Welfare Effects of Patent Protection in a Growth Model with R&D and Capital Accumulation

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Abstract

This study explores the welfare effects of patent protection in a Romer-type expanding variety model in which R&D and capital accumulation are both engines of growth. It shows that the comparison between the productivity of R&D and that of capital plays an important role in the welfare analysis. When the relative productivity of R&D compared to capital is high (low), social welfare takes an inverted-U shape for (is decreasing in) the strength of patent protection, and the welfare-maximizing degree of patent protection is no greater than (identical to) the growth-maximizing degree. Moreover, the model is calibrated to the US economy and the numerical results support these welfare implications.

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1 Introduction

In this study, we explore the effects of patent protection on economic growth and social welfare in a Romer-type expanding variety model. We consider research and development (R&D) and capital accumulation as non-complementary growth engines and allow the growth rates of these factors to be determined independently, given that both innovation growth and capital growth significantly contribute to output growth, as shown in growth accounting studies.[1] We find that the relative productivity of R&D compared to capital is particularly crucial for the effect of tightening intellectual property rights (IPR) protection on not only economic growth but also on social welfare. According to Park (2008), patent protection in many countries has strengthened since the agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), but patent protection indexes remain considerably lower than the upper bound; even high-income countries that are considered innovation-driven economies, such as the US, Japan, and Germany, do not pursue the strongest patent rights index.[2] Therefore, this study attempts to provide a new rationale from the viewpoint of welfare analysis to explain why countries may not prefer a high degree of patent protection.[3]

Although R&D and capital accumulation are both considered key driving forces of economic growth, Iwaisako and Futagami (2013) develop a theoretical framework to show that these two growth engines are affected very differently by a tightening of IPR that better protects innovations. Specifically, on the one hand, strengthening patent protection enables firms with market power to charge higher prices through enlarging markups. This increases the amount of profits generated by the invention and positively affects R&D activities, which stimulate economic growth. On the other hand, strengthening patent protection leads firms with market power to decrease their production volume because of larger markups. This reduces the demand for capital inputs used in manufacturing and negatively affects capital accumulation, which impedes economic growth. Summing up the two opposing effects, tightening patent rights lowers (raises) the rate of economic growth if the relative productivity of R&D compared to capital is low (high), that is, if the engine of innovation growth is less (more) efficient than the engine of capital growth. Accordingly, the lowest (highest) degree of patent protection tends to maximize economic growth in this framework.

In addition, strengthening patent protection has important implications for social welfare in this two-engine growth model. Stronger patent protection leads to a negative welfare effect stemming from a lower steady-state level of consumption, since the demand for capital inputs and, therefore, the amount of production declines. When the relative productivity of R&D is low, tightening IPR protection depresses economic growth, yielding a negative welfare effect that reinforces the negative welfare effect by decreased consumption. Thus, social welfare monotonically decreases in the strength of patent rights and is maximized by the same degree of growth-maximizing patent protection. By contrast, when the relative productivity of R&D is high, tightening IPR protection enhances economic growth, yielding a positive welfare effect. For low levels of IPR protection, this

[1] Barro and Sala-I-Martin (2004) (Table 10.1) illustrate that in OECD countries, the contribution of total factor productivity growth and that of capital growth as a fraction of GDP growth are similarly large.

[2] The measure of patent protection in Park (2008), known as the Ginarte-Park index, includes 122 countries and sets a scale from 0 to 5. The average scale of the Ginarte-Park index is 3.34 in 2005, showing that a number of countries, especially developing ones, do not have a considerably high degree of IPR protection.

[3] Iwaisako (2013) indicates that the degree of patent protection that maximizes social welfare is relatively weak in a country where public services are limited.

positive effect tends to overwhelm the negative welfare effect of lower consumption, which makes the welfare level positively correlated with the strength of patent rights. Nevertheless, for high levels of IPR protection, the former positive effect tends to be dominated by the latter negative welfare effect, which instead makes the welfare level negatively correlated with the strength of patent rights. Overall, social welfare exhibits an inverted-U shape in the degree of patent protection, and thus the welfare-maximizing degree cannot exceed the growth-maximizing degree in this case. Consequently, it seems that a patent policy that is growth-enhancing could worsen social welfare, and our analysis demonstrates that this may be true only for an economy with a more productive innovation-growth engine relative to the capital-growth engine (e.g., innovation-driven economies), provided that the initial enforcement of IPR is sufficiently strict; these economies will not tend to choose the strongest patent rights. To sum up, from the perspective of social welfare, the argument that "stronger is always better" for patent protection does not apply in either case of the relative R&D productivity.[4]

This study is closely related to the literature on dynamic general equilibrium (DGE) models that examine the effects of patent protection on economic growth and social welfare. The seminal work in this literature is Judd (1985), who reveals that infinite patent length maximizes social welfare. Subsequent studies apply variants of the endogenous growth model to show that strengthening patent protection in the form of different instruments could generate a negative or non-monotonic effect on economic growth and social welfare (see Goh and Olivier (2002) and Pan, Zhang, and Zou (2018) on patent breadth; Futagami and Iwaisako (2007) and Lin and Shampine (2018) on patent length; Furukawa (2007) on patent protection against imitation; O'Donoghue and Zweimüller (2004), Chu (2009), Chu, Cozzi, and Galli (2012), Cozzi and Galli (2014), and Yang (2018) on blocking patents). However, these studies rely on R&D-based growth models in which either capital accumulation is absent or capital accumulation and R&D complement one another (namely, the growth rate of capital is determined such that it is equal to the growth rate of innovations), and these settings differ from the empirical findings of the growth accounting studies. Therefore, the present analysis complements the above papers by analyzing the growth and welfare effects of IPR protection in a DGE model with two engines of growth, where capital growth and innovation growth are disproportionally determined, that is, they are negatively correlated.

Furthermore, this study contributes to a small but growing literature that explores the growth and welfare implications for various policy tools in a two-engine growth model. Iwaisako and Futagami (2013) present a two-engine growth model to show that the relationship between patent protection and economic growth is mixed (i.e., positive, negative, or non-monotonic), depending on the relative productivity of R&D compared to capital. In addition to the growth effects, it is important to consider the welfare effects of patent protection in such a framework. Thus, our study fills this gap by analytically and quantitatively investigating the welfare implications of patent protection according to the impacts on input allocations (in this study, labor allocations). Chen, Chu, and Lai (2015) examine the growth effects of subsidy policy in a similar two-engine growth model and show that subsidizing the R&D sector may be growth-retarding since it has opposing impacts on research and capital. Our study complements theirs by focusing on patent policy and including a welfare analysis. Recently, Chu, Lai, and Liao (2018) examine the growth and welfare

[4] This argument for patent protection does not apply from the economic-growth perspective either. See Thompson and Rushing (1996), Park (2005), Falvey, Foster, and Greenaway (2006), and Horii and Iwaisako (2007) for theoretical analysis and empirical evidence that present a mixed relationship between patent protection and economic growth.

effects of the interaction between monetary and patent policies in a growth model with R&D and capital accumulation. They show that when capital is the only input in intermediate-goods production, strengthening patent protection may lead to a non-monotonic effect on social welfare in the presence of spillovers across the R&D and capital-producing sectors. Nevertheless, the current study reveals that in the absence of cross-sector spillovers, when capital and labor are both inputs for producing intermediate goods, stricter patent rights can continue to yield a non-monotonic effect on social welfare and render the welfare-maximizing solution for patent protection different from (below) the growth-maximizing one.

The remainder of this study is organized as follows. Section 2 presents the expanding variety model with two growth engines. Moreover, this section investigates the welfare implications of patent protection by deriving and comparing the growth- and welfare-maximizing level of patent protection. Section 3 calibrates the model and conducts the numerical analysis. Section 4 considers two potential extensions of the model. Section 5 concludes.

2 The Model

To analyze the growth and welfare effects of patent protection in a DGE framework in which innovations and capital accumulation are both engines of growth, we follow Iwaisako and Futagami (2013) to adopt a Romer (1990) expanding variety model by incorporating a capital-producing sector in addition to an innovation-producing one. Furthermore, the level of patent breadth that affects the degree of firms' market power is influenced by the patent authority's policy, which reflects the strength of IPR protection.

2.1 Households

Suppose that the economy admits a unit continuum of identical households, and their utility function is given by

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt, \tag{1}$$

where $\rho > 0$ represents the discount rate, and C_t is the households' consumption of final goods at time t, whose price is normalized to unity. Assume that there is no population growth in the economy. Each household is endowed with one unit of time for labor.[5] Therefore, the value of households' total assets evolves according to

$$\dot{V}_t = R_t V_t + W_t - C_t, \tag{2}$$

[5]One can introduce the labor supply of households L_t by assuming a separable utility function $u(C_t, L_t) = \ln C_t + \theta \ln(1 - L_t)$, where the intertemporal elasticity of substitution for consumption equals that for leisure (i.e., $1 - L_t$). Nevertheless, the equilibrium labor supply L will be unaffected by the patent breadth μ in this setting, and therefore the qualitative result in the main text is robust to this alternative setting. See Subsection 4.1 for the detail.

where V_t is the real value of households' assets, W_t denotes the real wage rate, and R_t is the real interest rate. Then, the standard dynamic optimization implies the usual Euler equation

$$\frac{\dot{C}_t}{C_t} = R_t - \rho. \tag{3}$$

Moreover, the households own a balanced portfolio of all firms in the economy.

2.2 Final Goods

Following Vandenbussche, Aghion, and Meghir (2006), final goods Y_t are competitively produced by using a continuum of intermediate goods $X_t(j)$ for $j \in [0, N_t]$ and a fixed input factor S_t (this could indicate natural resources such as land) according to

$$Y_t = (S_t)^{1-\alpha} \int_0^{N_t} [X_t(j)]^\alpha \, dj, \tag{4}$$

where N_t is the number of varieties for intermediate goods. This production function is in line with the assumptions of constant returns to scale to inputs at the firm level and of constant returns to scale to the reproducible factor N_t, so unbounded growth on Y_t is possible. Notice that since S_t is assumed to be fixed, S_t equals its initial level S_0 and is constant over time. Because of free entry into the final-goods sector, the conditional demand function for intermediate goods is given by

$$P_t(j) = \alpha \left[S_0/X_t(j)\right]^{1-\alpha}, \tag{5}$$

where $P_t(j)$ is the price of $X_t(j)$ relative to the final goods.

2.3 Intermediate Goods

In each variety $j \in [0, N_t]$, intermediate goods are manufactured by a monopolist who holds a patent on the invention of this variety. This monopolist rents capital and hires labor for production according to a standard Cobb-Douglas function given by

$$X_t(j) = A \left[K_t(j)\right]^\gamma \left[L_{x,t}(j)\right]^{1-\gamma}, \tag{6}$$

where $K_t(j)$ is the amount of capital inputs, $L_{x,t}(j)$ is the employment level of production labor, and γ is the share parameter of capital. Then, by applying cost minimization to (6), the marginal cost of producing intermediate goods for the monopolist for variety j is

$$MC_t(j) = \frac{1}{A} \left(\frac{Q_t}{\gamma}\right)^\gamma \left(\frac{W_t}{1-\gamma}\right)^{1-\gamma}, \tag{7}$$

where Q_t denotes the rental price of capital.

To consider the degree of patent protection, the monopolist is allowed to charge a markup over the marginal production cost for profit maximization. Following previous studies such as Li (2001), Goh and Olivier (2002), and Iwaisako and Futagami (2013), we assume that markup $\mu_t \in (1, 1/\alpha]$

is represented by the strength of patent breadth, which is a policy instrument that can be set by the patent authority.[6] The upper bound of μ_t is the unconstrained markup value in the existing literature (e.g., Barro and Sala-I-Martin (2004)). Therefore, the profit-maximizing price is given by

$$P_t(j) = \mu_t MC_t(j). \tag{8}$$

Given $MC_t(j)$ and the pricing strategy, the monopolist chooses $K_t(j)$ and $L_{x,t}(j)$ to maximize her profit subject to (5) and (6). Hence, the monopolist's profit is

$$\Pi_{x,t}(j) = \left(\frac{\mu_t - 1}{\mu_t}\right) P_t(j)X_t(j). \tag{9}$$

The factor payments for labor and capital inputs employed in the intermediate-goods production are given by

$$W_t L_{x,t}(j) = \left(\frac{1-\gamma}{\mu_t}\right) P_t(j)X_t(j), \tag{10}$$

$$Q_t K_t(j) = \left(\frac{\gamma}{\mu_t}\right) P_t(j)X_t(j). \tag{11}$$

These equations are conditions that determine the input allocations in this sector.

2.4 Inventions and R&D

The value of invented variety j is denoted as $V_{n,t}(j)$. Following the standard literature (e.g., Cozzi, Giordani, and Zamparelli (2007)), we focus on the symmetric equilibrium such that $\Pi_{x,t}(j) = \Pi_{x,t}$ and $V_{n,t}(j) = V_{n,t}$. Then, the familiar no-arbitrage condition for the asset value is

$$R_t V_{n,t} = \Pi_{x,t} + \dot{V}_{n,t}, \tag{12}$$

which implies that the return on this asset $R_t V_{n,t}$ equals the sum of flow profits as a monopolist $\Pi_{x,t}$ and the potential capital gain $\dot{V}_{n,t}$.

New innovations for each variety are invented by a unit continuum of R&D firms indexed by $\iota \in [0,1]$. Each of these firms employs R&D labor $L_{r,t}(\iota)$ to produce inventions. The expected profit of the ι-th R&D firm is

$$\Pi_{r,t}(\iota) = V_{n,t}\dot{N}_t(\iota) - W_t L_{r,t}(\iota), \tag{13}$$

where $\dot{N}_t(\iota) = \varphi N_t L_{r,t}(\iota)$ is the number of inventions created by firm ι, depending on the existing number of varieties. φ is R&D productivity at time t. In equilibrium, the number of inventions occurring at the aggregate level equals the counterpart at the firm level for each variety, namely, $\dot{N}_t = \dot{N}_t(\iota)$. Then, free entry into the R&D sector implies the following zero-expected-profit

[6] This setting follows the endogenous R&D-based growth literature to assume that the monopolist with the invention for a new intermediate good receives a perpetual patent on this intermediate variety, namely, patent length is infinite. Nevertheless, a recent study by Lin and Shampine (2018) allows patent length to be finite in a scale-free R&D-based growth model and shows that the welfare loss of switching from the baseline 20-year patent length to the long-run optimal patent length is small if transitional impacts are taken into account.

condition:

$$\varphi N_t V_{n,t} = W_t. \tag{14}$$

This equation is the condition that determines the labor allocation for R&D.

2.5 Capital Production

The value of one unit of capital in variety j is denoted as $V_{k,t}(j)$. Similarly, symmetry across varieties implies $V_{k,t}(j) = V_{k,t}$. Then, the no-arbitrage condition for the capital asset is

$$R_t V_{k,t} = Q_t + \dot{V}_{k,t}. \tag{15}$$

Here as well, this equation implies that the return on asset $R_t V_{k,t}$ equals the sum of the rental price of capital Q_t and capital gain $\dot{V}_{k,t}$.

Capital goods for each variety are produced by a unit continuum of capital-producing firms indexed $\nu \in [0,1]$. Each of these firms employs capital-producing labor $L_{k,t}(\nu)$ for the production. The expected profit of the ν-th capital-producing firm is

$$\Pi_{k,t}(\nu) = V_{k,t}\dot{K}_t(\nu) - W_t L_{k,t}(\nu), \tag{16}$$

where $\dot{K}_t(\nu) = \phi A_{k,t} L_{k,t}(\nu)$ is the amount of capital goods produced by firm ν. $\phi A_{k,t}$ denotes the effectiveness of capital production at time t. Following Romer (1986), Iwaisako and Futagami (2013), and Chu, Lai, and Liao (2018), it is assumed that $A_{k,t} = K_t$; this effectiveness increases in the accumulated capital stock to capture the usual capital externality as in the AK model, enabling sustainable growth for physical capital.[7] In equilibrium, the amount of capital goods created at the aggregate level equals the counterpart at the firm level for each variety, namely, $\dot{K}_t = \dot{K}_t(\nu)$. Then, free entry into the capital-producing sector implies the following zero-expected-profit condition:

$$\phi K_t V_{k,t} = W_t. \tag{17}$$

This equation is the condition that determines the labor allocation for capital accumulation.

2.6 Decentralized Equilibrium

An equilibrium consists of a sequence of allocations $[C_t, Y_t, X_t(j), K_t(j), L_{x,t}, L_{r,t}, L_{k,t}]_{t=0}^{\infty}$, a sequence of prices $[P_t(j), R_t, W_t, Q_t, V_{n,t}, V_{k,t}]_{t=0}^{\infty}$, and a sequence of policies $[\mu_t]_{t=0}^{\infty}$. Moreover, in each instant of time,

- households choose $[C_t]$ to maximize their utility taking $[R_t, W_t]$ as given;
- final-goods firms produce $[Y_t]$ and choose $[X_t(j)]$ to maximize profits taking $[W_t, P_t(j)]$ as given;
- the intermediate-goods monopolist in industry $j \in [0, N_t]$ produces $[X_t(j)]$ and chooses $[K_t(j), L_{x,t}(j)]$ to maximize profits taking $[Q_t, W_t]$ as given;

[7] As will be shown, capital accumulation and variety expansion serve as non-complementary engines of growth in this model, because the growth of physical capital is pinned down independently of the growth of the number of varieties.

- R&D firms choose $[L_{r,t}]$ to maximize profits taking $[W_t, V_{n,t}]$ as given;
- capital-producing firms choose $[L_{k,t}]$ to maximize profits taking $[W_t, V_{k,t}]$ as given;
- the final-goods market clears such that $C_t = Y_t$;
- the labor market clears such that $L_{x,t} + L_{r,t} + L_{k,t} = 1$, where $L_{x,t} = \int_0^{N_t} L_{x,t}(j)dj$;
- the capital-goods market clears such that $K_t = \int_0^{N_t} K_t(j)dj$; and
- the values of intangible and tangible assets add up to households' assets value such that $V_{n,t}N_t + V_{k,t}K_t = V_t$.

In this subsection, we derive the equilibrium labor allocations. First, we obtain the following result that characterizes the dynamics of the model.

Lemma 1. *Holding μ constant, the economy immediately jumps to a unique and stable balanced growth path.*

Proof. See Appendix A. □

Lemma 1 implies that there is no transitional dynamics in this model; the economy jumps from one steady state to another when the policy instrument μ changes from one stationary value to another. Given a stationary path of μ_t, the equilibrium labor allocations are stationary. Define the transformed variables $\Psi_{n,t} \equiv Y_t/(V_{n,t}N_t)$ and $\Psi_{k,t} \equiv Y_t/(V_{k,t}K_t)$. Then, imposing balanced growth on (A.4) and (A.5) in Appendix A yields

$$L_{r,t} = \frac{\alpha}{\varphi}\left(\frac{\mu - 1}{\mu}\right)\Psi_{n,t} - \frac{\rho}{\varphi}, \tag{18}$$

$$L_{k,t} = \frac{\alpha}{\phi}\left(\frac{\gamma}{\mu}\right)\Psi_{k,t} - \frac{\rho}{\phi}, \tag{19}$$

which are the two equations that solve for $\{L_x, L_r, L_k\}$ given the steady-state value of Ψ_n in (A.12) and that of Ψ_k in (A.8). The last equation is simply the labor-market-clearing condition such that

$$L_{x,t} + L_{r,t} + L_{k,t} = 1. \tag{20}$$

Solving (18)-(20) with Ψ_n and Ψ_k, the equilibrium labor allocations are given by

$$L_x = \frac{1-\gamma}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right), \tag{21}$$

$$L_r = \frac{\mu - 1}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) - \frac{\rho}{\varphi}, \tag{22}$$

$$L_k = \frac{\gamma}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) - \frac{\rho}{\phi}. \tag{23}$$

The allocations in (22) and (23) show that R&D labor L_r is increasing in patent breadth μ, whereas capital-producing labor L_k is decreasing in it.[8] These opposite effects of μ on L_r and L_k are based on our setting that follows Iwaisako and Futagami (2013), as discussed in the Introduction.

[8]The parameter space is restricted to ensure that L_r and L_k are bounded between 0 and 1.

2.7 Growth-Maximizing Patent Breadth

This subsection considers the growth effects of patent breadth. Using symmetry across varieties in (4) yields $Y_t = (S_t)^{\alpha}(N_t)^{1-\alpha}(X_t)^{1-\alpha}$. Substituting (6) into this equation and differentiating with respect to t yields

$$g_y \equiv \frac{\dot{Y}_t}{Y_t} = (1-\alpha)\frac{\dot{N}_t}{N_t} + \alpha\gamma\frac{\dot{K}_t}{K_t} = (1-\alpha)\varphi L_r + \alpha\gamma\phi L_k, \tag{24}$$

where we use the fact that S_t is constant over time. In (24), the growth rate of the number of varieties $\dot{N}_t/N_t = \varphi L_r$ and that of physical capital $\dot{K}_t/K_t = \phi L_k$ are differently determined. Substituting (22) and (23) into (24) yields $g_y = (1+\rho/\varphi+\rho/\phi)[(1-\alpha)\varphi(1-1/\mu)+\alpha\gamma^2\phi/\mu]$. Taking the derivative of g_y with respect to μ implies that if the relative productivity of R&D compared to capital is high (low), namely, $\varphi/\phi > (<)\alpha\gamma^2/(1-\alpha)$, then g_y is positively (negatively) correlated with μ. This result is consistent with the existing empirical evidence that documents a mixed relationship between patent protection and economic growth.[9]

Lemma 2. *A larger patent breadth increases R&D but decreases capital production. Strengthening patent protection is growth-enhancing (retarding) for a high (low) relative R&D productivity, i.e.,* $\varphi/\phi > (<)\alpha\gamma^2/(1-\alpha)$.

Proof. Proven in the text. □

Given that the range of patent breadth is $\mu \in (1, 1/\alpha]$, it is straightforward to derive the level of patent protection that maximizes the growth rate of outputs, denoted by μ_g, as follows.

Proposition 1. *The growth-maximizing degree of patent protection is given by (i)* $\mu_g = (1+\rho/\varphi+\rho/\phi)/(1+\rho/\phi)$ *if* $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$ *and (ii)* $\mu_g = 1/\alpha$ *if* $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$.

Proof. When $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, g_y is monotonically decreasing in μ. Moreover, an increase in μ increases L_r but decreases L_x and L_k. Thus, the lowest level of μ that is feasible to attain is to make $L_r \to 0$. Then, setting (22) to zero yields the result in (i).

When $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$, g_y is monotonically increasing in μ. Thus, the highest level of μ that is feasible to attain is to set μ at its upper bound $1/\alpha$, yielding the result in (ii). □

In other words, when the relative productivity of R&D is high (low), the economy allocates more labor to the R&D- (capital-)producing sector to maximize economic growth by strengthening (weakening) the level of patent breadth as much as possible.

2.8 Welfare-Maximizing Patent Breadth

This section considers the welfare effects of patent breadth. In addition, we compare growth- and welfare-maximizing patent breadth and show the condition under which the former differs from the latter.

[9] It is obvious that when $\varphi/\phi = \alpha\gamma^2/(1-\alpha)$, adjusting patent breadth has no impact on the growth of outputs.

Given the balanced-growth behavior of the economy, the households' lifetime utility (1) can be reexpressed by

$$U = \frac{1}{\rho}\left(\ln C_0 + \frac{g_c}{\rho}\right), \tag{25}$$

where $g_c = g_y$ (the growth rate of consumption equals that of outputs) and $C_0 = Y_0 = S_0^{1-\alpha} N_0^{1-\alpha} X_0^{\alpha} = S_0^{1-\alpha} N_0^{1-\alpha}\left(AK_0^{\gamma} L_x^{1-\gamma}\right)^{\alpha}$ by using (4) and (6). Dropping the exogenous terms and substituting the growth rate of outputs in (24) yield

$$U = \frac{\alpha(1-\gamma)}{\rho}\ln L_x + \frac{1}{\rho^2}\left[(1-\alpha)\varphi L_r + \alpha\gamma\phi L_k\right], \tag{26}$$

where the labor allocations follow (21)-(23). As Lemma 1 shows, this model does not feature transitional dynamics, and thus the effects of altering the level of patent protection on welfare only apply to those on the steady-state welfare (i.e., long-run welfare). Then, taking the derivative of (26) with respect to μ yields

$$\frac{\partial U}{\partial \mu} = -\frac{\alpha(1-\gamma)}{\rho\mu} + \frac{1}{\rho^2\mu^2}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right)\left[(1-\alpha)\varphi - \alpha\gamma^2\phi\right]. \tag{27}$$

Therefore, it can be seen that the sign of $\partial U/\partial\mu$ is determined by that of $(1-\alpha)\varphi - \alpha\gamma^2\phi$. There are two cases to be considered on the basis of the relative productivity of R&D compared to that of capital, i.e., φ/ϕ. If $\varphi/\phi \le \alpha\gamma^2/(1-\alpha)$, we then obtain $\partial U/\partial\mu < 0$, implying that social welfare is monotonically decreasing in the level of patent breadth. By contrast, if $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$, then whether a strengthening of patent protection increases or decreases social welfare depends on the level of patent breadth. In particular, the threshold value

$$\tilde{\mu} = \frac{1}{\rho(1-\gamma)}\left(1 + \frac{\rho}{\phi} + \frac{\rho}{\varphi}\right)\left[\varphi\left(\frac{1-\alpha}{\alpha}\right) - \gamma^2\phi\right] \tag{28}$$

exists such that the welfare level rises (declines) as patent protection becomes stricter if $\mu < (>)\tilde{\mu}$. Therefore, it is straightforward to obtain the following result.

Lemma 3. *Social welfare (i) monotonically decreases in the degree of patent protection for* $\varphi/\phi \le \alpha\gamma^2/(1-\alpha)$ *and (ii) has an inverted-U shape for the degree of patent protection for* $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$.

Proof. Proven in the text. □

Notice that the critical value of relative R&D productivity that pins down the welfare effect of patent protection is identical to the one that pins down the growth effect as shown in Lemma 1 (namely, $\alpha\gamma^2/(1-\alpha)$). Consequently, Lemma 3 reveals that when the relative R&D productivity is low, strengthening patent protection impedes not only economic growth but also social welfare. A larger patent breadth always decreases production labor L_x, leading to a negative welfare effect (i.e., the first term on the RHS of (27)) because the initial (steady-state) level of consumption C_0 decreases. When $\varphi/\phi \le \alpha\gamma^2/(1-\alpha)$, the welfare effect of broadening patent breadth through economic growth (i.e., the second term on the RHS of (27)) also becomes negative since the negative impact on depressing capital accumulation (i.e., capital labor L_k) dominates the positive counterpart on enhancing innovations (i.e., R&D labor L_r). The above two welfare effects reinforce each other to make U decrease in μ; U is maximized at the lowest possible level of μ. This result may be able to partially explain why certain developing countries in which capital accumulation is a more effective engine of growth do not pursue a high degree of patent protection.

On the other hand, when the relative productivity of R&D is high (recall $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$), the positive impact of broadening patent breadth on L_r overwhelms the negative counterpart on L_k, which yields a positive welfare effect through economic growth. In such a case, if patent breadth is initially narrow (i.e., $\mu < \tilde{\mu}$), then this positive welfare effect by economic growth strictly dominates the negative welfare effect of smaller consumption by the decrease in L_x, and therefore strengthening patent protection simultaneously stimulates economic growth and increases social welfare. Alternatively, if patent breadth is initially broad (i.e., $\mu > \tilde{\mu}$), then the positive welfare effect by economic growth is dominated by the negative welfare effect by the decrease in L_x, and thus strengthening patent protection continues to promote economic growth but tends to diminish social welfare. In other words, U is maximized at the level of $\tilde{\mu}$. This result is consistent with the argument that strengthening patent protection to the highest level may not be optimal, which is the current situation in many developed economies.

Hence, the above analysis implies that extending patent breadth is not always beneficial in terms of promoting economic growth and raising social welfare, because the relative efficiency of the two growth engines determines which growth engine plays a more crucial role in the growth and welfare implications of implementing patent policy. Notice that the inverted-U shape of social welfare on patent protection only applies when $\tilde{\mu} < 1/\alpha$. If $\tilde{\mu} \geq 1/\alpha$, there does not exist a level of patent breadth that satisfies $\mu > \tilde{\mu}$ since μ cannot exceed $1/\alpha$. In this case, only the increasing part of the hump shape is valid in broadening patent breadth, implying that the positive welfare effect by economic growth strictly dominates the negative welfare effect by the decline in L_x for all possible ranges of μ. Consequently, social welfare becomes monotonically increasing in patent protection as in the case of economic growth.

In addition, the previous positive analysis for the welfare effect of patent breadth yields the design for the level of patent protection that maximizes social welfare, denoted by μ_w, which can be summarized by the following result.

Proposition 2. *The welfare-maximizing degree of patent protection is given by (i)* $\mu_w = (1+\rho/\varphi+\rho/\phi)/(1+\rho/\phi) = \mu_g$ *if* $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, *and (ii)* $\mu_w = \tilde{\mu} \leq \mu_g$ *if* $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$.

Proof. When $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, U is monotonically decreasing in μ, so the derivation of μ_w is the same as that of μ_g; setting (22) to zero yields the result in (i).

When $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$, U has a hump shape on μ. Thus, U is maximized at $\mu = \tilde{\mu}$. Also, denote $\mu_m \equiv 1/\alpha = \mu_g$. Then, using $\tilde{\mu}$ in (28) and μ_m, (27) can be rewritten as

$$\frac{\partial U}{\partial \mu} = -\frac{1-\gamma}{\rho\mu^2}\left(\alpha\mu - \frac{\tilde{\mu}}{\mu_m}\right). \tag{29}$$

Given that obtaining $\tilde{\mu}$ in (27) (namely, μ_w) requires $\partial U/\partial\mu = 0$, it is easy to show that $\mu_w/\mu_g = \tilde{\mu}/\mu_m = \alpha\mu \leq 1$ since $\mu \leq 1/\alpha$. This yields the result in (ii). □

Table 1: Calibrated parameter values.

ρ	μ	γ	φ	ϕ	α
0.02	1.3	0.3333	0.0708	0.1386	0.7142

3 Numerical Analysis

In this section, we calibrate the model to the US economy to numerically evaluate the growth and welfare effects of tightening patent protection in this two-engine growth model. This analysis also enables us to compare the growth-maximizing level of patent breadth and the welfare-maximizing level, which verifies the analytical results.

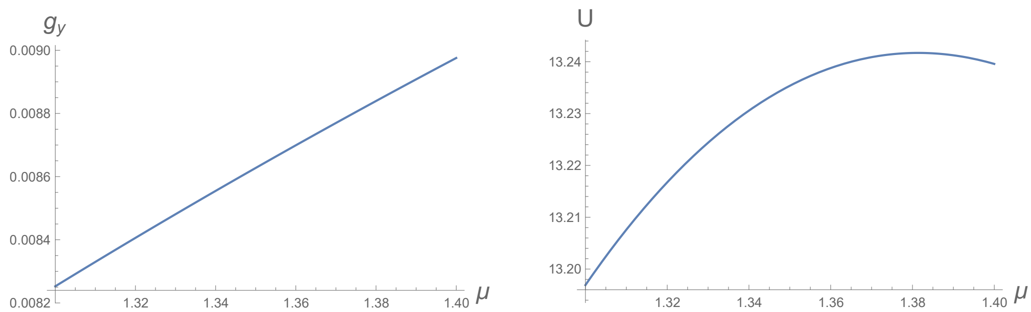
To perform this numerical exercise, we assign steady-state values to the following structural parameters $\{\rho, \mu, \gamma, \alpha, \varphi, \phi\}$. As for the discount rate ρ, we follow Grossmann, Steger, and Trimborn (2013) to set it to 0.02. As for patent breadth μ, we focus on values ranging from 1.3 to 1.4, where the former is consistent with the empirical estimate of the average markup reported in Norrbin (1993) and the latter is consistent with the upper bound of the markup reported in Jones and Williams (2000). We use $\mu = 1.3$ as the market level and gradually increase the magnitude of μ to strengthen the degree of patent protection. As for the factor share of capital γ, we set it to the conventional value of 1/3. As for the factor share of intermediate goods α, it is set to the value that allows the whole range of μ to satisfy the restriction such that $\alpha \leq 1/\mu$. To calibrate the remaining parameters, we follow Chu, Lai, and Liao (2018) to use the growth rate of capital g_k in the US from 1999 to 2010 (i.e., 3.07%). Furthermore, Comin (2004) shows that the contribution of R&D investment drives only a fraction of the long-run economic growth in the US. Hence, we follow Chu (2010) to set the ratio of the growth rate of innovations g_n to the output growth rate g_y to 0.4. The above moments, therefore, yield the calibrated values of R&D productivity φ and capital productivity ϕ, as shown in Table 1.

3.1 Results

In this calibrated economy, as the degree of patent protection μ rises from 1.3 to 1.4, R&D labor L_r increases from 0.0467 to 0.1251, while production labor L_x and capital labor L_k decrease from 0.7318 to 0.6795 and from 0.2252 to 0.1954, respectively. Consequently, the growth rate of innovations g_n increases from 0.3301% to 0.8849%, whereas the capital growth rate g_k decreases from 3.07% to 2.7079%. These changes are the implications of labor reallocations as shown in (21)-(23).

More importantly, the combination of the calibrated values in Table 1 implies that the relative productivity of R&D compared to capital (i.e., φ/ϕ) in the benchmark is above the threshold value that pins down the choice of growth- and welfare-maximizing patent breadth as specified in Propositions 1 and 2 (i.e., $\alpha\gamma^2/(1-\alpha) = 0.2778$). In this case, the output growth rate g_y is monotonically increasing in μ from 0.8253% at $\mu = 1.3$ to 0.8976% at $\mu = 1.4$, implying that the positive R&D channel for economic growth overwhelms the negative capital channel. Hence, the growth-maximizing patent breadth is obtained by the corner solution such that $\mu_g = 1/\alpha = 1.4$.

Furthermore, social welfare U has an inverted-U shape for μ, which first increases from 13.1969 at $\mu = 1.3$ to 13.2417 at $\mu = 1.3814$ and then decreases to 13.2396 at $\mu = 1.4$. This result implies that given a high level of efficiency in R&D relative to capital accumulation, recalling (26) shows that at the low levels of μ, the positive welfare effect through economic growth is sufficient to dominate the negative welfare effect through smaller consumption (due to the decline in manufacturing labor). Nevertheless, as μ increases, this domination gradually weakens, and finally the negative welfare effect dominates the positive one. Hence, the welfare-maximizing patent breadth is given by an interior solution such that $\mu_w = \tilde{\mu} = 1.3814$. This implies that in patent-protected and R&D-intensive industries in the US, the degree of patent protection that makes the resulting markup rate and profit share of firms equal approximately 38% maximizes welfare. In addition, it is obvious to see that $u_w < u_g$, which is in line with the analysis in Proposition 2. The above results are displayed in Figure 1, and the magnitudes of labor allocations, growth rates, and welfare levels for particular values of μ are summarized in Table 2.



(a) Economic Growth (b) Social Welfare

Figure 1: The growth and welfare effects of a strengthening of patent protection: benchmark

Table 2: Growth and welfare effects of patent protection: benchmark.

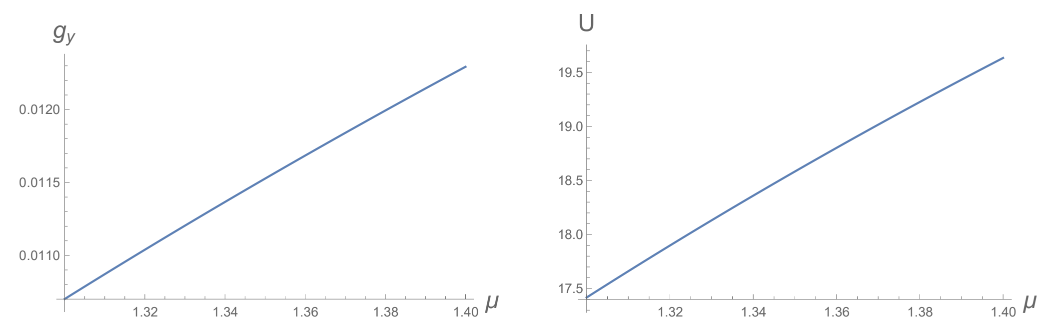
$\varphi/\phi = 0.5107$						
μ	1.3	1.32	1.34	1.36	1.38	1.4
L_x	0.7318	0.7207	0.7099	0.6995	0.6894	0.6795
L_r	0.0467	0.0633	0.0794	0.0951	0.1103	0.1251
L_k	0.2252	0.2160	0.2106	0.2054	0.2003	0.1954
g_n	0.3301%	0.4478%	0.5620%	0.6728%	0.7804%	0.8849%
g_k	3.0700%	2.9932%	2.9187%	2.8463%	2.7761%	2.7079%
g_y	0.8253%	0.8406%	0.8555%	0.8699%	0.8839%	0.8976%
U	13.1969	13.2167	13.2306	13.2388	13.2417	13.2396

In addition, it can be shown that the welfare-maximizing level of patent protection μ_w leads to a socially suboptimal outcome that results in a welfare loss. This is because given the steady-state level of relative productivity of R&D as calibrated, from the social point of view, the first-best outcome requires shutting down the weaker growth engine (namely, R&D) and only using the stronger growth engine (namely, capital accumulation) in the economy.[10] [11] However, the two growth engines are simultaneously used under μ_w as displayed in Table 2. Hence, μ_w does not achieve the first-best optimal allocations.

3.2 Sensitivity Checks: Relative Productivity

In this subsection, we conduct two sensitivity checks on our numerical exercise to examine the extent to which the quantitative results would change under a different level of relative productivity of R&D compared to capital.

First, we consider an alternative value of $\varphi/\phi = 0.8334$ by raising the value of φ while maintaining that of ϕ, such that the value of φ/ϕ is greater than the threshold value $\alpha\gamma^2/(1-\alpha)$, which is similar to the previous subsection. Figure 2 and Table 3 summarize the changes in the growth and welfare implications of tightening patent protection in this case. Under this value of φ/ϕ, both the growth rate of outputs g_y and the level of welfare U monotonically increase in patent protection μ. Accordingly, growth- and welfare-maximizing patent breadth coincide with each other, and are represented by the broadest patent breadth $\mu_g = \mu_w = 1.4$. This result is still consistent with the implication of Proposition 2 (i.e., $\mu_w \leq \mu_g$).[12]



(a) Economic Growth (b) Social Welfare

Figure 2: Growth and welfare effects of strengthening patent protection: high φ/ϕ

Furthermore, the steady-state value φ/ϕ in the previous exercises is calibrated to be higher than the threshold value $\alpha\gamma^2/(1-\alpha)$, which implies that strengthening patent protection is always growth-enhancing but has an inverted-U or a positive effect on social welfare. Here, we consider an alternative value of $\varphi/\phi = 0.2776$ by lowering the value of φ while keeping the value of ϕ unchanged, such that this value is below the aforementioned threshold value. Figure 3 and Table 4 summarize the variations in the impacts of patent protection on economic growth and social

[10] Under this calibration, the first-best optimal labor allocations denoted by the superscript $*$ are given by $L_x^* = 0.2887$, $L_r^* = 0$, and $L_k^* = 0.7113$, yielding an innovation growth rate of $g_n^* = 0\%$, a capital growth rate of $g_k^* = 9.8564\%$, an output growth rate of $g_y^* = 2.3468\%$, and a welfare level of 29.0870.

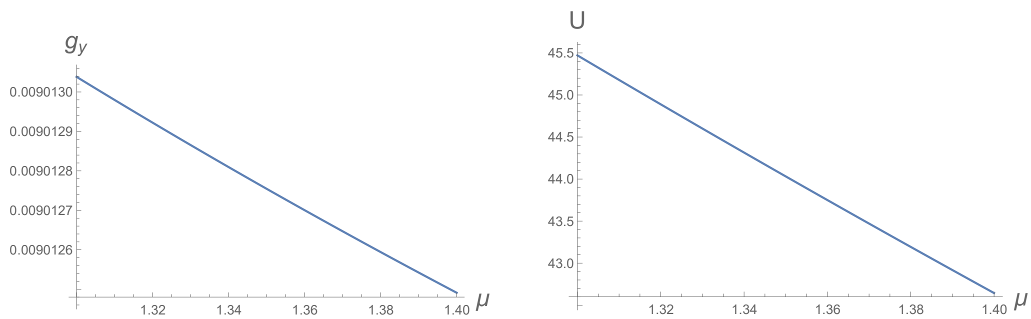
[11] See the Online Appendix for the derivation of the first-best allocations. The computation for the numerical result can be seen in the complementary *Mathematica* file, which is available upon request.

[12] Under $\varphi/\phi = 0.8334$ and $\phi = 0.1386$, the welfare-maximizing patent breadth implied by Proposition 2 is given by $\tilde{\mu}$ in (28) (i.e., 3.0431), which exceeds the upper bound of the calibrated value of μ. Thus, a corner solution continues to be used for μ_w, equaling the growth-maximizing patent breadth μ_g.

Table 3: Growth and welfare effects of patent protection: high relative productivity of R&D.

$\varphi/\phi = 0.8334$						
μ	1.3	1.32	1.34	1.36	1.38	1.4
L_x	0.6757	0.6554	0.6555	0.6458	0.6365	0.6274
L_r	0.1309	0.1462	0.1611	0.1756	0.1896	0.2032
L_k	0.1935	0.1884	0.1834	0.1786	0.1739	0.1694
g_n	1.5111%	1.6884%	1.8605%	2.0274%	2.1896%	2.3471%
g_k	2.6811%	2.6102%	2.5414%	2.4746%	2.4097%	2.3467%
g_y	1.0701%	1.1039%	1.1367%	1.1685%	1.1993%	1.2293%
U	17.4174	17.8983	18.3595	18.8019	19.2264	19.6339

welfare in this case.[13] Under this value of φ/ϕ, both the growth rate of outputs g_y and the level of welfare U become monotonically decreasing in patent protection μ. Accordingly, the growth- and welfare-maximizing levels of patent breadth coincide with each other; both are represented by the narrowest patent breadth such that $\mu_g = \mu_w = 1.3$, which again supports the implication of Proposition 2 (i.e., $\mu_w = \mu_g$).[14]



(a) Economic Growth (b) Social Welfare

Figure 3: Growth and welfare effects of strengthening patent protection: low φ/ϕ

Finally, when φ/ϕ is high (i.e., 0.8334), the welfare-maximizing patent breadth $\mu_w = 1.4$ cannot attain the first-best outcome (see Footnote 11 and Table 3), since the optimum now tends to shut down the socially weak growth engine of capital accumulation and instead relies on only the socially strong growth engine of R&D, which is obviously not the case with $\mu_w = 1.4$ under this high value of φ/ϕ.[15] Moreover, when the relative productivity of R&D φ/ϕ is low (i.e., 0.2776), the first-best

[13] In this sensitivity check for a low φ/ϕ, to ensure that the level of R&D labor L_r is strictly positive, the value of the discount rate ρ is reduced to 0.0125, which is close to the calibrated value in Chu, Lai, and Liao (2018) (i.e., 0.0132).

[14] In fact, under $\varphi/\phi = 0.2776$ and $\phi = 0.1386$, the growth- and welfare-maximizing levels of patent breadth implied by Propositions 1 and 2 are given by $(1 + \rho/\varphi + \rho/\phi)/(1 + \rho) = 1.1514$, which is smaller than the lower bound of the calibrated value of μ. Thus, the corner solution that $\mu_g = \mu_w = 1.3$ is used in this case.

[15] In the case of a high φ/ϕ, the first-best labor allocations are given by $L_x^* = 0.2887$, $L_r^* = 0.7113$, and $L_k^* = 0$, yielding an innovation growth rate of $g_n^* = 8.2146\%$, a capital growth rate of $g_k^* = 0\%$, an output growth rate of $g_y^* = 2.3470\%$, and a welfare level of 29.0917.

Table 4: Growth and welfare effects of patent protection: low relative productivity of R&D.

$\varphi/\phi = 0.2776$						
μ	1.3	1.32	1.34	1.36	1.38	1.4
L_x	0.7257	0.7147	0.7041	0.6937	0.6837	0.6739
L_r	0.0016	0.0181	0.0341	0.0496	0.0647	0.0794
L_k	0.2727	0.2672	0.2618	0.2566	0.2516	0.2467
g_n	0.0062%	0.0696%	0.1312%	0.1909%	0.2489%	0.3053%
g_k	3.7780%	3.7019%	3.6279%	3.5562%	3.4866%	3.4189%
g_y	0.9013%	0.9013%	0.9013%	0.9013%	0.9013%	0.9012%
U	45.4711	44.8887	44.3151	43.7500	43.1932	42.6444

allocations that correspond to this case exhibit an analogous pattern to those for the benchmark case, such that the growth engine of R&D should be abandoned in the social optimum. Therefore, the welfare-maximizing level of patent protection $\mu_w = 1.3$, as shown in Table 4, does not achieve the first-best allocations either.[16]

3.3 Capital as an Input for Reproducing New Capital

In this subsection, we consider a general case in which capital is used as an input in addition to labor to produce new capital. We examine the underlying effects of patent protection on economic growth and social welfare and contrast these effects with those in the benchmark case.

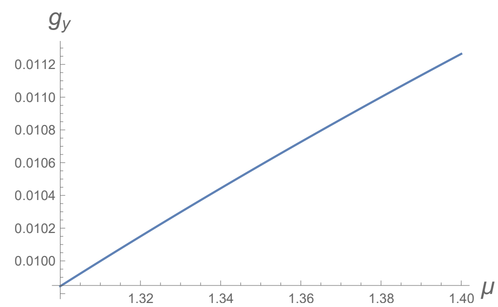
We follow Iwaisako and Futagami (2013) to modify the production function for capital to $\dot{K}_t = \phi(K_{k,t})^{\delta}(K_t L_{k,t})^{1-\delta}$, where $K_{k,t}$ is the amount of capital that is used for reproducing new capital and δ is the parameter for the capital share. Therefore, the capital-goods-market-clearing condition becomes $\int_0^{N_t} K_t(j)dj + K_{k,t} = K_t$. Moreover, we replace the Cobb-Douglas production function of final goods in (4) by the CES production function given by $Y_t = \{\int_0^{N_t}[X_t(j)]^{\varepsilon}dj\}^{1/\varepsilon}$, where the parameter $\epsilon \in (0,1)$ determines the elasticity of substitution between intermediate goods (i.e., $\epsilon \equiv 1/(1-\varepsilon)$). Thus, the growth rate of capital is given by $g_k = \delta\phi(1-\delta)^{(1-\delta)/\delta}(\phi/\varphi)^{(1-\delta)/\delta}(V_k/V)^{(1-\delta)/\delta} - \rho$, where V_k and V are the steady-state values of aggregate capital and aggregate patents (i.e., $V_{k,t}K_t$ and $V_{n,t}N_t$), respectively.[17] The growth rate of the number of varieties is given by $g_n = \varphi - [V_k/(\gamma V)][(1-\delta)g_k + (1-\gamma)\rho]$ and the growth rate of final goods is given by $g_y = [(1-\varepsilon)/\varepsilon]g_n + \gamma g_k$. Finally, the steady-state level of welfare is given by $U = [\ln(1/\mu) - (1-\gamma)\ln(\varphi V) - \gamma \ln M]/\rho + g_y/\rho^2$.

There are two parameters $\{\delta, \varepsilon\}$ that are new in this numerical exercise. First, we set the parameter δ to 1/3 so that the share of capital in the production function for capital is identical to the one for intermediate goods. Second, we use $\epsilon = 3.5$ as the value for the elasticity of substitution between intermediate goods, since this value implies that $\varepsilon = 0.7143$, preserving the same upper bound of patent breadth μ as in the benchmark case, given that the restriction on the range of

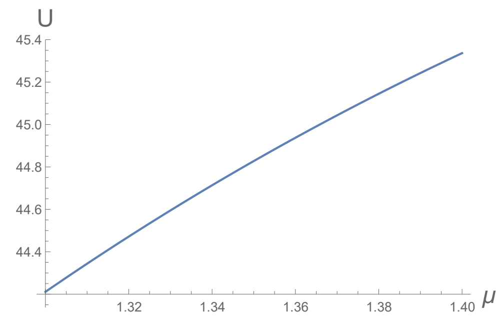
[16] In the case of a low φ/ϕ, the first-best labor allocations are given by $L_x^* = 0.1804$, $L_r^* = 0$, and $L_k^* = 0.8196$, yielding an innovation growth rate of $g_n^* = 0$, a capital growth rate of $g_k^* = 11.3564\%$, an output growth rate of $g_y^* = 2.7039\%$, and a welfare level of 107.8140.

[17] Iwaisako and Futagami (2013) show that the steady-state values of V_k and V are pinned down by two equations such that $\rho V_k + [(1-\delta)/\delta]M = \gamma/(\delta\mu)$ and $\rho V_k + (\varphi+\rho)V = 1$, where $M = \delta(1-\delta)^{(1-\delta)/\delta}(\phi V_k)^{1/\delta}(\varphi V)^{(\delta-1)/\delta}$.

patent breadth $\mu \leq 1/\varepsilon$ must hold in this exercise. The calibrated values of other parameters remain the same as before. Figure 4 displays the effects of a strengthening of patent protection on economic growth and social welfare, and Table 5 shows the labor allocations, growth rates, and welfare levels under some specific degrees of patent breadth, accordingly.



(a) Economic Growth



(b) Social Welfare

Figure 4: Growth and welfare effects of strengthening patent protection: using capital for reproducing new capital

It can be seen that in this general case, an increase in μ from 1.3 to 1.4 leads to a rise in economic growth g_y from 0.9845% to 1.1264%, implying that the R&D channel dominates the capital channel as in the benchmark case shown in Figure 1, so that the growth-maximizing level of patent protection is given by the broadest patent breadth $\mu_g = 1.4$. More importantly, the level of welfare U is monotonically increasing in μ, implying that the welfare-maximizing level of patent protection is also given by $\mu_w = 1.4$. This result can be explained as follows. Introducing capital as an input into the production function of capital reduces the dependence on the use of labor. This is revealed by lower levels of capital-producing labor L_k in Table 5 as compared to the benchmark counterparts in Table 2. Therefore, Table 5 shows that relative to the baseline model, tightening patent protection reallocates more manufacturing labor L_x to enhance the volume of intermediate-goods production and more R&D labor L_r to stimulate the growth rate of innovations. Based on these two forces, along the process of increasing μ, the positive effect on welfare through economic growth reinforces the positive effect on welfare through more consumption, making U strictly increase in μ. As a result, under the same relative productivity of R&D as in the benchmark (i.e., $\varphi/\phi = 0.5107$), the welfare-maximizing degree of patent breadth in this exercise is given by the upper bound of μ, analogous to the case in Figure 2.[18]

[18]In this general model, if the upper bound of the range of patent breadth is broadened to $\mu \in [1.3, 3]$ by decreasing ε to 1/3 and the relative productivity of R&D compared to capital is lowered to $\varphi/\phi = 0.049$ by increasing ϕ to 1.4441, then an inverted-U relationship between patent protection and output growth will arise as shown in Iwaisako and Futagami (2013), yielding an interior solution for the growth-maximizing level of patent breadth such that $\mu_g = 2.9011$. Moreover, in this case, an inverted-U relationship between patent protection and social welfare will also arise, yielding an interior solution for the welfare-maximizing level of patent breadth such that $\mu_w = 1.8871$.

Table 5: Growth and welfare effects of patent protection: using capital for reproducing new capital.

μ	1.3	1.32	1.34	1.36	1.38	1.4
L_x	0.7664	0.7542	0.7423	0.7308	0.7196	0.7087
L_r	0.0622	0.0794	0.0959	0.1120	0.1275	0.1426
L_k	0.1713	0.1665	0.1618	0.1573	0.1529	0.1487
g_n	0.4405%	0.5615%	0.6787%	0.7922%	0.9022%	1.0089%
g_k	2.4251%	2.3711%	2.3185%	2.2673%	2.2173%	2.1686%
g_y	0.9845%	1.0150%	1.0443%	1.0727%	1.1000%	1.1264%
U	44.2110	44.4716	44.7134	44.9375	45.1449	45.3366

4 Discussion

In this section, we discuss two directions according to which the baseline model can be extended. In Subsection 4.1, we introduce the households' leisure-consumption decision by allowing for elastic labor supply. In Subsection 4.2, we propose two settings that can help eliminate the problem of scale effects.

4.1 Elastic Labor Supply

We now assume that households derive utility from both consumption and leisure. Under this assumption, the utility function (1) is modified to

$$U = \int_0^\infty e^{-\rho t}[\ln C_t + \theta \ln(1 - L_t)]dt, \tag{30}$$

where L_t is the amount of labor supplied and the parameter $\theta > 0$ determines the intensity of leisure preference relative to consumption. Therefore, in addition to the Euler equation (3), the standard optimization yields the leisure-consumption decision given by

$$W_t(1 - L_t) = \theta C_t, \tag{31}$$

where $1 - L_t$ captures the level of leisure. To solve for the equilibrium labor allocations, combining (31) and the labor-market-clearing condition with (A.4), (A.7), and (A.10) in Appendix A yields

$$L_x = \frac{1-\gamma}{\mu}\left(\frac{\alpha}{\alpha+\theta}\right)\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right), \tag{32}$$

$$L_r = \frac{\mu-1}{\mu}\left(\frac{\alpha}{\alpha+\theta}\right)\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) - \frac{\rho}{\varphi}, \tag{33}$$

$$L_k = \frac{\gamma}{\mu}\left(\frac{\alpha}{\alpha+\theta}\right)\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) - \frac{\rho}{\phi}, \tag{34}$$

$$L = 1 - \frac{\theta}{\alpha+\theta}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right), \tag{35}$$

where the equilibrium labor allocations of L_x, L_r, and L_k are a function of patent breadth μ, whereas the labor supply L is not. Using (32) and (35) yields the ratio of production labor to leisure $L_x/(1-L) = \alpha(1-\gamma)/(\theta\mu)$. This implies that when an elastic labor supply is incorporated, patent protection creates a distortion on the allocation between production and leisure, such that a higher μ decreases $L_x/(1-L)$;[19] this impact of patent breadth on the ratio of manufacturing labor to leisure is consistent with that in Yang (2018). Moreover, the levels of labor allocations in production, R&D, and capital-producing in this case (i.e., (32)-(34)) are lower than their counterparts in the baseline model (i.e., (21)-(23)).

The qualitative pattern regarding the growth and welfare effects of μ in the baseline model continues to hold in this extended model. If the relative productivity of R&D compared to capital is high (low), i.e., $\varphi/\phi > (<)\alpha\gamma^2/(1-\alpha)$, a stronger patent protection μ is growth-enhancing (retarding), whereas it generates an inverted-U shape on (decreases in) the welfare level U. Thus, under a low value of φ/ϕ, the growth- and welfare-maximizing level of patent protection is given by the lowest level of μ that maintains a positive L_r. By contrast, under a high value of φ/ϕ, the growth-maximizing level of patent protection is given by $\mu_g = 1/\alpha$, and the welfare-maximizing level of patent protection is given by $\hat{\mu} = [\alpha/(\alpha+\theta)](1+\rho/\varphi+\rho/\phi)[\varphi(1-\alpha)/\alpha - \gamma^2\phi]/[\rho(1-\gamma)]$.

Notice that under an elastic labor supply, the interior solution for the welfare-maximizing level of patent protection $\hat{\mu}$ is strictly smaller than the baseline counterpart $\tilde{\mu}$. This is because an elastic labor supply decreases the equilibrium labor allocations of R&D L_r and capital L_k, which, under a high relative productivity of R&D, mitigates the positive impact of growth on welfare. Therefore, the optimal policy response is to decrease the degree of patent protection to reduce the distortion in the ratio of production labor to leisure.

4.2 Scale Effects

It is known that the baseline model is subject to the scale-effect problem such that the growth rate of outputs g_y depends on the sizes of research labor L_r and capital-producing labor L_k, which are determined by the level of population. Therefore, we follow Cozzi (2017a,b) to propose two approaches to remove the scale effect and discuss how these approaches affect the results on the growth and welfare effects of patent protection.[20]

The first approach that eliminates the scale effect is the *fully endogenous* solution, as in Peretto (1998), Young (1998), and Howitt (1999). In the context of this study, suppose that the total amount of labor equals L_t instead of unity in the baseline model. Then, the (aggregate) production functions for varieties and capital are modified to $\dot{N}_t = \bar{\varphi}N_t L_{r,t}$ and $\dot{K}_t = \bar{\phi}K_t L_{k,t}$, where the productivity parameters are given by $\bar{\varphi} = \varphi/L_t$ and $\bar{\phi} = \phi/L_t$, respectively. This specification implies that the steady-state growth rate of innovations g_n (capital g_k) depends on the fraction of labor employed in R&D $l_{r,t} \equiv L_{r,t}/L_t$ (capital production $l_{k,t} \equiv L_{K,t}/L_t$). Following similar derivations as in Subsection 2.6, the per capita labor allocations in manufacturing ($l_{x,t} \equiv L_{x,t}/L_t$),

[19] In the presence of patent protection μ, the ratio of manufacturing labor to leisure in equilibrium is always lower than the socially optimal counterpart, namely, $L_x/(1-L) < L_x^*/(1-L^*) = \alpha(1-\gamma)/\theta$.

[20] A more complete welfare analysis of policy changes should take into account the dynamic transition of households' utility from the initial steady state to the final one. See Peretto (2007, 2011) for such an examination in a fully endogenous growth model and Iwaisako (2018) in a semi-endogenous growth model. For simplicity, the analysis in this subsection follows Acemoglu and Akcigit (2012) to only focus on the effect on steady-state welfare.

R&D, and capital-producing $\{l_x, l_r, l_k\}$ in equilibrium are still given by the same expressions as in (21)-(23). In this case, the steady-state growth rate of outputs is given by $g_y = (1-\alpha)\varphi l_r + \alpha\gamma\phi l_k$ and the steady-state level of welfare is given by $U = [\alpha(1-\gamma)/\rho]\ln l_x + g_y/\rho$, respectively. Therefore, as compared to the baseline model, the analytical results regarding the growth and welfare implications of patent protection μ do not change under this approach.

The second approach that eliminates the scale effect is the *semi-endogenous* solution, as in Jones (1995), Kortum (1997), and Segerstrom (1998). In the context of this study, suppose that the total amount of labor L_t grows at a rate of $n > 0$. Then, the (aggregate) production functions for varieties and capital are modified to $\dot{N}_t = \varphi N_t^{\delta_r} L_{r,t}^{\lambda_r}$ and $\dot{K}_t = \phi K_t^{\delta_k} L_{k,t}^{\lambda_k}$, respectively, where $\lambda_r \in (0,1)$ ($\lambda_k \in (0,1)$) measures the degree of diminishing returns in R&D labor (capital-producing labor) and $\delta_r < 1$ ($\delta_k < 1$) measures the degree of limited spillovers from the accumulated knowledge stock (capital stock). This specification implies that the steady-state growth rate of innovations (capital) is given by $g_n = \lambda_r n/(1-\delta_r)$ ($g_k = \lambda_k n/(1-\delta_k)$). This yields the classical semi-endogenous prediction that government policies (i.e., patent policy μ in this study) affecting labor allocations $\{L_{x,t}, L_{r,t}, L_{k,t}\}$ cannot affect g_n and g_k, differing from the significant growth effects of μ in the baseline model. Moreover, the impact of μ on steady-state welfare operates only through the balanced-growth level of consumption C_0, which is a function of the manufacturing labor $L_{x,0}$ and the balanced-growth levels of innovations N_0 and capital K_0, where $N_0 = [\varphi L_{r,0}^{\lambda_r}(1-\delta_r)/(\lambda_r n)]^{1/(1-\delta_r)}$ and $K_0 = [\phi L_{k,0}^{\lambda_k}(1-\delta_k)/(\lambda_k n)]^{1/(1-\delta_k)}$. Accordingly, the welfare effect of μ depends on the labor reallocation between $L_{r,0}$ and $L_{k,0}$, which is expected to be critically determined by the ratio of φ/ϕ. In other words, similar to Lemma 3 and Proposition 2, the relative productivity of R&D φ/ϕ under semi-endogenous growth would continue to play an important role in designing the welfare-maximizing level of patent protection μ_w.

5 Conclusion

This study explores the impacts of patent protection on economic growth and social welfare in a Romer-type expanding variety model in which R&D and capital accumulation are both engines of growth. The current setup allows the growth rates of innovations and capital to be determined independently. Previous studies on this type of two-engine growth model, such as Iwaisako and Futagami (2013), reveal that the comparison of productivity between R&D and capital is a major determinant of the growth effects of IPR protection. However, the findings of this study suggest that such a comparison continues to be important for the welfare effects of IPR protection. In particular, our results show that when the relative productivity of R&D is low, economic growth and social welfare both decrease in the strength of IPR protection, implying that the growth- and welfare-maximizing levels of patent protection coincide with each other at the lowest level. By contrast, when the relative productivity of R&D is high, economic growth increases in the strength of IPR protection, whereas social welfare has an inverted-U shape, implying that the welfare-maximizing degree of patent protection would be below the growth-maximizing degree. Therefore, this study provides a novel channel through welfare analysis to explain why countries, with either high or low productivity in innovations, may not prefer significantly strong protection in IPR, even though stricter enforcement of patent rights could be growth-stimulating.

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Appendix A

Proof of Lemma 1

In this proof, we examine the stability of this model given a stationary path of μ_t. First, define the transformed variables $\Psi_{n,t} \equiv Y_t/(V_{n,t}N_t)$ and $\Psi_{k,t} \equiv Y_t/(V_{k,t}K_t)$. Then, differentiating $\Psi_{n,t}$ with respect to time yields

$$\frac{\dot{\Psi}_{n,t}}{\Psi_{n,t}} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{V}_{n,t}}{V_{n,t}} - \frac{\dot{N}_t}{N_t}. \tag{A.1}$$

From the final-goods resource constraint $Y_t = C_t$, the law of motion for Y_t is given by

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = R_t - \rho, \tag{A.2}$$

where the second equality stems from the Euler equation in (3). From (12), the law of motion for $V_{n,t}$ is

$$\frac{\dot{V}_{n,t}}{V_{n,t}} = R_t - \frac{\Pi_{x,t}}{V_{n,t}}. \tag{A.3}$$

where $\Pi_{x,t} = \alpha(\mu - 1)Y_t/(\mu_t N_t)$, which is obtained by applying symmetry across varieties in (4) to rewrite (5) as $\alpha Y_t/N_t = P_t(j)X_t(j)$ and substituting it into (9). Combining (A.1)-(A.3) yields

$$\frac{\dot{\Psi}_{n,t}}{\Psi_{n,t}} = \alpha\left(\frac{\mu - 1}{\mu}\right)\Psi_{n,t} - \varphi L_{r,t} - \rho, \tag{A.4}$$

where we use the fact that $\dot{N}_t/N_t = \varphi L_{r,t}$.

Using the same logic, differentiating $\Psi_{k,t}$ with respect to time yields

$$\frac{\dot{\Psi}_{k,t}}{\Psi_{k,t}} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{V}_{k,t}}{V_{k,t}} - \frac{\dot{K}_t}{K_t}. \tag{A.5}$$

From (15), the law of motion for $V_{k,t}$ is

$$\frac{\dot{V}_{k,t}}{V_{k,t}} = R_t - \frac{Q_t}{V_{k,t}}, \tag{A.6}$$

where $Q_t = \alpha\gamma Y_t/(\mu_t K_t)$, which is obtained by applying symmetry across varieties in (4) to rewrite (5) as $\alpha Y_t/N_t = P_t(j)X_t(j)$ and substituting it into (11). Combining (A.2), (A.5), and (A.6) yields

$$\frac{\dot{\Psi}_{k,t}}{\Psi_{k,t}} = \alpha\left(\frac{\gamma}{\mu}\right)\Psi_{k,t} - \phi L_{k,t} - \rho, \tag{A.7}$$

where we use the fact that $\dot{K}_t/K_t = \phi L_{k,t}$.

Furthermore, combining (14) and (17) yields $\varphi V_{n,t}N_t = \phi V_{k,t}K_t$, which implies

$$\frac{\Psi_{n,t}}{\varphi} = \frac{\Psi_{k,t}}{\phi}, \tag{A.8}$$

and also $\dot{\Psi}_{n,t}/\Psi_{n,t} = \dot{\Psi}_{k,t}/\Psi_{k,t}$. Using this result and (A.8), we rewrite (A.7) to make $L_{k,t}$ a function of $\dot{\Psi}_{n,t}/\Psi_{n,t}$ and $\Psi_{n,t}$ such that

$$L_{k,t} = -\frac{1}{\phi}\left[\frac{\dot{\Psi}_{n,t}}{\Psi_{n,t}} - \alpha\left(\frac{\gamma}{\mu}\right)\left(\frac{\phi}{\varphi}\right)\Psi_{n,t} + \rho\right]. \tag{A.9}$$

Then, we use (10) to derive

$$L_{x,t} = \int_0^{N_t} L_{x,t}(j)dj = \frac{\left(\frac{1-\gamma}{\mu}\right)\int_0^{N_t} P_t(j)X_t(j)dj}{W_t} = \frac{\left(\frac{1-\gamma}{\mu}\right)\alpha Y_t}{W_t} = \frac{\alpha}{\varphi}\left(\frac{1-\gamma}{\mu}\right)\Psi_{n,t}, \tag{A.10}$$

where (4) and (5) are used in the third equality and (14) is used in the fourth equality.

Finally, substituting (A.9), (A.10), and the labor-market-clearing condition $L_{x,t}+L_{r,t}+L_{k,t} = 1$ into (A.4), a few steps of manipulation yield a one-dimensional differential equation in $\Psi_{n,t}$:

$$\frac{\dot{\Psi}_{n,t}}{\Psi_{n,t}} = \left(1+\frac{\varphi}{\phi}\right)^{-1}\left[\alpha\Psi_{n,t} - \varphi\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)\right]. \tag{A.11}$$

Therefore, the dynamics of $\Psi_{n,t}$ is characterized by saddle-point stability such that $\Psi_{n,t}$ jumps immediately to its interior steady-state value given by

$$\Psi_n = \frac{\varphi}{\alpha}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right). \tag{A.12}$$

Then, (A.4), (A.7), and (A.10) reveal that when Ψ_n and μ are stationary, L_r, L_k, and L_x must also be stationary, respectively.

Appendix B : Not for Publication

Socially Optimal Allocations

For the first-best optimal allocations in this model, we rewrite the final-goods constraint as $C_t = Y_t = (S_t)^{1-\alpha}(N_t)^{1-\alpha}[A(K_t)^{\gamma}(L_{x,t})^{1-\gamma}]^{\alpha}$. Using this equation, the welfare maximization problem for the social planner is given by the following current-value Hamiltonian:

$$\begin{aligned} H_t =& (1-\alpha)\ln S_0 + (1-\alpha)\ln N_t + \alpha\gamma \ln K_t + \alpha(1-\gamma)\ln L_{x,t} \\ &+ \eta_{1,t}(\varphi N_t L_{r,t}) + \eta_{2,t}(\phi K_t L_{k,t}) + \eta_{3,t}(1 - L_{x,t} - L_{r,t} - L_{k,t}), \end{aligned} \tag{B.1}$$

where $\eta_{1,t}$, $\eta_{2,t}$, and $\eta_{3,t}$ are the costate variables associated with the law of motion for R&D technology, capital production, and the labor market, respectively, and we use the fact that $S_t = S_0$ over time. Then, the first-order conditions for $L_{x,t}$, $L_{r,t}$, and $L_{k,t}$ are given by

$$\frac{\partial H_t}{\partial L_{x,t}} = \frac{\alpha(1-\gamma)}{L_{x,t}} - \eta_{3,t} = 0; \tag{B.2}$$

$$\frac{\partial H_t}{\partial L_{r,t}} = \eta_{1,t}\varphi N_t - \eta_{3,t} \leq 0; \tag{B.3}$$

$$\frac{\partial H_t}{\partial L_{k,t}} = \eta_{2,t}\phi K_t - \eta_{3,t} \leq 0; \tag{B.4}$$

$$\frac{\partial H_t}{\partial N_t} = \frac{1-\alpha}{N_t} + \eta_{1,t}\varphi L_{r,t} = \rho\eta_{1,t} - \dot{\eta}_{1,t}; \tag{B.5}$$

$$\frac{\partial H_t}{\partial K_t} = \frac{\alpha\gamma}{K_t} + \eta_{2,t}\phi L_{k,t} = \rho\eta_{2,t} - \dot{\eta}_{2,t}. \tag{B.6}$$

Manipulating (B.5) and (B.6) yields two differential equations such that $\eta_{1,t}\dot{N}_t + \dot{\eta}_{1,t}N_t = \rho\eta_{1,t}N_t - (1-\alpha)$ and $\eta_{2,t}\dot{K}_t + \dot{\eta}_{2,t}K_t = \rho\eta_{2,t}K_t - \alpha\gamma$, implying that $\eta_{1,t}N_t$ and $\eta_{2,t}K_t$ must jump to their steady-state values given by $(1-\alpha)/\rho$ and $\alpha\gamma/\rho$. In this case, the above autonomous dynamic system of either $\eta_{1,t}N_t$ or $\eta_{2,t}K_t$ indicates that saddle-point stability is satisfied. Therefore, this saddle-point stability implies that the underlying welfare level under the present maximization problem is equivalent to the (steady-state) level of welfare under the maximization problem with respect to the lifetime utility of households along the balanced growth path (namely, the maximization of (25) subject to the resource constraint of labor.).

If $\varphi/\phi > \alpha\gamma/(1-\alpha)$, then substituting the conditions that $\eta_{1,t}N_t = (1-\alpha)/\rho$ and $\eta_{2,t}K_t = \alpha\gamma/\rho$ into (B.3) and (B.4) yields

$$\eta_{3,t} = \eta_{1,t}\varphi N_t = \frac{\varphi(1-\alpha)}{\rho} > \frac{\alpha\gamma\phi}{\rho} = \eta_{2,t}\phi K_t. \tag{B.7}$$

Substituting (B.7) into (B.2) and (B.4) yields the first-best production labor $L_x^* = \rho\alpha(1-\gamma)/[\varphi(1-\alpha)]$ and the first-best capital labor $L_k^* = 0$. Combining L_x^* and L_k^* with $L_{x,t} + L_{r,t} + L_{k,t} = 1$ yields the first-best R&D labor $L_r^* = 1 - \rho\alpha(1-\gamma)/[\varphi(1-\alpha)]$.

Similarly, if $\varphi/\phi < \alpha\gamma/(1-\alpha)$, then substituting the conditions that $\eta_{1,t}N_t = (1-\alpha)/\rho$ and

$\eta_{2,t}K_t = \alpha\gamma/\rho$ into (B.5) and (B.6) yields

$$\eta_{1,t}\varphi N_t = \frac{\varphi(1-\alpha)}{\rho} < \frac{\alpha\gamma\phi}{\rho} = \eta_{2,t}\phi K_t = \eta_{3,t}. \tag{B.8}$$

Substituting (B.8) into (B.2) and (B.3) yields the first-best production labor $L_x^* = \rho(1-\gamma)/(\varphi\gamma)$ and the first-best R&D labor $L_r^* = 0$. Combining L_x^* and L_r^* with $L_{x,t} + L_{r,t} + L_{k,t} = 1$ yields the first-best capital labor $L_k^* = 1 - \rho(1-\gamma)/(\varphi\gamma)$.

Finally, if $\varphi/\phi = \alpha\gamma/(1-\alpha)$, then we obtain $\eta_{3,t} = \varphi(1-\alpha)/\rho = \alpha\gamma\phi/\rho$, implying that $L_{x,t}$ can equal either $\rho\alpha(1-\gamma)/[\varphi(1-\alpha)]$ or $\rho(1-\gamma)/(\varphi\gamma)$. In this case, the equation system (B.2)-(B.6) loses one condition to pin down the relationship between $L_{r,t}$ and $L_{k,t}$, such that any combination of $\{L_{r,t}, L_{k,t}\}$ satisfying $L_{r,t} + L_{k,t} = 1 - \alpha(1-\gamma)(\rho/\varphi)/(1-\alpha) = 1 - (1-\gamma)(\rho/\phi)/(1-\alpha)$ can be a solution. Without loss of generality, it is assumed that φ/ϕ is either large or small to facilitate the welfare analysis that follows, and thus the possibility of $\varphi/\phi = \alpha\gamma/(1-\alpha)$ is excluded.

Consequently, the above labor allocations by dynamic welfare maximization imply a corner solution in the first-best allocations. The first-best allocations are efficient in terms of assigning labor to the growth engine that has a larger effect on welfare. In other words, when the impact of the growth engine through R&D (capital accumulation) on welfare is stronger, that is $(1-\alpha)\varphi >$ $(<)\alpha\gamma\phi$, no labor is allocated to the weaker growth engine, yielding $L_k^* = 0$ $(L_r^* = 0)$.