t) = \frac{s\pi(t)}{\rho - n + I(\lambda) + I(\lambda^2)}. \tag{S.16}$$

One-step quality leaders earn flow profits net of licensing payments $(1-s)\pi(t)$, hire specialized RPAs labor to impede competitor innovation, and hire R&D labor in pursuit of further innovation. The associated no-arbitrage condition is, $r(t)V_1(\lambda, t) = (1-s)\pi(t) - w_x(t)L_x(\lambda, t) - L_I(\lambda^2) - I(\lambda, t)[V_1(\lambda, t) - V_2(t)] + I(\lambda^2, t)[V_1(\lambda^2, t) - V_1(\lambda, t)] + \dot{V}_1(\lambda, t)$. Each one-step leader chooses both $L_x(\lambda, t)$ and $L_I(\lambda^2)$ to maximize their value. This optimization results in the demand functions given by (34) and (35) in the main text and results in the following equilibrium expression

$$V_1(\lambda, t) = \frac{(1-s)\pi(t) + [1+\eta_x]I(\lambda)V_2(t) + (1-\beta)I(\lambda^2)V_1(\lambda^2, t)}{\rho - n + [1+\eta_x]I(\lambda) + (1-\beta)I(\lambda^2)}. \tag{S.17}$$

Two-step leaders also hire RPAs labor to defend their market position, but do not pay licensing fees and have no incentive to hire R&D labor for further innovation. The associated no-arbitrage condition is, $r(t)V_1(\lambda^2, t) = \pi(t) - w_x(t)L_x(\lambda^2, t) - I(\lambda, t)[V_1(\lambda^2, t) - V_2(t)] + \dot{V}_1(\lambda^2, t)$. The value maximizing choice of $L_x(\lambda^2, t)$ is given by (34) in the main text. In equilibrium, we have

$$V_1(\lambda^2, t) = \frac{\pi(t) + [1+\eta_x]I(\lambda)V_2(t)}{\rho - n + [1+\eta_x]I(\lambda)}. \tag{S.18}$$

In equations (S.17) and (S.18), we have used the fact that η_x is constant in a steady-state equilibrium in which $I(\lambda)$, $I(\lambda^2)$ and $n(\lambda)$ are constant. This follows from the labor market clearing condition for specialized RPAs labor and the definition of entrant innovation difficulty as a stock variable with $\dot{D}(t) = \gamma[\delta L_x(j,t) + (1-\delta)N(t)] - \kappa D(t)$, where $j \in \{\lambda, \lambda^2\}$. The stock formulation allows for a steady-state equilibrium in which all industries share a common rate of entrant innovation even though the level of RPAs employment at a particular time t differs across industries depending if the industry has a one or two-step leader. As in the baseline model, a constant rate of entrant innovation requires that the equilibrium growth rate of $D(t)$ equals the population growth rate, n, so that $D(t)/N(t)$ is constant. For each industry, average RPAs employment is given by $\bar{L}_x(t) \equiv n(\lambda)L_x(\lambda,t) + [1-n(\lambda)]L_x(\lambda^2,t)$. The specialized labor market clearing condition requires that $\bar{L}_x(t) = \alpha N(t)$. Thus, $\dot{D}(t)/D(t) = n$ implies that the equilibrium entrant innovation difficulty is given by

$$\frac{D(t)}{N(t)} = \frac{\gamma[\delta\alpha + (1-\delta)]}{n+\kappa}, \tag{S.19}$$

which corresponds directly to (18) in the main text, with the $(n+\kappa)$ term in the denominator capturing the accumulation of innovation difficulty over time given the stock formulation. The marginal effect of RPAs labor in each industry is given by

$$\frac{\partial I(\lambda)}{\partial L_x(j,t)} = \left| \frac{\partial I(\lambda)}{\partial D(t)} \frac{\partial D(t)}{\partial L_x(j,t)} \right| = \frac{I(\lambda)\gamma\delta}{D(t)} \tag{S.20}$$

Since $D(t)$ is symmetric across industries, so is the effectiveness of RPAs. Using (S.19) and $\bar{L}_x(t) = \alpha N(t)$, η_x is constant and given by

$$\eta_x = \frac{\delta\alpha(n+\kappa)}{\delta\alpha + 1 - \delta}, \tag{S.21}$$

which corresponds to equation (17) in the main text.

We characterize the equilibrium using three equilibrium conditions in three endogenous variables $I(\lambda)$, $I(\lambda^2)$ and c. First, using the definition of incumbent innovation given by (33), one-step leaders' optimal choice of $L_I(\lambda^2, t)$ given by (35), and the value of each type of incumbent firm given by (S.16), (S.17), and (S.18), we can derive the following "incumbent R&D condition,"

$$\frac{\phi I(\lambda^2)^{\frac{1-\beta}{\beta}}}{\beta} = \left[\frac{c(\theta-1)}{\theta}\right] \left\{ \frac{s}{\rho - n + [1+\eta_x]I(\lambda) + (1-\beta)I(\lambda^2)} \right\}. \tag{S.22}$$

This is identical to (36) in the main text. Second, using (S.16), (S.17) and (S.19), we derive the "free-entry condition" given by (37) in the main text,

$$\frac{\gamma[\delta\alpha + (1-\delta)]}{n+\kappa} = \left\{ \frac{c(\theta-1)}{\theta[\rho - n + [1+\eta_x]I(\lambda)]} \right\} \left\{ 1 + \frac{s[I(\lambda)\eta_x - (\rho-n) - I(\lambda^2)\Omega]}{\rho - n + I(\lambda) + I(\lambda^2)} \right\}, \tag{S.23}$$

where

$$\Omega \equiv \frac{\beta(\rho-n) + (\beta+\eta_x)I(\lambda)}{\rho - n + (1+\eta_x)I(\lambda) + (1-\beta)I(\lambda^2)} > 0.$$

The final equilibrium condition comes from non-specialized labor market clearing, which requires $(1-\alpha)N(t) = L_z(t) + L_I(\lambda, t) + n(\lambda)L_I(\lambda^2, t)$. Note that incumbent innovation occurs only in the $n(\lambda)$ with a one-step leader. At each point in time, the flow of industries that transition from a two-step leader to a one-step leader is $[1 - n(\lambda)]I(\lambda)$, and the flow of industries that transition from a one to two-step leader is $n(\lambda)I(\lambda^2)$. Thus, the constant equilibrium proportion of industries with a one-step leader is

$$n(\lambda) = \frac{I(\lambda)}{I(\lambda) + I(\lambda^2)}. \tag{S.24}$$

Using (33), we can write the total labor used in incumbent R&D as

$$n(\lambda)L_I(\lambda^2, t) = \phi N(t) I(\lambda^2)^{1/\beta} \left[\frac{I(\lambda)}{I(\lambda) + I(\lambda^2)}\right]. \tag{S.25}$$

Since all leaders charge the same price of θ, total employment in production remains $L_z(t) = cN(t)/\theta$. Using (33) and (S.19), $L_I(\lambda) = I(\lambda)D(t) = I(\lambda)N(t)\gamma[\delta\alpha + (1-\delta)]/(n+\kappa)$. This yields the following "labor market clearing condition"

$$1 - \alpha = \frac{c}{\theta} + \left\{\frac{\gamma[\delta\alpha + (1-\delta)]}{n+\kappa}\right\} I(\lambda) + \phi I(\lambda^2)^{1/\beta}\left[\frac{I(\lambda)}{I(\lambda) + I(\lambda^2)}\right]. \tag{S.26}$$

Finally, since both entrants and incumbents innovate, the equilibrium rate of economic growth is given by,

$$g = \ln\lambda[I(\lambda) + n(\lambda)I(\lambda^2)] = I(\lambda)\ln\lambda\left[1 + \frac{I(\lambda^2)}{I(\lambda) + I(\lambda^2)}\right], \tag{S.27}$$

where $[I(\lambda) + n(\lambda)I(\lambda^2)]$ represents the aggregate innovation arrival rate.

S.6 Numerical Analysis: Incumbent R&D

We now provide a quantitative illustration regarding the impact of forward patent protection when incumbent firms conduct R&D. We calibrate the model to the aggregate US economy following the same approach as our baseline numerical analysis. In addition to the two policy parameters $\{\theta_0, s_0\}$, the model now features nine structural parameters $\{\rho, n, \lambda, \alpha, \delta, \gamma, \beta, \phi, \kappa\}$. Following our baseline calibration, we externally set $\theta_0 = 1.08$, $s_0 = 0.15$, $\rho = 0.05$, $n = 0.01$ and $\alpha = 0.0075$. We set the degree of diminishing returns to incumbent R&D to $\beta = 0.35$ following Acemoglu and Akcigit (2012). The parameter $0 < \kappa < 1$ determines the depreciation rate of entrant R&D difficulty. However, its only actual impact on the model's equilibrium is to scale $D(t)/N(t)$ and η_x. In particular, $(n+\kappa)$ sets the upper bound of η_x as seen in (S.21). We set $\kappa = 0.85$ so that we are able to examine a case with a relatively high value of η_x. We note that the flow form of $D(t)$ used in the baseline model corresponds to full instantaneous depreciation.

We jointly calibrate the remaining four parameters λ, δ, γ and ϕ internally. As before, we target a 2% rate of economic growth and an expected patent lifespan of 11 years. Patents continue to remain profitable until two subsequent innovations occur. Since equation (S.27) shows that economic growth remains equal to $ln\lambda$ multiplied by the aggregate innovation arrival rate, the same value of $\lambda = 1.1163$ used in the baseline calibration ensures that an expected patent lifespan of 11 years corresponds to 2% growth. We again calibrate δ to match a target firm average for RPAs expenditure as a share of revenue. We separately consider the same four targets as in Section 5 in the main text, namely 0.0%, 0.5%, 1.5% and 2.5%. Note that all firms earn the same revenue of $cN(t)$ and pay the same wage w_x to specialized RPAs labor. This implies that average RPAs expenditure across one and two-step incumbents as a proportion of revenue is given by $w_x \bar{L}_x(t)/(cN(t))$. Finally, we calibrate the incumbent innovation difficulty parameter ϕ to match a target for a share of economic growth that is attributable to incumbent innovation. Empirical estimates generally indicate that most innovation comes from incumbent firms (Garcia-Macia *et al.*, 2019). However, our model specifically isolates incumbent innovation that is motivated by an ability to escape patent infringement and associated licensing fees. For the illustrative purpose of this exercise, we choose a target of 20%. In general, the main pattern of results that we highlight here are not sensitive to this choice of initial target. We summarize our calibration approach in Table S.2.

Table S.2: Incumbent R&D calibration summary

Parameter	**Description**	**Value**	**Source/Target**
External			
ρ	Discount factor	0.05	Standard
n	Population growth rate	0.01	US labor force growth, BLS
α	Proportion RPAs Labor	0.0075	Various, see fn. 14
β	Incumbent R&D dim. returns	0.35	Acemoglu and Akcigit (2012)
κ	Depreciation rate of $D(t)$	0.85	Imposed
s_0	Initial forward protection	0.15	Chu (2009)
θ_0	Initial backward protection	1.08	Basu (2019), see fn. 13
Internal			
δ	Innovation difficulty, RPAs	0.9908	RPAs exp. / revenue (1.5%)
γ	Innovation difficulty, overall	16.150	Economic growth (2%)
ϕ	Incumbent innovation difficulty	4.0000	Incumbent growth share (20%)
λ	Innovation size	1.1163	Patent lifespan (11 years)
$\eta_x(\alpha, \delta)$	Implied elasticity of I w.r.t. RPAs	0.3843	–

The results of our numerical analysis are displayed in Figure S.2. As discussed in the main text, our first primary result is that strengthened forward protection can still stimulate entrant innovation. Through the same cost-saving mechanism emphasized in the baseline model and illustrated in Panel (e), entrant innovation remains most likely to increase in s when initial demand for RPAs is high. The presence of incumbent innovation does introduce the innovation-reducing escape infringement effect of forward protection on entrant R&D incentives as described in the main text. This is most easily seen in the $\eta_x = 0.384$ case where average RPAs expenditure is equal to 1.5% of firm revenue. In our numerical analysis of this case in the baseline model without incumbent innovation, we found entrant innovation to increase monotonically with s. Here however, entrant innovation increases in s only when s is very low. As s increases, $I(\lambda^2)$

increases, the escape infringement effect becomes more important and entrant innovation begins to decrease in s. In the high η_x case where RPAs expenditure equals 2.5% of revenue, we continue to find that the cost-saving effect is sufficiently important such that entrant innovation increases monotonically in s.

Our second main result is that the potential for a differential impact of forward protection on entrant and incumbent innovation can produce an inverted U-shaped relationship between forward protection and economic growth. This is because incumbent innovation increases most rapidly in s when s is low. In cases where entrant innovation declines in s, the rapid increase in $I(\lambda^2)$ can still initially boost growth. When the decline in entrant innovation is sufficiently large at higher values of s, economic growth begins to decrease in s. This pattern occurs even in the $\eta_x = 0$ case, where the RPAs cost-saving mechanism is removed entirely. In fact, this result directly corresponds to Chu and Pan (2013), who find a similar inverted U-shaped relationship when firms have an incentive to pursue larger innovations in order to avoid licensing fees. In their model, all R&D is conducted by potential market entrants that endogenously select an innovation size. The inverted U-shaped relationship arises in their model due to the competing growth-reducing backloading effect of forward protection and growth-enhancing effect of incentivizing R&D firms to pursue larger innovations. Although incumbents pursue further innovation to escape licensing fees in our model, the same essential forces are at work. Our analysis shows that incumbents firms' use of RPAs introduces an additional growth-enhancing role for forward protection. When η_x is high, the cost-saving effect from RPAs can be sufficiently important such that entrant innovation, incumbent innovation and economic growth all monotonically increase in s.

Figure S.2: Numerical analysis: incumbent R&D

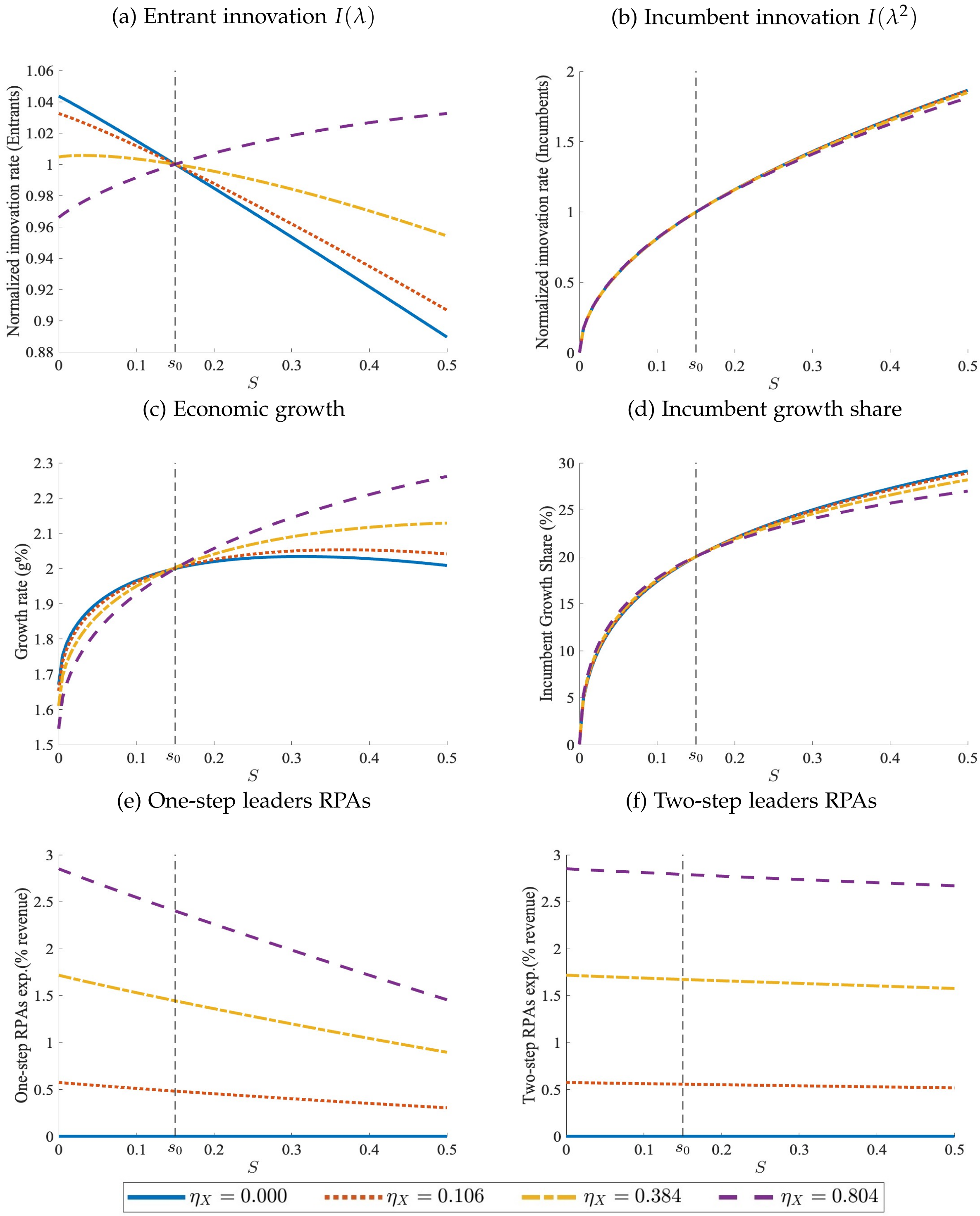

Figure S.2 displays the impact of forward protection in four initial equilibria corresponding to average RPAs expenditure equal to a [0%, 0.5%, 1.5%, 2.5%] share of revenue respectively. The associated parameter values are $\delta = [0.0, 0.9495, 0.9908, 0.9995]$, $\gamma = [0.335, 5.432, 16.15, 27.97]$, $\phi = [5.00, 4.67, 4.00, 3.30]$ and $\eta_x = [0.0, 0.106, 0.384, 0.804]$. All other parameters remain as reported in Table S.2.

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