# Patent protection and income inequality in a model with two growth engines [*]

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**Abstract**

Recent evidence suggests that the degrees of both patent protection and income inequality have increased significantly. Existing literature mainly analyzes the patent-inequality relationship in a growth-theoretic framework with a sole growth engine. This study explores the effect of patent policy on income inequality in a variety-expansion model, in which R&D and capital accumulation are non-complementary engines of growth. We find that patent protection affects income inequality only through the *interest-rate* channel, which depends on the magnitude of R&D productivity relative to capital productivity. If R&D productivity relative to capital is high (low), stronger patent protection intensifies (mitigates) income inequality. Moreover, we calibrate the model to the US economy, and the numerical results support the implications of patent protection on economic growth and income inequality. This result is consistent with our empirical findings using cross-country panel data on OECD countries.

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# 1 Introduction

Motivated by recent evidence in Piketty (2014) and Saez and Zucman (2016), income inequality has received increasing attention in academic research and policy advice. Given that income inequality is determined endogenously, it would be interesting to understand how income inequality is affected by exogenous policy regimes. Since the signing of the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) in 1994, patent protection in many countries has strengthened, and patent policy has become an important issue to be explored in macroeconomic studies. Recent empirical evidence (Chu *et al.* 2021) suggests that income inequality correlates strongly with the measures of patent strength across countries, and an increasing number of studies (e.g., Chu 2010, Chu and Cozzi 2018, and Kiedaisch 2021) have attempted to investigate the patent-inequality relationship in endogenous growth frameworks, which focus on innovation as the sole growth engine. Given the significant contribution of innovation and physical capital growth to output growth as highlighted in growth accounting studies,[1] however, how would the patent-inequality relationship alter in an economic environment where these non-complementary engines, namely research and development (R&D) and capital accumulation, are both driving economic growth? These observations motivate us to revisit the underlying economic forces and mechanisms that shape the relationship between patent policies and the distribution of income in a growth-theoretic framework.

In this study, we first exploit cross-country panel data on 38 OECD countries to empirically assess the relationship between patent strength and income inequality. We document the stylized fact that the effect of a strengthened patent policy on income inequality is contingent on the ratio of R&D expenditure to gross capital formation. We argue that the ratio of R&D expenditure to capital formation can be interpreted as a proxy for the relative productivity of R&D to capital. Based on the long-run average of the relative productivity measure in our sample, we categorize OECD countries into two groups, namely, high relative and low relative productivity countries, and estimate cross-country regressions for each group independently. We find that the estimated effect of increased patent protection on income inequality among countries with high relative productivity is strongly positive. In sharp contrast, strengthening patent protection in countries with low relative productivity tends to generate an inequality-mitigating effect.

To rationalize these interesting empirical findings within a unified growth-theoretic framework, this study follows Iwaisako and Futagami (2013) and Chu *et al.* (2019) and considers an endogenous growth model with innovation and physical capital accumulation. Differing from the canonical endogenous growth framework in which the growth engines of innovation and physical capital are complementary, the current framework allows these two growth engines to work separately. Moreover, this model captures the protection of intellectual property rights

[1] For example, Barro and Sala-I-Martin (2004) shows that capital growth accounts for an equally large fraction of GDP growth as total factor productivity growth.

(IPR) through patent breadth, which determines the market power of monopolistic firms. On one hand, a larger patent breadth leads to a higher markup that increases firms' profitability and generates a positive effect on economic growth by stimulating incentives in R&D. However, a larger patent breadth depresses capital accumulation and in turn, economic growth, because firms' production volume decreases with less demand for capital inputs. Accordingly, strengthening patent protection raises (lowers) the rate of economic growth if R&D productivity relative to capital is high (low).[2] [3]

The novel contribution of this study is that we introduce heterogeneous households with different holdings of assets into the two-engine growth framework.[4] Given that wage income is equally distributed among households, income inequality originates from the unequal distribution of households' asset income, which can be further decomposed into the rate of return on assets (i.e., the *interest-rate* effect) and the value of assets relative to the value of wage (i.e., the *asset-value* effect). Nevertheless, given the production settings of innovation and capital production, the free-entry conditions in these sectors show that the ratio of asset to wage value depends on sectoral productivity, implying that patent breadth does not play a role (i.e., the asset-value effect is absent). In this case, the impact of strengthening patent protection on income inequality operates only through the interest-rate effect; the impact of patent policy on the equilibrium growth rate is completely transferred to the impact on the interest rate through the Euler equation of households. Consequently, strengthening patent protection increases (decreases) the degree of income inequality if the relative R&D productivity to capital is high (low).[5] The current two-engine growth model provides a theoretical rationale to justify the observation for the mixed patent-inequality nexus by highlighting the important role of the relative productivity between R&D and capital. Finally, we calibrate the model to the US economy to perform a quantitative analysis. This result supports the growth- and inequality-implications of patent breadth, as previously mentioned, and continues to hold in the robustness exercises.

This study relates to the literature that examines the patent-growth relationship, taking into account various forms of patent protection within a dynamic general equilibrium model. For example, Goh and Olivier (2002) and Pan *et al.* (2018) focus on patent breadth; Futagami and Iwaisako (2007) and Lin and Shampine (2018) introduce patent protection via patent length; Furukawa (2007) discusses the role of patent policies against imitation; and Cozzi and Galli (2014) and Yang (2018) analyze the effect of blocking patents on economic growth. These studies exploit

[2] In addition, Chu *et al.* (2019) consider the interactive effects of monetary policy and patent protection on economic growth. They found that increasing patent breadth strengthens (weakens) the effect of money growth on economic growth if the relative productivity of R&D to capital is sufficiently high (low).

[3] See Yang (2021) for the welfare analysis of patent protection in this two-engine growth model.

[4] See Piketty *et al.* (2014) for evidence showing that wealth inequality is a critical determining factor of income inequality.

[5] In Section 6, we consider three analytical extensions of the baseline model. The baseline results still hold when (a) using a CES final-good production function, (b) considering an iso-elastic utility function of consumption and leisure, and (c) introducing liquidity constraints on the innovating and capital-producing sectors.

R&D-based growth models with homogeneous households or a representative household, and their focus is not on the effect of patent protection on income inequality. Complementing the above studies, we exploit a framework of heterogeneous households to investigate simultaneously the patent-inequality relationship.

This study also contributes to the literature on income inequality and innovation in the R&D-based growth model; see, for example, Zweimüller (2000), Foellmi and Zweimüller (2006), Grossman and Helpman (2018), and Aghion *et al.* (2019), focusing on the innovation-inequality relationship. Our study relates to these interesting studies by exploring the role of patent protection in shaping the innovation-inequality relationship. In addition, Chu (2010) and Chu and Cozzi (2018) find that the effect of patent policy on income inequality is positive within a quality-ladder growth model. A recent study by Chu *et al.* (2021) explores the same issue by endogenizing the market structure, showing that the effect of patent protection on income inequality in the short run (i.e., a positive or an inverted-U effect) differs from that in the long run (i.e., a negative effect). However, the previously mentioned studies on the patent-inequality nexus are based on a framework in which the engine of growth is only innovation.[6] Our study is in contrast to their studies by considering a framework with two growth engines (i.e., innovation and capital accumulation), which work independently. We find that the two-engine growth framework leads to a novel implication: the impact of patent protection on income inequality depends on the relative productivity of the two growth engines, which is supported by our empirical finding using panel data from OECD countries.

The remainder of this paper is organized as follows. Section 2 presents the empirical analysis and documents the relative productivity-contingent effect of patent protection on income inequality. Section 3 presents the basic theoretical model and explores the growth effects of patent protections. Section 4 investigates the effects of patent protection on income inequality. Section 5 presents a quantitative analysis of the results. Section 6 considers extensions of the baseline model, and the final section concludes the paper.

## 2 Stylized facts

This section exploits panel data on OECD countries and investigates the empirical relationship between patent policies and income inequality. We document the stylized fact that the overall effect of patent strength on income inequality hinges on the R&D productivity relative to capital. In particular, we find that the increasing patent strength amplifies (mitigates) income inequality in countries with a high (low) productivity of R&D relative to capital.

[6] In addition, Chan *et al.* (2022) investigate the patent-inequality relationship in a Schumpeterian growth model with heterogeneous wealth and skills, and Lu and Lai (2022) explore this effect in a Schumpeterian framework featuring endogenous quality increments. However, similar to the above studies, R&D is the only growth engines in these two studies.

Given that these productivity measures are not directly observable at the aggregate level, to avoid empirical difficulties in statistically inferring them from cross-country macroeconomic data, we assume that R&D productivity and capital productivity are positively correlated with the shares of resources devoted to R&D activities and capital formation in total output, respectively.[7] Following our assumption, the ratio of R&D intensity (or the share of R&D expenditure in GDP) to the share of gross capital formation in total output provides an instant proxy for the relative productivity. It is worth noting that this proxy reduces to the ratio of total R&D expenditure to gross capital formation, which is henceforth referred to as the relative productivity measure for conciseness.

Exploring a dataset covering 38 OECD countries, we compute the sample mean of the ratio of R&D expenditure to gross capital formation within each country, and report the ranking in a descending order in Table 1. The cross-country mean of the relative productivity measure was around 6.39%. Based on the ranking list, we categorize the investigated countries into two groups: High Relative Productivity Countries (HRPC) and Low Relative Productivity Countries (LRPC) by choosing a cutoff value $\alpha^*$. We then estimate the following cross-country panel regression for each group independently:

$$INE_{i,t} = \beta_1^j IPR_{i,t-1} + \beta_2^j IPR_{i,t-1} \times RP_{i,t-1} + H^j X_{i,t-1} + \gamma_i^j + \lambda_t^j + \varepsilon_{i,t}, \tag{1}$$

where $INE$ denotes income inequality; $IPR$ and $RP$ denote the strength of patent protection and relative productivity, respectively; $X$ denotes a vector of control variables, and $H$ is the coefficient matrix; $\gamma$ and $\lambda$ are country- and year-fixed effects, respectively; and $i$, $j$, and $t$ are country, group and time indices, respectively. We incorporated the products of $IPR$ and $RP$ to capture their potential interaction effects. In the baseline regressions, the cutoff value of $\alpha^*$ was set to 8%, which is 25% higher than the previously mentioned cross-country average (6.39%). Consequently, the top 13 countries (from Israel to the Netherlands), which account for around one-third of the total number of investigated countries, fall into the HRPC category. As an alternative practice, we choose a higher standard for HRPC by setting $\alpha^*$ to 8.5%. As shown later, our baseline results are largely robust to the alternative grouping criteria.

Our control variables are standard in the empirical growth literature. First, we incorporate variables closely related to economic growth, such as government spending to GDP ratio, R&D intensity, human capital, inflation rate, and trade openness. Second, following the financial development literature that highlights the impact of credit expansion on inequality, we add the ratio of total credit to the private non-financial sector to the control vector.[8] In addition, several

[7] While seemingly strong, our assumption is not necessarily implausible. For example, using standard production functions, Comin (2004) shows that the growth rate of R&D-driven technologies is positively correlated with R&D intensity.

[8] For instance, recent studies, such as Beck *et al.* (2007), Delis *et al.* (2014), Jauch and Watzka (2016), and others, report statistically significant effects of financial development on income inequality. Representative works of the theoretical exposition of the finance-inequality relationship include Galor and Zeira (1993), Galor and Moav (2004),

Table 1: Average R&D expenditure to gross capital formation ratio for OECD countries (%)

| | | | | |
|---|---|---|---|---|
| 1.IL: 14.81 | 9.FR: 9.11 | 17.GB: 7.21 | 25.EE: 4.26 | 33.TR: 2.59 |
| 2.SE: 13.07 | 10.IS: 8.89 | 18.BE: 6.81 | 26.LT: 4.25 | 34.LV: 2.25 |
| 3.FI: 11.79 | 11.AT: 8.33 | 19.SI: 6.01 | 27.LU: 4.20 | 35.CR: 2.22 |
| 4.JP: 11.60 | 12.KR: 8.07 | 20.NZ: 5.18 | 28.ES: 3.94 | 36.MX: 1.94 |
| 5.DE: 11.47 | 13.NL: 8.06 | 21.IE: 4.94 | 29.PT: 3.87 | 37.CL: 1.42 |
| 6.US: 10.85 | 14.NO: 7.36 | 22.CZ: 4.74 | 30.PL: 3.70 | 38.CO: 1.02 |
| 7.DK: 10.00 | 15.CA: 7.33 | 23.HU: 4.43 | 31.GR: 2.98 | |
| 8.CH: 9.40 | 16.AU: 7.30 | 24.IT: 4.41 | 32.SK: 2.95 | |

*Notes*: OECD countries are labeled using the Alpha-2 code as described in the ISO international standard. Missing observations were removed when computing the long-run average. The sample period is 1996-2015.

model specifications in our empirical practice incorporate lagged GDP per capita to control for economic development.[9]

Accurately approximating the strength of patent protection can be challenging because *IPR* measures are generally too few, and each of them has limitations. In this study, we exploit the Protection of Property Rights (PPR) index, which is published under the *Legal System and Property Rights* category in the *Economic Freedom of the World* Report by Fraser Institute, as an approximation for *IPR*.[10] Compared with the Ginarte-Park index (in Ginarte and Park 1997 and Park 2008), the benefit of the PPR index is twofold. First, the Ginarte-Park index primarily focuses on the strength of statutory protection, and hence, does not sufficiently capture the enforcement of patent laws.[11] Second, different from the Ginarte-Park index which is only available at the quinquennial frequency, annual data on PPR helps avoid the difficulty in mixed-frequency data estimation or the loss of information induced by substantially reduced number of observations.[12]

and Ghossoub and Reed (2017), and so forth.

[9]The intension is to alleviate the concern that the empirical patent-inequality relationship is contingent on a country's level of economic development, instead of relative productivity. First, the highly developed countries and members of the HRPC in our study did not significantly overlap. For example, according to the United Nations Human Development Index, Australia, Great Britain, and Belgium ranked high in the report and all fall into the LRPC group. Moreover, according to our calculation, the long-run averages of GDP per capita from 1980 to 2015 in Australia and Italy were higher than those in France, Finland, Japan, and Korea. However, based on the relative productivity measure, countries with higher (lower) GDP per capita are considered as LRPC (HRPC). In addition, we show that the empirical results are robust when GDP per capita is incorporated into the regressions.

[10]The PPR index was computed as the average of two components. The first component is based on the Global Competitiveness Report question, surveying whether property rights are clearly defined and well protected by law, using a scale from 1 to 7. (Source: World Economic Forum) The second component is the Property Rights and Rule-Based Governance from Country Policy and Institutional Assessment, evaluating the extent to which property and contract rights are reliably respected and enforced in a legal system and governance structure. (Source: World Bank Group, CPIA Database)

[11]For detailed discussion, see Hu and Png (2013).

[12]Hu and Png (2013) gauge the level of effective patent rights by the composition of the Ginarte-Park index and the previously mentioned data on the Legal System and Property Rights in the Fraser index. We do not adopt this

Using the PPR index to approximate $IPR$ is a choice resulting from the limited availability of data. While not necessarily inconsistent with systematic evidence, it relies on the assumption that the degree of patent protection is positively correlated with the protection of property rights within a broader scope.[13] [14]

In this study, we consider three measures of income inequality. Our empirical analysis focuses on the Gini coefficient published in the World Income Inequality Database (WIID). As WIID also reports the income share of decile, we construct two additional inequality measures $T10/B10$ and $T20/B20$, where the former denotes the ratio of the income share of the richest decile to that of the poorest decile, and the latter denotes the income share ratio of the top 20% of households to that of the bottom 20%.[15] For control variables, we collect data on R&D to GDP ratio, and import and export shares in GDP from World Bank Indicators (WDI). The data on total credit to the private non-financial sector are from the Bank for International Settlements (BIS), and other macroeconomic data are mainly from the Penn World Table (PWT). The detailed data description is provided in Appendix A.1.

Table 2 reports the estimation results using the full sample. When we include country- and year-fixed effects but exclude the interaction of $IPR$ and relative productivity, Columns (3), (5), and (7) suggest an overall negative effect of patent strength on Gini coefficient. In particular, the estimated coefficients of $IPR$ under the Gini index (WIID) and income ratio measures are all statistically significant at the 10% level. When $IPR \times RP$ is added to the regressions, the estimated inequality-mitigating effect of patent protection becomes stronger and statistically significant at the 5% level across all measures of income inequality. In addition, while small in magnitude, the coefficient estimates of $IPR \times RP$ under $T10/B10$ and $T20/B20$ are both positive and significant, indicating that the effect of $IPR$ on narrowing income dispersion might weaken when the relative productivity increases.

Next, we demonstrate that the estimated effects of patent policy on income inequality among HRPC and LRPC are remarkably distinct. Table 3 presents the empirical findings under the baseline estimation for HRPC, where the cutoff value $\alpha$ was set to 8%. First, Column (1) suggests that, excluding the interaction term, the point estimate of the coefficient on $IPR$ is approximately 0.14, but this is not statistically significant. Second, Column (2) shows that considering the interaction term ($IPR \times RP$) yields a strong positive effect of patent protection under the Gini coefficient (WIID) measure. However, this positive effect tends to weaken slightly as relative

approach, because it does not resolve the issue of a reduced number of observations.

[13]Other possible measures of $IPR$ include the Patent Systems Strength index by Papageorgiadis *et al.* (2014), and the Patent Enforcement index by Papageorgiadis and Sofka (2020).

[14]Using published data on the Patent Enforcement index in Papageorgiadis and Sofka (2020), our empirical analysis yields similar qualitative results. The results are available upon request.

[15]Another popular measure of income inequality is the Gini index published by Solt (2009)'s Standardized World Income Inequality Database (SWIID). We do not exploit the Gini index from SWIID because statistical inference based on this measure can be subject to caveat owing to embedded measurement errors. For a detailed discussion, please refer to Jenkins (2015).

Table 2: Effect of $IPR$ on income inequality – full sample

| | Gini (WIID) | | | | $T10/B10$ | | $T20/B20$ | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $IPR$ | -0.39** | -0.21 | -0.38* | -0.64** | -0.83* | -1.37** | -0.24* | -0.38** |
| | (0.18) | (0.21) | (0.22) | (0.29) | (0.46) | (0.68) | (0.13) | (0.18) |
| $IPR \times RP$ | | | | 0.04 | | 0.10* | | 0.03* |
| | | | | (0.03) | | (0.06) | | (0.01) |
| Control | No | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year-Fixed Effect | Yes | No | Yes | Yes | Yes | Yes | Yes | Yes |
| Obs. | 634 | 476 | 476 | 476 | 461 | 461 | 461 | 461 |
| $R^2$ | 0.02 | 0.09 | 0.07 | 0.09 | 0.05 | 0.07 | 0.09 | 0.12 |

*Notes*: The estimation results reported in the table are based on the full sample. Gini (WIID) denotes the Gini coefficient published in the World Income Inequality Database. $T10/B10$ denotes the ratio of the income share of the 10th decile (top) to that of the 1st decile (bottom), and $T20/B20$ refers to the ratio of the income share of the top 20% of households to that of the bottom 20%. The sample period is 2000-2018. The control variables were all lagged by one period. The government spending to GDP ratio, R&D to GDP ratio, and credit to GDP ratio are in logarithm. The estimation using the full sample excluded lagged GDP per capita. Robust standard errors clustered by country are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

productivity increases. The statistical inference that increasing the degree of patent protection widens the income gap among countries with high relative productivity is robust to adding GDP per capita, which we use to control for economic development (see Column (3)). The estimated positive effect of $IPR$ on income inequality contrasts sharply to the regression results based on the full sample. These empirical findings for HRPC are robust to alternative income inequality measures. We find that the estimates of $\beta_1$ become more significant when income inequality is proxied by income ratios. However, the regressions under Columns (5), (6) and (9) do not yield statistically significant point estimates of $\beta_2$, implying that increased relative productivity does not necessarily reduce the inequality-amplifying effect of strengthened patent protection.

Table 4 reports the regression results for countries with low relative productivity. Under the Gini coefficient (WIID) measure, Columns (1) and (2) indicate that the estimated effects of patent protection are significantly negative with and without the interaction term $IPR \times RP$. As shown in Column (3), the regression results remain largely the same when GDP per capita is added. In addition, across all measures of income inequality, our regressions consistently yield a strong negative effect of $IPR$ among LRPC, the qualitative pattern of which is similar to the estimation using the full sample but substantially distinct from that of the HRPC group. Comparing Table 4 with Table 2, we also find that the magnitude of the estimated coefficient on $IPR$ among LRPC is systematically larger than its counterpart using the full sample, which might be an indicator that regressions mixing HRPC with LRPC not only incorrectly capture the effect of patent protection on income distribution in high relative productivity countries, but also tend to underestimate its effect on narrowing the income dispersion among countries with low relative productivity.

Considering that the grouping criterion under the baseline regression might be arbitrary, we perform an additional robustness check by raising the threshold value $\alpha^*$ to 8.5%, which effectively shortens the list of HRPC to 10 countries (from Israel to Iceland in the list). As reported in Tables 6 and 7 in Appendix A.2, the qualitative effects of $IPR$ for both groups were consistent with those under the baseline estimation. In the following section, we propose a growth-theoretic model to rationalize the relative-productivity-contingent relationship between patent strength and income gap.[16]

# 3 Model

In this section, we follow Iwaisako and Futagami (2013) and Chu *et al.* (2019) to extend the Romer (1990)'s variety-expansion model by allowing two independent growth engines: technological innovation and physical capital accumulation. To investigate the linkages between patent protection and income dispersion, we (a) introduce heterogeneous households in terms of asset holdings as in García-Peñalosa and Turnovsky (2006) as the source of income inequality, and (b) incorporate patent breadth in terms of the degree of firms' market power, as in Goh and Olivier (2002), as the patent policy instrument.

## 3.1 Households

The economy is populated by a unit continuum of infinitely-lived households, each indexed by $h \in [0,1]$. Households have identical preferences for consumption $C_s(h)$, but differ in asset holdings. At time $t$, the lifetime utility of household $h$ is as follows:

$$U_t = \int_t^{\infty} e^{-\rho(s-t)} \ln C_s(h) ds, \tag{2}$$

where $C_s(h)$ is the consumption of final good at an instant of time $s$, and $\rho > 0$ represents the subjective discount factor. Each household earns the wage income $W_t$ via inelastically supplying one unit of labor.[17] Similar to Chu *et al.* (2021) and Yang (2021), the evolution of household $h$'s

[16]It is worth explaining that we also add squared-$IPR$ to the regressions and examine the possible nonlinear relationship between patent protection and income inequality in the full sample. Under the measure of Gini coefficient (WIID), we identify a potential U-shaped $IPR$-inequality relation, which indicates that the effect of $IPR$ might be contingent upon the degree of patent protection in a country. Therefore, if countries with a high degree of patent protection happen to exhibit high relative productivity, then the relative productivity scenario may not hold. Two observations, however, alleviate this concern. First, as shown in Table 8 in Appendix A.2, the nonlinear effect of patent strength is not robust to measures of the income ratio (namely $T10/B10$ and $T20/B20$), whereas the relative productivity scenario is robust to all inequality measures. Second, Table 9 in Appendix A.2 clearly shows that the group of countries exhibiting high average PPR is different from the HRPC group.

[17]For tractability, our baseline analysis only considers the framework with inelastic labor. In subsection 6.2, we show that extending the baseline model to allow for an iso-elastic function of consumption and leisure yields similar analytical results.

Table 3: Effect of $IPR$ on income inequality in HRPC – baseline estimation

| | Gini (WIID) | | | $T10/B10$ | | | $T20/B20$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $IPR$ | 0.14 | 0.59* | 0.56* | 0.35** | 0.50*** | 0.48*** | 0.12* | 0.20*** | 0.18** |
| | (0.34) | (0.30) | (0.31) | (0.14) | (0.11) | (0.14) | (0.06) | (0.06) | (0.07) |
| $IPR \times RP$ | | -0.05*** | -0.05*** | | -0.02 | -0.02 | | -0.01* | -0.01 |
| | | (0.01) | (0.01) | | (0.01) | (0.01) | | (0.01) | (0.01) |
| Trade Openness | -0.05* | -0.06** | -0.06** | -0.03** | -0.03** | -0.03** | -0.01 | -0.01* | -0.01* |
| | (0.03) | (0.02) | (0.02) | (0.01) | (0.02) | (0.01) | (0.01) | (0.01) | (0.01) |
| Human Capital | -12.86** | -12.73*** | -13.73*** | -8.61*** | -8.58*** | -9.20*** | -3.64*** | -3.63*** | -4.02*** |
| | (4.84) | (4.63) | (5.16) | (1.98) | (1.90) | (2.30) | (0.75) | (1.71) | (0.91) |
| Gov. Spending to GDP Ratio | -0.55 | -0.34 | -0.86 | -2.23 | -2.21 | -2.61 | -0.63 | -0.62 | -0.88 |
| | (3.52) | (3.25) | (2.97) | (1.57) | (1.61) | (1.96) | (0.60) | (0.64) | (0.73) |
| Inflation | -0.03 | -0.02 | -0.03 | -0.02* | -0.02* | -0.02* | -0.01 | -0.01 | 0.01 |
| | (0.03) | (0.03) | (0.03) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) |
| R&D to GDP Ratio | 6.89*** | 10.93*** | 10.57*** | 3.26*** | 4.64*** | 4.47*** | 1.08*** | 1.79*** | 1.68*** |
| | (2.14) | (1.71) | (1.51) | (1.10) | (1.17) | (1.41) | (0.39) | (0.44) | (0.54) |
| Credit to GDP ratio | 3.96* | 2.61 | 2.59 | 3.03*** | 2.61** | 2.60** | 1.02*** | 0.81** | 0.80*** |
| | (2.30) | (2.14) | (2.13) | (1.02) | (1.09) | (1.09) | (0.39) | (0.32) | (0.31) |
| GDP per capita | | | 1.78 | | | 1.50 | | | 0.96 |
| | | | (4.43) | | | (2.68) | | | (1.02) |
| Control | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Obs. | 185 | 185 | 185 | 175 | 175 | 175 | 175 | 175 | 175 |
| $R^2$ | 0.17 | 0.21 | 0.21 | 0.11 | 0.12 | 0.12 | 0.17 | 0.19 | 0.19 |

*Notes*: Estimation results reported in the table are based on the subsample of HRPC. The grouping criterion sets $\alpha^* = 8\%$, under which the top 13 countries (from IL to NL) in Table 1 fall into the HRPC category. Gini (WIID) denotes the Gini coefficient published in the World Income Inequality Database. $T10/B10$ denotes the ratio of income share of the 10th decile (top) to that of the 1st decile (bottom); and $T20/B20$ refers to the ratio of income share of the top 20% households to that of the bottom 20% households. The sample period is 2000-2018. GDP per capita, government spending to GDP ratio, R&D to GDP ratio and credit to GDP ratio are in logarithm. GDP per capita is lagged by five periods, and other control variables are lagged by one period. Robust standard errors clustered by country are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

assets equals the sum of the return on assets and the labor wage net of consumption. Therefore, household $h$'s asset-accumulation equation is as follows:

$$\dot{A}_t(h) = R_t A_t(h) + W_t - C_t(h), \tag{3}$$

Table 4: Effect of $IPR$ on income inequality in LRPC – baseline estimation

| | Gini (WIID) | | | $T10/B10$ | | | $T20/B20$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $IPR$ | -0.48* | -0.85*** | -0.83*** | -1.22* | -2.30** | -2.17** | -0.33* | -0.61** | -0.58*** |
| | (0.27) | (0.30) | (0.30) | (0.68) | (0.98) | (0.89) | (0.18) | (0.24) | (0.22) |
| $IPR \times RP$ | | 0.09*** | 0.09*** | | 0.28** | 0.26** | | 0.07** | 0.07** |
| | | (0.04) | (0.04) | | (0.13) | (0.12) | | (0.03) | (0.03) |
| Trade Openness | -0.01 | 0.01 | 0.01 | 0.02 | -0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| | (0.01) | (0.01) | (0.01) | (0.02) | (0.01) | (0.02) | (0.01) | (0.01) | (0.01) |
| Human Capital | 0.01 | 2.03 | 1.89 | -5.01 | -0.95 | -2.01 | -0.91 | 0.12 | -0.08 |
| | (3.54) | (3.11) | (3.15) | (7.79) | (5.17) | (5.81) | (2.34) | (1.77) | (1.87) |
| Gov. Spending to GDP Ratio | 1.52 | 0.96 | 0.94 | -7.62 | -9.25* | -9.43* | -1.97 | -2.38 | -2.41 |
| | (2.49) | (2.42) | (2.36) | (5.74) | (5.51) | (4.95) | (1.64) | (1.57) | (1.46) |
| Inflation | -0.05** | -0.05** | -0.05** | -0.04 | -0.02 | -0.03 | -0.01 | -0.01 | -0.01 |
| | (0.02) | (0.02) | (0.02) | (0.02) | (0.03) | (0.03) | (0.01) | (0.01) | (0.01) |
| R&D to GDP Ratio | -3.15* | -5.48*** | -5.33*** | -6.66 | -13.72* | -12.39* | -1.60 | -3.40** | -3.15** |
| | (1.63) | (1.87) | (1.82) | (5.12) | (7.17) | (6.41) | (1.22) | (1.64) | (1.51) |
| Credit to GDP ratio | 1.01 | 1.16 | 1.46 | 1.76 | 2.46 | 4.97 | 0.36 | 0.53 | 1.01 |
| | (1.32) | (1.08) | (1.36) | (2.15) | (2.05) | (3.05) | (0.69) | (0.61) | (0.84) |
| GDP per capita | | | -1.24 | | | -10.79* | | | -2.05 |
| | | | (3.35) | | | (6.10) | | | (1.42) |
| Control | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Obs. | 291 | 291 | 291 | 286 | 286 | 286 | 286 | 286 | 286 |
| $R^2$ | 0.11 | 0.15 | 0.15 | 0.06 | 0.10 | 0.11 | 0.10 | 0.18 | 0.19 |

*Notes*: Estimation results reported in the table are based on the subsample of LRPC. The grouping criterion sets $\alpha^* = 8\%$, under which the bottom 25 countries (from NO to CO) in Table 1 fall into the LRPC category. The sample period is 2000-2018. Measures of dependent and independent variables are the same as those in Table 3. Robust standard errors clustered by country are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

where $A_t(h)$ denotes the total value of innovation assets (i.e., the share of monopolistic firms) and capital assets (i.e., the value of capital stock) possessed by household $h$. $R_t$ represents the rate of interest. We define by $\theta_{A,t}(h) \equiv A_t(h)/A_t$ the relative wealth owned by household $h$ at time $t$, where $A_t \equiv \int_0^1 A_t(h)$ is the average (aggregate) value of assets. At any instant of time $t$, the relative asset has mean 1, and its distribution has a standard deviation of $\sigma_{A,t} > 0$, with the initial and exogenously given dispersion $\sigma_{A,0}$.

Solving household $h$'s utility-maximization problem yields the familiar Euler equation, such that

$$\frac{\dot{C}_t(h)}{C_t(h)} = R_t - \rho. \tag{4}$$

Equation (4) reveals that the consumption growth rate is identical across all households, regardless of their asset holdings. Define by $C_t \equiv \int_0^1 C_t(h)dh$ the average economy-wide consumption. We then obtain $\dot{C}_t/C_t = \dot{C}_t(h)/C_t(h) = R_t - \rho$.

### 3.2 Final good

The economy produces a unique final good $Y_t$ that is all for consumption. We assume that $Y_t$ is produced by a mass of perfectly competitive firms using the following production function:

$$Y_t = (Z_t)^{1-\alpha} \int_0^{N_t} [X_t(j)]^\alpha \, dj, \tag{5}$$

where $X_t(j)$ is the quantity of intermediate goods in industry $j \in [0, N_t]$; $Z_t$ denotes a fixed input factor such as land and other natural resources; $N_t$ measures the variety of intermediate goods; and $0 < \alpha < 1$ governs the elasticity of demand for $X_t(j)$. As $Z_t$ is assumed to be fixed, its level over time always equals the initial endowment $Z_0$. Accordingly, the conditional demand function for $X_t(j)$ is given by

$$P_t(j) = \alpha \left[ \frac{Z_0}{X_t(j)} \right]^{1-\alpha}, \tag{6}$$

where $P_t(j)$ is the price of $X_t(j)$ relative to the final good, chosen as the numeraire in the economy.

### 3.3 Intermediate goods

Intermediate goods in industry $j$ are manufactured by a monopolist with a patent for the state-of-the-art technology. The monopolistic firm in industry $j$ uses physical capital $K_t(j)$ and labor $L_{x,t}(j)$ for production, and adopts the following technology:

$$X_t(j) = \beta [K_t(j)]^\gamma [L_{x,t}(j)]^{1-\gamma}, \tag{7}$$

where $\beta > 0$ is the productivity parameter and $\gamma \in (0, 1)$ governs the share of capital in production. Solving the cost-minimization problem faced by the intermediate-good producer for variety $j$ yields the function of marginal cost:

$$MC_t(j) = \frac{1}{\beta} \left( \frac{Q_t}{\gamma} \right)^\gamma \left( \frac{W_t}{1-\gamma} \right)^{1-\gamma}, \tag{8}$$

where $Q_t$ denotes the rental rate of capital.

Following previous studies such as Li (2001), Iwaisako and Futagami (2013), and Chu *et al.*

(2019), we assume that patent breadth is incomplete, which implies that the price of the intermediate good $j$ charged by the monopolistic producer will be limited.[18] To be specific, $\mu \in (1, 1/\alpha]$ captures the degree of patent breadth, which in turn determines the price markup.[19] In this case, the profit-maximizing price is $P_t(j) = \mu MC_t(j)$.[20] Conditional on this pricing strategy, along with the marginal cost function in (8), the incumbent maximizes her profit by appropriately choosing $K_t(j)$ and $L_{x,t}(j)$, subject to equations (6) and (7). Therefore, the profit function of the intermediate-good producer $j$ is as follows:

$$\Pi_{x,t}(j) = \left(\frac{\mu - 1}{\mu}\right) P_t(j) X_t(j). \tag{9}$$

For labor and capital inputs, the factor payments satisfy

$$W_t L_{x,t}(j) = \left(\frac{1-\gamma}{\mu}\right) P_t(j) X_t(j), \tag{10}$$

$$Q_t K_t(j) = \left(\frac{\gamma}{\mu}\right) P_t(j) X_t(j). \tag{11}$$

### 3.4 Inventions and R&D

Let $V_{n,t}(j)$ denote the value of a firm in variety $j$. Following the conventional literature (e.g., Cozzi *et al.* 2007), we restrict attention to the symmetric equilibrium, where $\Pi_{x,t}(j) = \Pi_{x,t}$ and $V_{n,t}(j) = V_{n,t}$. For asset value $V_{n,t}$, applying the no-arbitrage condition yields

$$R_t V_{n,t} = \Pi_{x,t} + \dot{V}_{n,t}, \tag{12}$$

which indicates that the asset return $R_t V_{n,t}$ should equal the sum of the monopolistic profit flow $\Pi_{x,t}$ and the potential capital gain $\dot{V}_{n,t}$.

There are a mass of competitive R&D firms inventing new innovations for each variety. Assume that inventions are produced by R&D labor $L_{r,t}$. The R&D technology is specified as $\dot{N}_t = \varphi N_t L_{r,t}$, where the parameter $\varphi > 0$ governs the innovation productivity. The expected profit of R&D firms is $\Pi_{r,t} = V_{n,t}\dot{N}_t - W_t L_{r,t}$. Free entry into the R&D sector ensures the following zero-expected profit condition:

$$\varphi N_t V_{n,t} = W_t. \tag{13}$$

[18] When patent breadth is incomplete, the incumbent in industry $j$ faces potential profit-driven entry from competitive fringes. Assuming that entrants' marginal cost of production (implied by the imitation cost) is higher than that of the incumbents, these incumbents will charge a monopoly price that is limited by the fringes' costs.

[19] Note that the maximum value of $\mu$, namely $1/\alpha$, is given by the unconstrained markup charged by the monopolistic firms.

[20] See Appendix B.1 for details.

## 3.5 Capital production

Let $V_{k,t}(j)$ denote the value of per-unit capital in variety $j$. In a symmetric equilibrium, $V_{k,t}(j) = V_{k,t}$ holds. For $V_{k,t}$, the no-arbitrage condition regulates that

$$R_t V_{k,t} = Q_t + \dot{V}_{k,t} \tag{14}$$

Equation (14) suggests that the asset return $R_t V_{k,t}$ equals the sum of the potential capital gain $\dot{V}_{k,t}$ and the rental rate of capital $Q_t$.

A mass of capital-producing firms produce capital goods for each variety. These firms employ capital-producing labor $L_{k,t}$ for production. The expected profit of firms providing capital goods is $\Pi_{k,t} = V_{k,t}\dot{K}_t - W_t L_{k,t}$, where $\dot{K}_t = \phi A_{k,t} L_{k,t}$ is the quantity of these firms' output depending on the existing knowledge of $A_{k,t}$ in production, and $\phi > 0$ is the productivity parameter. Following Romer (1986), Iwaisako and Futagami (2013), and Chu *et al.* (2019), we assume that the existing knowledge in production equals the current stock of capital such that $A_{k,t} = K_t$. Hence, the zero-expected-profit condition resulting from free entry into the capital-producing sector suggests

$$\phi K_t V_{k,t} = W_t. \tag{15}$$

## 3.6 Decentralized equilibrium

Let $L_{x,t} = \int_0^1 L_{x,t}(j)dj$ and $A_t = \int_0^1 A_t(h)dh$, respectively, denote the aggregate demand of manufacturing labor and asset holdings. We define the general equilibrium as follows:

**Definition 1.** *The general equilibrium consists of the sequences of aggregate variables* $\{A_t, C_t, Y_t, X_t, L_{x,t}, L_{r,t}, L_{k,t}\}_{t=0}^{\infty}$ *and aggregate prices* $\{P_t(j), W_t, R_t, V_{n,t}, V_{k,t}\}_{t=0}^{\infty}$*, for* $j \in [0,1]$*. At each instant in time, households maximize their utility, and firms, including the final-good, intermediate-good, R&D, and capital-producing firms, maximize their profits, and all markets are clear. Specifically, the market-clearing conditions for the final good and capital goods are* $C_t = Y_t$ *and* $K_t = \int_0^{N_t} K_t(j)dj$*, respectively. The asset and labor market clear such that*

$$V_{n,t}N_t + V_{k,t}K_t = A_t, \tag{16}$$

*and*

$$L_{x,t} + L_{r,t} + L_{k,t} = 1, \tag{17}$$

*respectively.*

The dynamics of the model is characterized by the following lemma.

**Lemma 1.** *Holding the level of patent breadth* $\mu$ *constant, the economy immediately jumps to a unique* *and stable balanced growth path. Moreover, the equilibrium allocations are stationary and are given by*

$$L_x = \frac{(1-\gamma)(1+\rho/\varphi+\rho/\phi)}{\mu}, \tag{18}$$

$$L_r = \frac{(\mu-1)(1+\rho/\varphi+\rho/\phi)}{\mu} - \frac{\rho}{\varphi}, \tag{19}$$

$$L_k = \frac{\gamma(1+\rho/\varphi+\rho/\phi)}{\mu} - \frac{\rho}{\phi}. \tag{20}$$

*Proof.* See Appendix B.2. □

Equations (19) and (20) show that R&D labor $L_r$ is increasing in patent breadth $\mu$, whereas capital-producing labor $L_k$ is decreasing in $\mu$.

### 3.7 Growth effect of patent protection

Substituting equation (7) into equation (5) yields the equilibrium production function for final good such that $Y_t = Z_t^\alpha N_t^{1-\alpha} X_t^{1-\alpha}$. Differentiating its log with respect to time $t$ yields

$$g_y \equiv \frac{\dot{Y}_t}{Y_t} = (1-\alpha)\frac{\dot{N}_t}{N_t} + \alpha\gamma\frac{\dot{K}_t}{K_t} = (1-\alpha)\varphi L_r + \alpha\gamma\phi L_k, \tag{21}$$

where we have applied the condition that $Z_t = Z_0$ is time invariant. The growth effect of strengthened patent protection is similar to that of Iwaisako and Futagami (2013), Chu *et al.* (2019), and Yang (2021), such that

$$\frac{\partial g_y}{\partial \mu} = \frac{(1+\rho/\varphi+\rho/\phi)[\varphi(1-\alpha)-\alpha\gamma^2\phi]}{\mu^2} \gtrless 0 \Leftrightarrow \frac{\varphi}{\phi} \gtrless \frac{\alpha\gamma^2}{1-\alpha}. \tag{22}$$

Intuitively, R&D investment increases in response to a broadened patent breadth, which reduces capital production at the same time. Consequently, strengthening patent protection enhances (retards) economic growth when the relative productivity of R&D to capital is high (low). The above results are in line with the mixed empirical evidence on the relationship between growth and patent policies in the literature.[21]

**Proposition 1.** *If the relative R&D productivity is high (low), that is, $\varphi/\phi > (<)\alpha\gamma^2/(1-\alpha)$, broadening the patent breadth $\mu$ is growth-enhancing (retarding).*

*Proof.* Proven in the text. □

[21]See Yang (2021) for a survey about the patent-and-growth empirical relation.

# 4 Patent protection and income inequality

Following García-Peñalosa and Turnovsky (2006) and Chu and Cozzi (2018), this section first shows the stationarity of households' wealth distribution along the balanced growth path, and then examines the effect of patent breadth on income inequality.

## 4.1 Wealth distribution

Aggregating equation (3) by $h$ yields

$$\dot{A}_t = R_t A_t + W_t - C_t. \tag{23}$$

Combining equation (3) with equation (23) yields the dynamics of $\theta_{A,t}(h)$, such that

$$\dot{\theta}_{A,t}(h) = \left(\frac{\dot{A}_t(h)}{A_t(h)} - \frac{\dot{A}_t}{A_t}\right)\theta_{A,t}(h) = \left(\frac{C_t - W_t}{A_t}\right)\theta_{A,t}(h) - \frac{C_t\theta_{C,t}(h) - W_t}{A_t}, \tag{24}$$

where $\theta_{C,t}(h) \equiv C_t(h)/C_t$ is the relative consumption of household $h$ at time $t$. The distribution of wealth is shown to be stationary over time through the following lemma:

**Lemma 2.** *For any given patent breadth $\mu$, the relative wealth of each household $h$ is constant over time and exogenously given at time 0 such that $\theta_{A,t}(h) = \theta_{A,0}(h)$ for all time $t$, which is achieved by $\theta_{C,t}(h)$ jumping to its steady-state value $\theta_{C,0}(h)$ given by*[22]

$$\theta_{C,0}(h) = 1 + \frac{\rho/\varphi + \rho/\phi}{1 + \rho/\varphi + \rho/\phi}\left[\theta_{A,0}(h) - 1\right]. \tag{25}$$

*Proof.* See Appendix B.3. □

## 4.2 Income distribution

It is straightforward to see that the total income of any individual household $h$ is $I_t(h) = R_t A_t(h) + W_t$, whereas the average income of all households in the model economy is $I_t = R_t A_t + W_t$. Hence, the relative income of household $h$ is given by

$$\theta_{I,t}(h) \equiv \frac{I_t(h)}{I_t} = \frac{R_t A_t(h) + W_t}{R_t A_t + W_t} = \frac{R_t A_t \theta_{A,0}(h)}{R_t A_t + W_t} + \frac{W_t}{R_t A_t + W_t}, \tag{26}$$

[22] Equation (25) implies that consumption inequality, defined as $\sigma_C \equiv \sqrt{\int_0^1 [\theta_{C,t}(h) - 1]^2 dh} = \frac{\rho(1/\varphi+1/\phi)}{1+\rho(1/\varphi+1/\phi)}\sigma_A$ is unaffected by the patent lever $\mu$ in this model.

where we have used $\theta_{A,t}(h) = \theta_{A,0}(h)$. In particular, $\theta_{I,t}(h)$ is interpreted as the relative income distribution function, the mean of which is unity. The standard deviation of $\theta_{I,t}(h)$ is given by

$$\sigma_{I,t} = \sigma_I \equiv \sqrt{\int_0^1 [\theta_{I,t}(h) - 1]^2 dh} = \frac{R_t A_t / W_t}{1 + R_t A_t / W_t} \sigma_A. \tag{27}$$

According to equation (27), an increase in the ratio of asset income $R_t A_t$ to wage income $W_t$ leads to a higher $\sigma_I$, which widens the income dispersion. This finding is similar to the analytical result in Chu and Cozzi (2018). At the aggregate level, our model suggests that the degree of patent protection shapes the income distribution through two channels, namely the asset-value (i.e., $A_t/W_t$) channel and the interest-rate (i.e., $R_t$) channel.

By substituting equations (13) and (15) into (16), we derive the following asset-wage ratio:

$$\frac{A_t}{W_t} = \frac{1}{\varphi} + \frac{1}{\phi}. \tag{28}$$

Together with $R_t = \rho + g$ in equation (4), we can reexpress equation (27) as

$$\sigma_I = \frac{(\rho + g)(1/\varphi + 1/\phi)}{1 + (\rho + g)(1/\varphi + 1/\phi)} \sigma_A. \tag{29}$$

Equation (28) shows that the asset-wage ratio is invariant to the change in patent breadth parameter $\mu$, which implies that patent breadth only affects the distribution of income through the interest-rate channel. This is because equations (18)-(20) show that patent breadth $\mu$ affects the labor allocation between manufacturing and the growth engines, which is associated with the aggregate growth effect and also the interest rate according to the Euler equation (4). However, $\mu$ does not affect the values of innovation assets and capital assets (relative to the wage); in fact, the free-entry conditions in equations (13) and (15) reveal that the value of innovation assets relative to that of capital assets is contingent on sectoral productivity. In this case, the effect of $\mu$ on economic growth reduces its effect on income inequality through the interest-rate channel. Consequently, according to Proposition 1, increasing $\mu$ increases (decreases) the rate of economic growth $g$ and, thereby, the degree of income inequality $\sigma_A$ when the relative R&D productivity $\varphi/\phi$ is sufficiently high (low). Proposition 2 summarizes the above results.

**Proposition 2.** *The degree of income inequality increases (decreases) in patent breadth $\mu$ if $\varphi/\phi > (<) \alpha\gamma^2/(1-\alpha)$.*

*Proof.* Proven in text. □

Accordingly, the implication of Proposition 2 provides a theoretical rationale for the co-movement between patent breadth and income inequality in the US and other industrialized countries (e.g., countries in the OECD), as shown by the results in Section 2.

# 5 Quantitative analysis

Calibrating the model to the US data, we quantitatively investigate the effects of strengthening patent protection policy on economic growth and income inequality, respectively. To perform a robustness analysis, we redo the quantitative exercises by altering the key structural parameters and empirical moments.

## 5.1 Calibration

The six structural parameters that we intend to calibrate in this model are $\{\rho, \mu, \alpha, \gamma, \varphi, \phi\}$.[23] The subjective discount rate $\rho$ is set to 0.02, which is standard in the literature. Consistent with the average markup ratio estimate in Bils and Klenow (2004), we set the level of patent breadth $\mu$ to 1.2.

For the remaining four parameters, we simultaneously choose their values to match the relevant statistics. Following Akcigit and Kerr (2018), we minimize the distance between the sample moments and the model-implied counterparts according to the following criterion:

$$\min \sum_{\iota=1}^{4} \frac{|model(\iota) - data(\iota)|}{|model(\iota)|/2 + |data(\iota)|/2},$$

where we index each moment by $\iota$. The four moments we target are as follows: (1) According to Chu *et al.* (2019), the capital growth rate was 3.07% from 1999 to 2010; (2) Real GDP per capita grew averagely at an annual rate of 1.2% during the 2000-2019 time window; (3) According to the data on researchers per 1000 employees from the OECD Research and Development database and the data on the number of manufacturing workers from the Bureau of Labor Statistics, we compute the R&D-to-manufacturing labor ratio in the US, which was 8.38% from 2000 to 2018;[24] (4) The average US R&D investment to GDP ratio was around 2.71% per annum during the period of 2000-2019, which corresponds to the ratio of $w_t L_r / (w_t L_r + w_t L_x + Y_t)$ in our model. Exploiting this procedure, we report the calibrated values and the model-implied moments in Table 5.[25]

## 5.2 Benchmark simulation

Given the above mentioned parameter values, Figure 1a shows that strengthening patent protection induces a higher economic growth rate. In this benchmark analysis, the relative productiv-

[23] Since neither the steady-state growth rate nor the income distribution is affected by $\beta$, we do not need to pin down its value.

[24] The data on on researchers per 1000 employees is available before 2018.

[25] Constrained by the ranges that certain parameters must fall into, the model-implied moments did not perfectly match the data. The constraints include that (a) all parameters must be non-negative; (b) the intermediate goods elasticity $\alpha$ must satisfy $\mu \leq 1/\alpha$; and (c) the capital share of intermediate goods $\gamma$ must be in the $[0, 1]$ interval. Otherwise, the economic interpretation can hardly be meaningful.

Table 5: Benchmark parameter values and model fit

| Parameters taken from external sources | | |
|---|---|---|
| Parameters | Interpretation | Value |
| $\rho$ | Subjective discount rate | 0.02 |
| $\mu$ | Price markup | 1.2 |

| Calibrated parameters and model fit | | | | |
|---|---|---|---|---|
| Parameters | Value | Moments | Data | Model |
| Intermediate goods elasticity, $\alpha$ | 0.803 | Capital growth rate | 3.07% | 3.11% |
| Capital share in intermediate goods, $\gamma$ | 0.419 | Per capita economic growth rate | 1.2% | 1.17% |
| R&D productivity, $\varphi$ | 0.117 | R&D-to-manufacturing labor ratio | 8.38% | 8.39% |
| Capital productivity, $\phi$ | 0.108 | R&D-to-GDP ratio | 2.71% | 2.71% |

ity of R&D to capital (i.e., $\varphi/\phi$) exceeds the threshold value that determines the sign of the impact of patent breadth on economic growth as specified in equation (34) (i.e., $\alpha\gamma^2/(1-\alpha) = 0.716$). In this case, the negative growth effect of patent protection through the capital channel is dominated by the positive effect from through the innovation channel. In addition, Proposition 2 suggests that strengthening patent protection amplifies income inequality when it is growth enhancing. As depicted in Figure 1b, raising the level of patent breadth from 1.05 to 1.2 causes a rise in the degree of income inequality by 2.65%.[26]

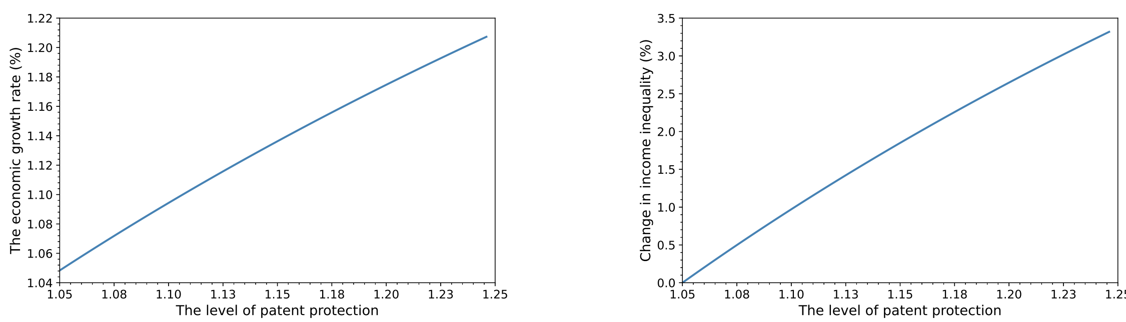


Figure 1: (a) Economic growth and patent protection; (b) Income inequality and patent protection.

Following Yang (2021), we hold $\phi$ constant and reduce the value of $\varphi$ such that $\varphi/\phi = 0.5$, which falls below the threshold $\alpha\gamma^2/(1-\alpha) = 0.716$. Figure 2a and 2b show that increased degree of patent protection reduces the economic growth rate and mitigates income inequality. This is because under this alternative value of $\varphi/\phi$, the growth-retarding effect of patent protection via the capital channel dominates the growth-enhancing effect of patent protection via the R&D channel. This negative growth effect in turn leads to a negative interest-rate effect. Consequently,

[26]Our policy experiments focus on $\mu \in (1, 1.245]$, since the patent breadth parameter must satisfy $\mu \leq 1/\alpha = 1.245$.

the income dispersion becomes narrowed in response to strengthened patent protection, which is still consistent with the implication of Proposition 2. In this case, raising the level of patent breadth from 1.05 to 1.2 causes the degree of income inequality to fall by 1.38%.

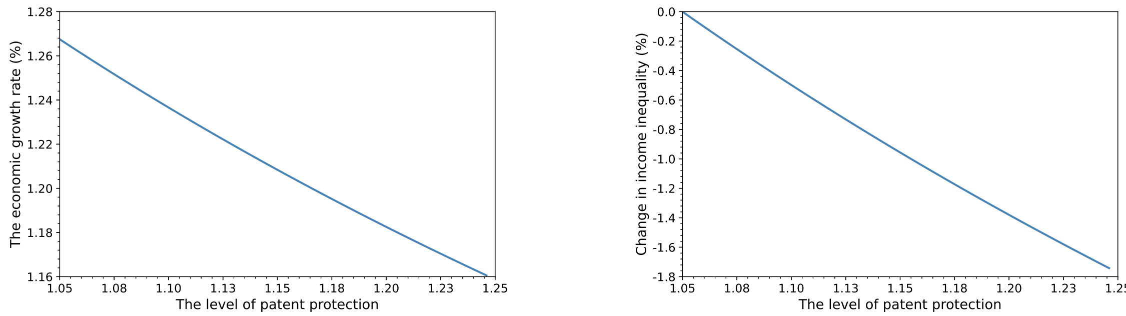


Figure 2: (a) Economic growth and patent protection; (b) Income inequality and patent protection.

## 5.3 Robustness checks

This subsection presents two additional numerical exercises for the purpose of robustness check. First, we assign an alternative value to the capital growth rate. Second, we investigate the quantitative results when R&D intensity takes other reasonable values.

### 5.3.1 Capital growth rate

Preserving the rest of empirical moments as in the benchmark, we recalibrate the parameter values by setting the capital growth rate $g_k$ to 3.4%, according to the Conference Board of the Total Economy Database. In this case, the recalibrated parameter values were given by $\{\alpha = 0.77, \gamma = 0.397, \varphi = 0.121, \phi = 0.123\}$. Under this new set of parameter values, the ratio of R&D to capital productivity is still above the threshold value, that is, $\varphi/\phi = 0.984 > \alpha\gamma^2/(1-\alpha) = 0.528$. Therefore, as shown in Figures 3a and 3b, the economic growth rate and the degree of income inequality are increasing in patent breadth, the qualitative pattern of which is similar to the benchmark results.

### 5.3.2 R&D-to-GDP ratio

Comin (2004) points out that the true value of US firms' expenditure on innovation-related activities is likely to be higher than the one captured by the data. Given this concern, we use a slightly larger value (3.0%) to re-calibrate the model.[27] In this case, the recalibrated parameter

[27]We find that choosing a larger R&D-to-GDP ratio leads the calibrated value of $\alpha$ to approach unity, which narrows the parameter space of patent breadth, given that the condition of the markup range $\mu \leq 1/\alpha$ must be satisfied.

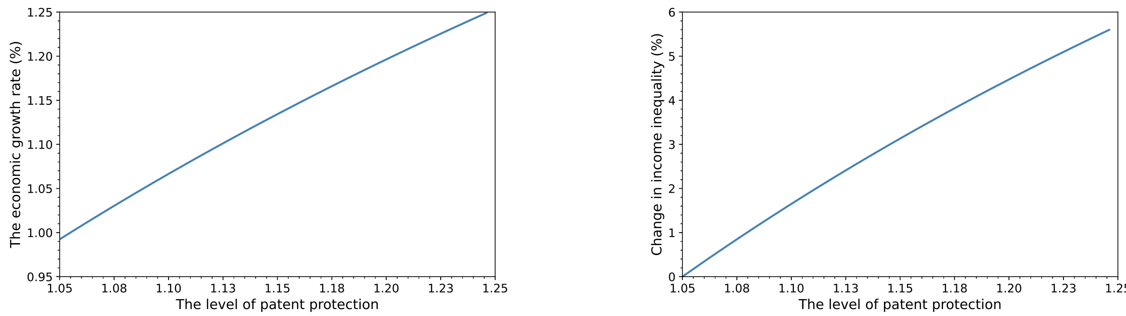


Figure 3: (a) Economic growth and patent protection; (b) Income inequality and patent protection.

values are given by $\{\alpha = 0.819, \gamma = 0.418, \varphi = 0.121, \phi = 0.108\}$. Under this new set of parameter values, we find that the relative productivity of R&D to capital is also above the threshold value, that is, $\varphi/\phi = 1.120 > \alpha\gamma^2/(1-\alpha) = 0.791$. Therefore, the positive growth effect of patent protection through the R&D channel continues to dominate the negative growth effect via the capital channel, leading the economic growth rate and income inequality still to be increasing in the level of patent protection, as shown in Figures 4a and 4b.

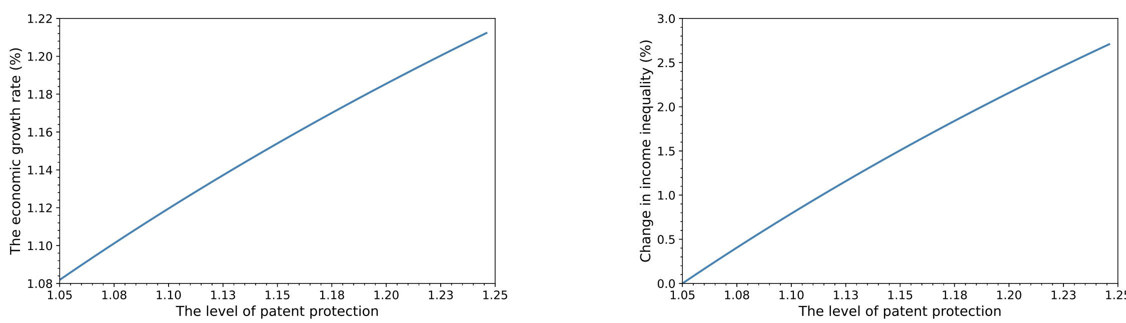


Figure 4: (a) Economic growth and patent protection; (b) Income inequality and patent protection.

# 6 Extensions

In this section, we extend the baseline model along three dimensions: (a) relaxing the assumption of the Cobb-Douglas final-good production function; (b) considering an iso-elastic utility function of consumption and leisure; and (c) introducing liquidity constraints on the innovating and capital-producing sectors. We show that incorporating these additional features into the model yields robust analytical results.

## 6.1 Final-good production specification

In this extension, we consider the CES production function of final goods. Specifically, the final-good production function takes the following form:

$$Y_t = \left[ \int_0^{N_t} X_t(j)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{30}$$

where $\sigma > 1$ is the elasticity of substitution between the intermediate inputs. Hence, the conditional demand function for $X_t(j)$ becomes:

$$X_t(j) = Y_t P_t(j)^{-\sigma}. \tag{31}$$

The settings in the intermediate-good, R&D, and capital-producing sectors remain unchanged, as in Section 3. Following the same logic as in the baseline model, we derive the equilibrium production function for the final good in this extended model such that

$$Y_t = \beta \left[ \int_0^{N_t} N_t \left( \frac{K_t}{N_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} \left( \frac{L_{x,t}}{N_t} \right)^{\frac{(1-\gamma)(\sigma-1)}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = \beta K_t^{\gamma} L_x^{1-\gamma} N_t^{\frac{1}{\sigma-1}}. \tag{32}$$

Then, log-differentiating equation (32) with respect to time $t$ yields

$$g = \left( \frac{1}{\sigma-1} \right) \frac{\dot{N}_t}{N_t} + \gamma \frac{\dot{K}_t}{K_t} = \frac{\varphi(\mu-1)}{\mu(\sigma-1)} \left( 1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi} \right) + \frac{\phi\gamma^2}{\mu} \left( 1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi} \right) - \rho \left( \gamma + \frac{1}{\sigma-1} \right). \tag{33}$$

Differentiating equation (33) with respect to patent breadth $\mu$ yields the growth effect of patent policy such that

$$\frac{\partial g}{\partial \mu} = \frac{1}{\mu^2} \left( 1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi} \right) \left( \frac{\varphi}{\sigma-1} - \phi\gamma^2 \right),$$

which implies

$$\frac{\partial g}{\partial \mu} \gtrless 0 \Leftrightarrow \frac{\varphi}{\phi} \gtrless \gamma^2(\sigma-1). \tag{34}$$

This result is identical to the implications of Proposition 1. Moreover, the baseline results of patent protection and income inequality continue to hold because this modification does not change the channels through which patent breadth shapes the distribution of income; the *interest-rate effect* continues to act in the patent-inequality relation.

## 6.2 Utility function

In this extension, we assume that leisure, in addition to consumption, enters households' utility function, and equation (2) is modified to an iso-elastic function of consumption and leisure for household $s$ such that

$$U_t = \int_t^{\infty} e^{-\rho(s-t)} \frac{\left[C_s(h) L_s^{\delta}(h)\right]^{1-\eta} - 1}{1-\eta} ds, \tag{35}$$

where $L_s(h)$ is the leisure of household $h$, the parameter $\delta > 0$ determines the intensity of leisure preference relative to consumption, and the parameter $\eta > 0$ (but $\eta \neq 1$) is the inverse of the intertemporal substitution elasticity. When $\eta = 1$, equation (35) is reduced to the log-utility case in equation (2). Accordingly, the asset-accumulation equation in (3) becomes

$$\dot{A}_t(h) = R_t A_t(h) + W_t[1 - L_t(h)] - C_t(h). \tag{36}$$

Solving the utility-maximization problem yields the consumption-leisure decision and the Euler equation such that

$$W_t L_t(h) = \delta C_t(h), \tag{37}$$

and

$$\frac{\dot{C}_t(h)}{C_t(h)} = \frac{R_t - \rho}{\eta - \delta(1-\eta)} - \frac{\delta(1-\eta)}{\eta - \delta(1-\eta)} \frac{\dot{W}_t}{W_t}. \tag{38}$$

This also implies that $\dot{C}_t(h)/C_t(h) = \dot{C}_t/C_t$.

The settings of the intermediate-good, R&D, and capital-producing sectors remain unchanged as in Section 3. Following the same logic as in the baseline model, we solve this extended model and derive the following key equilibrium variables. The equilibrium output-wage ratio is

$$\frac{Y_t}{W_t} = \frac{1 + \frac{\rho/\phi + \rho/\varphi}{\eta + \alpha(1-\eta)(1-\gamma)}}{\delta + \frac{\alpha\phi[\eta+\alpha(1-\eta)(1-\gamma)] + \alpha^2\gamma^2\phi(1-\eta)(1+\phi/\varphi) + \alpha\varphi(\mu-1)\{(1-\eta)(1-\alpha) + (\phi/\varphi)[1-\alpha\gamma(1-\eta)]\}}{\mu\phi[\eta+\alpha(1-\eta)(1-\gamma)]}}. \tag{39}$$

The total labor supply, defined as $1 - L_t = 1 - \int_0^1 L_t(h)dh$, and the equilibrium labor allocations are given by

$$1 - L = 1 - \frac{\delta Y_t}{W_t}, \tag{40}$$

$$L_x = \frac{\alpha(1-\gamma)}{\mu} \frac{Y_t}{W_t}, \tag{41}$$

$$L_r = \frac{\alpha^2\phi\gamma^2(1-\eta) + \alpha\varphi(\mu-1)[1-\alpha\gamma(1-\eta)]}{\mu\varphi[\eta+\alpha(1-\eta)(1-\gamma)]} \frac{Y_t}{W_t} - \frac{\rho}{\varphi[\eta+\alpha(1-\eta)(1-\gamma)]}, \tag{42}$$

$$L_k = \frac{\alpha\gamma\phi[\eta+\alpha(1-\eta)] + \alpha\varphi(\mu-1)(1-\alpha)(1-\eta)}{\mu\phi[\eta+\alpha(1-\eta)(1-\gamma)]} \frac{Y_t}{W_t} - \frac{\rho}{\phi[\eta+\alpha(1-\eta)(1-\gamma)]}. \tag{43}$$

When $\eta = 1$, equations (41)-(43) are reduced to equations (18)-(20). It is straightforward to verify that labor in the manufacturing sector ($L_x$) decreases in $\mu$. Under the condition of a sufficiently large $\phi/\varphi$, the R&D labor $L_r$ increases in $\mu$ whereas the capital-producing labor $L_k$ decreases in $\mu$, yielding results to those of the benchmark counterparts. Therefore, the qualitative pattern of the growth effect of $\mu$ in the extended model remains the same as its baseline counterpart.

Next, we examine the effect of $\mu$ on income inequality. Under an elastic labor supply, the relative income becomes

$$\theta_{I,t}(h) \equiv \frac{I_t(h)}{I_t} = \frac{R_t A_t(h) + W_t[1 - L_t(h)]}{R_t A_t + W_t(1 - L_t)}. \tag{44}$$

By applying the steady-state equilibrium growth rate of assets such that $\dot{A}_t/A_t = g$, we use equation (36) to obtain household $s$'s labor wage such that

$$W_t L_t(h) = \frac{\delta W_t}{1+\delta} + \frac{\delta(R-g)}{1+\delta} A_t(h), \tag{45}$$

where we have also applied equation (37). Aggregating this equation over $h$ yields

$$W_t L_t = \frac{\delta W_t}{1+\delta} + \frac{\delta(R-g)}{1+\delta} A_t. \tag{46}$$

Moreover, by using the steady-state equilibrium condition $\dot{C}_t/C_t = \dot{W}_t/W_t$, from equation (38) we can derive

$$R = \rho + \eta g. \tag{47}$$

Following the same logic as in the baseline model, together with (45), (46), and (47), we derive the distribution function of relative income $\theta_{I,t}(h)$ such that

$$\sigma_I \equiv \sqrt{\int_0^1 [\theta_{I,t}(h) - 1]^2 dh} = \frac{[\rho + (\eta+\delta)g] A_t/W_t}{1 + [\rho + (\eta+\delta)g] A_t/W_t} \sigma_A, \tag{48}$$

where $A_t/W_t$ remains unchanged, as in equation (28). Consequently, extending to a general utility function with a constant intertemporal elasticity of substitution and elastic labor supply does not qualitatively affect the impact of patent protection on income inequality.

### 6.3 Liquidity constraints

In this extension, we consider liquidity constraints on both innovating and capital-producing sectors to examine the role of the nominal interest rate on the patent-inequality relation.[28] Specifically, we follow Chu and Cozzi (2014) to model firms' liquidity constraints by introducing CIA constraints on both R&D and capital-producing activities. The expected profits of R&D and capital-producing firms then become $\Pi_{r,t} = V_{n,t}\dot{N}_t - W_t L_{r,t}(1 + \xi i_t)$ and $\Pi_{k,t} = V_{k,t}\dot{K}_t - W_t L_{k,t}(1 +$

[28]See Huang *et al.* (2017) for an analysis of the interaction between patent and monetary policies, along with its growth and welfare implications, in a quality-ladder model.

$\kappa i_t$), respectively. Here the terms $(1+\xi i_t)$ and $(1+\kappa i_t)$ capture firms' additional cost of borrowing money from households to facilitate their investments subject to the nominal interest rate. The parameters $\xi \in (0,1)$ and $\kappa \in (0,1)$ denote the strengths of the CIA constraint on R&D and capital production, respectively. Accordingly, the free-entry conditions in equations (13) and (15) become:

$$\varphi N_t V_{n,t} = W_t(1+\xi i_t), \tag{49}$$

and

$$\phi K_t V_{k,t} = W_t(1+\kappa i_t). \tag{50}$$

Moreover, household $h$'s budget constraint in equation (3) becomes

$$\dot{A}_t(h) + \dot{M}_t(h) = R_t A_t(h) + i_t B_t(h) + W_t - \pi_t M_t(h) - C_t(h) - T_t, \tag{51}$$

where $M_t(h)$ is the real money balance held by household $h$, with $\pi_t$, the inflation rate, determining the cost of holding money; $B_t(h)$ refers to the quantity of money that R&D and capital-producing firms borrow from household $h$, with $i_t$ denoting the return rate; and $T_t$ is the real lump-sum transfer from the government. Solving the utility-maximizing problem yields an additional no-arbitrage condition between real asset value and money holdings such that $i_t = \pi_t + R_t$ (i.e., the Fisher equation). Furthermore, we assume that the patent authority takes as given the nominal interest rates of nominal interest and inflation, which are controlled by monetary authority. Therefore, this assumption implies that $i_t = i$ and $\pi_t = \pi$.

We now solve this extended model and derive the equilibrium labor allocations as follows:

$$L_x = \frac{(1-\gamma)(1+\rho/\varphi+\rho/\phi)}{\mu + \gamma\frac{(\xi-\kappa)i}{1+\kappa i}}, \tag{52}$$

$$L_r = \frac{(\mu-1)(1+\rho/\varphi+\rho/\phi)}{\mu + \gamma\frac{(\xi-\kappa)i}{1+\kappa i}} - \frac{\rho}{\varphi}, \tag{53}$$

$$L_k = \frac{\gamma(1+\rho/\varphi+\rho/\phi)}{\mu\frac{1+\kappa i}{1+\xi i} + \gamma\frac{(\xi-\kappa)i}{1+\xi i}} - \frac{\rho}{\phi}. \tag{54}$$

Apparently, when the CIA constraints are absent, namely $\xi = \kappa = 0$, equations (52)-(54) are reduced to equations (18)-(20). In this extended model, the relationship between $g_y$ and $\mu$ captured in (22) becomes

$$\frac{\partial g_y}{\partial \mu} \gtrless 0 \Leftrightarrow \frac{\varphi}{\phi} \gtrless \frac{\alpha\gamma^2\left(\frac{1+\xi i}{1+\kappa i}\right)}{(1-\alpha)\left[1+\frac{\gamma(\xi-\kappa)i}{1+\kappa i}\right]}. \tag{55}$$

Therefore, the key model implication derived in the baseline model still holds. That is, an increased degree of patent protection is growth-enhancing (retarding) if the relatively productivity of R&D is high (low). Moreover, equation (55) implies that if R&D firms are more (less) cash-constrained than capital-producing firms, namely $\xi > \kappa$ ($\xi < \kappa$), an increase in the nominal interest rate $i$ raises (lowers) the threshold of relative R&D productivity, reinforcing (mitigating) the positive effect of patent breadth on economic growth.

Next, we analyze the relationship between patent breadth and income inequality. First, denote by $D_t(h) = A_t(h) + M_t(h)$ the total wealth of household $h$, comprising of financial assets and money holdings. At any point of time, the relative wealth has the mean of unity, and the standard deviation of the relative wealth distribution is $\sigma_D > 0$, with the initial and exogenously given standard deviation being $\sigma_{D,0}$. Following the same logic as in Lemma 1, we can prove that the distribution of $\theta_{D,t}(h)$ is stationary over time and is given by $\theta_{D,t}(h) = \theta_{D,0}(h)$ for all $t > 0$.[29] The relative total income of household $h$ becomes

$$\theta_{I,t}(h) = \frac{RD_t\theta_{D,0}(h) + W_t}{RD_t + W_t}. \tag{56}$$

Accordingly, the distribution of relative income features a mean of unity and the following standard deviation:

$$\sigma_I = \left(\frac{RD_t/W_t}{1 + RD_tW_t}\right)\sigma_D. \tag{57}$$

In equilibrium, the aggregate/average money holdings of all households are equal to the money borrowed by all R&D firms and capital-producing firms, such that $M_t = \xi W_t L_r + \kappa W_t L_k$. Therefore, the wealth-wage ratio is given by

$$\frac{D_t}{W_t} = \frac{A_t}{W_t} + \frac{M_t}{W_t} = \left(\frac{1+\xi i}{\varphi} + \frac{1+\kappa i}{\phi}\right) + \xi L_r + \kappa L_k, \tag{58}$$

where $L_r$ and $L_k$ are given by equations (53) and (54), respectively

Comparing equation (58) to (28) shows that the extended model with liquidity constraints introduces an additional channel through which adjustments in patent breadth can be translated into a change in income inequality. In particular, if R&D firms (i.e., $\xi$) are sufficiently more cash-constrained than capital-producing firms, which is supported by existing empirical studies such as Cooper and Haltiwanger (2006) and Brown and Petersen (2011),[30] the qualitative result regarding the patent-inequality relationship remains unchanged as in the baseline model. Intuitively, a larger patent breadth $\mu$ induces shifts of labor from the capital-producing sector to the R&D sector. Equation (58) reveals that this tends to increase the wealth-wage ratio $D_t/W_t$ when R&D firms face tighter CIA constraints (i.e., $\xi > \kappa$), because money holdings are components of households' assets. Since this *wealth-value* effect reinforces the interest-rate effect, the overall impact of patent strength on income dispersion still replies on the relative R&D productivity, as

[29] The proof is available upon request.

[30] Empirical findings in these studies show that firms have a much smaller demand on smoothing physical investment with costly cash holdings, partly because physical capital adjustment costs are relatively "modest".

in the baseline model. In this case, our baseline model implications on how patent patent-growth and patent-inequality relationship would continue to hold in this extended setting.

Nevertheless, suppose that monetary authority raises the nominal interest rate $i$. In the presence of a stronger CIA constraint on R&D (i.e., $\xi > \kappa$), equation (55) implies that an increased nominal interest rate $i$ raises the threshold of relative productivity, mitigating the positive interest-rate effect. In the meantime, equation (58) implies that a higher $i$ tends to reinforce the positive wealth-value effect. Hence, the overall impact of patent breadth on the distribution of income would become ambiguous, depending on the relative magnitude of these opposing forces. Specifically, a higher nominal interest rate leads patent protection to increase (decrease) the degree of income inequality if the reduction in the interest-rate effect is smaller (greater) than the increase in the wealth-value effect.

# 7 Conclusion

This study explores the effect of patent policy on income inequality in a variety-expansion model, in which R&D and capital accumulation are non-complementary engines of growth. In contrast to the related existing studies, the current study reveals that the impact of patent protection on income inequality is contingent upon the impact on economic growth through the interest-rate effect, which depends on the productivity of R&D relative to capital. Specifically, we analytically show that the degree of income inequality increases (decreases) in the strength of patent protection if the relative productivity of R&D to capital is high (low). We also consider several extensions to show the theoretical robustness of the analytical result. Furthermore, our analytical results are supported by our empirical evidence using data on OECD countries and our quantitative exercises by calibrating the model to US data.

Our results imply that for a country with a high (low) level of productivity in R&D relative to capital, strengthening patent protection tends to stimulate (stifle) economic growth in exchange for a larger (smaller) degree of income inequality. In this sense, when designing patent policy and considering the potential implications of income inequality, policymakers should be aware of the comparative advantages of different engines to promote economic growth.

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# Appendix A

## A.1 Data description

The empirical practice in this study focuses on 38 OECD countries. Yearly data on investigated variables are described as follows:

(1) $IPR$: Protection of Property Rights index, downloaded from the Economic Freedom of the World by Fraser Institute (2022 Annual Report).

(2) Gini coefficient (WIID): Gini coefficient, downloaded from the World Income Inequality Database (WIID 2021).

(3) Income shares: Income share of decile, downloaded from the World Income Inequality Database (WIID 2021).

(4) Import: The value of import share in GDP, downloaded from the World Bank Database; Series "NE.IMP.GNFS.ZS".

(5) Export: The value of export share in GDP, downloaded from the World Bank Database; Series "NE.EXP.GNFS.ZS".

(6) Government spending to GDP ratio: General government final consumption expenditure as a percentage of GDP, downloaded from Penn World Table version 10.0 (PWT 10).

(7) Gross capital formation to GDP ratio: Gross capital formation as a percentage of GDP, downloaded from Penn World Table version 10.0 (PWT 10).

(8) R&D intensity: Research and development expenditure as a percentage of GDP, downloaded from the World Bank Database; Series "GB.XPD.RSDV.GD.ZS"

(9) Human capital: Human capital index, based on years of schooling and returns to education, downloaded from Penn World Table version 10.0 (PWT 10).

(10) Private Credit: Ratio of total credit to private non-financial sector to GDP, downloaded from Bank for International Settlements (BIS).

(11) Real GDP: Real GDP at constant 2017 national prices (in millions of 2017 USD), downloaded from Penn World Table version 10.0 (PWT 10).

(12) Population: Population in millions, downloaded from Penn World Table version 10.0 (PWT 10).

Conditional on the series described above, trade openness is computed as the sum of import and export shares in GDP; real GDP per capita is calculated as real GDP divided by population. WIID occasionally reports multiple observations on the Gini coefficient for a particular country within a year, which are either collected from different sources or computed according to different criteria. Whenever it happens, our strategy of constructing the Gini coefficient series is to take the average of all available observations for country $i$ in year $t$.

Note that the total number of countries used for estimation is below 38, due to unavailable data on private credit in certain countries.

## A.2 Supplementary tables for empirical analysis

Table 6: Effect of $IPR$ on income inequality in HRPC – robustness check

| | Gini (WIID) | | | $T10/B10$ | | | $T20/B20$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $IPR$ | 0.03 | 0.50* | 0.69*** | 0.42** | 0.57*** | 0.54*** | 0.13* | 0.20*** | 0.17** |
| | (0.29) | (0.30) | (0.25) | (0.17) | (0.14) | (0.17) | (0.07) | (0.05) | (0.08) |
| $IPR \times RP$ | | -0.05*** | -0.06*** | | -0.02 | -0.02 | | -0.01 | -0.01 |
| | | (0.01) | (0.01) | | (0.02) | (0.02) | | (0.01) | (0.01) |
| Trade Openness | 0.02 | 0.02 | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 | 0.01 | 0.01 |
| | (0.04) | (0.03) | (0.02) | (0.01) | (0.01) | (0.02) | (0.01) | (0.01) | (0.01) |
| Human Capital | -14.53** | -13.84*** | -13.43*** | -7.54*** | -7.41*** | -7.50*** | -3.13*** | -3.07*** | -3.14*** |
| | (5.54) | (4.27) | (3.74) | (2.15) | (2.02) | (1.97) | (0.76) | (0.58) | (0.63) |
| Gov. Spending to GDP Ratio | -4.97 | -4.46 | -2.90 | -2.22 | -2.14 | -2.44 | -0.76 | -0.72 | -0.96 |
| | (3.10) | (2.80) | (2.47) | (2.07) | (2.13) | (2.45) | (0.69) | (0.75) | (0.88) |
| Inflation | -0.04 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | 0.01 |
| | (0.04) | (0.04) | (0.04) | (0.01) | (0.01) | (0.02) | (0.01) | (0.01) | (0.01) |
| R&D to GDP Ratio | 0.36 | 4.71** | 5.65*** | 2.42 | 3.88** | 3.73* | 0.45 | 1.17** | 1.06* |
| | (2.33) | (2.33) | (1.98) | (1.59) | (1.64) | (1.91) | (0.50) | (0.49) | (0.61) |
| Credit to GDP ratio | 3.93* | 2.45 | 2.52 | 3.08*** | 2.60** | 2.61** | 1.03** | 0.80** | 0.81** |
| | (2.12) | (1.92) | (1.85) | (1.01) | (1.13) | (1.11) | (0.40) | (0.34) | (0.33) |
| GDP per capita | | | -9.72*** | | | 1.47 | | | 1.15 |
| | | | (3.07) | | | (3.27) | | | (1.19) |
| Control | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Obs. | 136 | 136 | 136 | 134 | 134 | 134 | 134 | 134 | 134 |
| $R^2$ | 0.24 | 0.28 | 0.31 | 0.11 | 0.12 | 0.12 | 0.18 | 0.20 | 0.21 |

*Notes*: Estimation results reported in the table are based on the alternative grouping criterion for HRPC, where $\alpha^*$ is set to 8.5%. Hence, the top 10 countries (from IL to IS) in Table 1 fall into the HRPC category. The sample period is 2000-2018. Measures of dependent and independent variables are the same as those in the baseline estimation. Robust standard errors clustered by country are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Table 7: Effect of $IPR$ on income inequality in LRPC – robustness check

| | Gini (WIID) | | | $T10/B10$ | | | $T20/B20$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $IPR$ | -0.42 | -0.95*** | -0.95*** | -1.19* | -2.14** | -2.07** | -0.32* | -0.58** | -0.57*** |
| | (0.28) | (0.31) | (0.31) | (0.70) | (0.92) | (0.86) | (0.18) | (0.23) | (0.22) |
| $IPR \times RP$ | | 0.12*** | 0.12*** | | 0.23** | 0.22** | | 0.06** | 0.06*** |
| | | (0.04) | (0.04) | | (0.10) | (0.09) | | (0.02) | (0.02) |
| Trade Openness | -0.01 | 0.01 | 0.01 | 0.02 | -0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| | (0.01) | (0.01) | (0.01) | (0.02) | (0.02) | (0.02) | (0.01) | (0.01) | (0.01) |
| Human Capital | 3.35 | 3.70 | 3.60 | -4.07 | -2.92 | -3.19 | -0.67 | -0.36 | -0.41 |
| | (4.38) | (3.17) | (3.11) | (7.67) | (5.53) | (5.76) | (2.32) | (1.80) | (1.81) |
| Gov. Spending to GDP Ratio | 1.68 | 1.04 | 1.05 | -7.62 | -8.78* | -8.88* | -1.96 | -2.27 | -2.29 |
| | (2.73) | (2.46) | (2.50) | (5.72) | (5.23) | (4.72) | (1.64) | (1.51) | (1.41) |
| Inflation | -0.05*** | -0.05** | -0.05** | -0.03 | -0.02 | -0.03 | -0.01 | -0.01 | -0.01 |
| | (0.02) | (0.02) | (0.02) | (0.02) | (0.02) | (0.02) | (0.01) | (0.01) | (0.01) |
| R&D to GDP Ratio | -2.19 | -5.85*** | -5.95*** | -6.05 | -12.65* | -11.56* | -1.42 | -3.21** | -3.01** |
| | (1.83) | (1.96) | (1.95) | (4.98) | (6.74) | (6.10) | (1.19) | (1.58) | (1.47) |
| Credit to GDP ratio | -0.58 | 0.10 | -0.15 | 0.95 | 2.09 | 4.34 | 0.05 | 0.36 | 0.77 |
| | (1.64) | (1.23) | (1.37) | (2.11) | (2.06) | (3.06) | (0.70) | (0.64) | (0.85) |
| GDP per capita | | | 1.19 | | | -10.24* | | | -1.87 |
| | | | (3.10) | | | (5.93) | | | (1.35) |
| Control | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Obs. | 340 | 340 | 340 | 327 | 327 | 327 | 327 | 327 | 327 |
| $R^2$ | 0.07 | 0.17 | 0.17 | 0.06 | 0.09 | 0.10 | 0.10 | 0.17 | 0.18 |

*Notes*:Estimation results reported in the table are based on the alternative grouping criterion for LRPC, where $\alpha^*$ is set to 8.5%. Hence, the bottom 28 countries (from AT to CO) in Table 1 fall into the LRPC category. The sample period is 2000-2018. Measures of dependent and independent variables are the same as those in the baseline estimation. Robust standard errors clustered by country are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

# Appendix B

## B.1 Derivation of monopoly pricing

The monopolistic firm $j$ maximizes her profit $\Pi_{x,t}(j) = P_t(j)X_t(j) - MC_t(j)X_t(j)$ subject to the conditional demand (6) and the price constraint such that $P_t(j) \leq \mu MC_t(j)$. Therefore, the

Table 8: Nonlinear effect of $IPR$ on income inequality – full sample

| | Gini (WIID) | | $T10/B10$ | | $T20/B20$ | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $IPR$ | -2.10*** | -2.26*** | 1.83 | 1.33 | -0.32 | -0.45 |
| | (0.81) | (0.76) | (4.37) | (3.74) | (0.62) | (0.50) |
| $IPR^2$ | 0.16** | 0.15** | -0.14 | -0.14 | 0.02 | 0.02 |
| | (0.06) | (0.06) | (0.31) | (0.29) | (0.05) | (0.04) |
| Control | Yes | Yes | Yes | Yes | Yes | Yes |
| Country-Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes |
| Year-Fixed Effect | No | Yes | No | Yes | No | Yes |
| Obs. | 393 | 393 | 383 | 383 | 383 | 383 |
| $R^2$ | 0.13 | 0.08 | 0.03 | 0.03 | 0.05 | 0.06 |

*Notes*: Estimation results reported in the table are based on the full sample. The sample period is 2000-2015. The control variables are all lagged by one period. Government spending to GDP ratio, R&D to GDP ratio and credit to GDP ratio are in logarithm. Estimation using the full sample excludes lagged GDP per capita. Robust standard errors clustered by country are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Table 9: Average PPR for OECD countries

| | | | | |
|---|---|---|---|---|
| 1.CH: 8.91 | 9.NO: 8.12 | 17.EE: 6.84 | 25.IT: 5.80 | 33.LT: 5.20 |
| 2.FI: 8.89 | 10.FR: 8.04 | 18.IS: 6.84 | 26.ES: 5.77 | 34.PL: 4.96 |
| 3.SE: 8.49 | 11.IE: 7.95 | 19.PT: 6.58 | 27.CA: 5.69 | 35.TR: 4.95 |
| 4.NL: 8.48 | 12.US: 7.83 | 20.CL: 6.41 | 28.GR: 5.62 | 36.CR: 4.92 |
| 5.DK: 8.46 | 13.JP: 7.82 | 21.NZ: 6.24 | 29.HU: 5.46 | 37.MX: 4.91 |
| 6.GB: 8.46 | 14.BE: 7.59 | 22.LU: 6.21 | 30.CZ: 5.41 | 38.CO: 3.97 |
| 7.AT: 8.40 | 15.DE: 7.04 | 23.KR: 6.18 | 31.LV: 5.37 | |
| 8.AU: 8.16 | 16.IL: 6.89 | 24.SI: 5.89 | 32.SK: 5.27 | |

*Notes*: OECD countries are labeled by Alpha-2 code as described in the ISO international standard. Missing observations are removed when computing the long-run average. The sample period is 2000-2015.

current-value Hamiltonian for this firm is

$$H_t(j) = P_t(j)X_t(j) - MC_t(j)X_t(j) + \omega_t(j)[\mu MC_t(j) - P_t(j)], \tag{B.1}$$

where $\omega_t(j)$ is the costate variable associated with $P_t(j) \leq \mu MC_t(j)$. Substituting equation (6) into equation (B.1), we can derive

$$\frac{\partial H_t(j)}{\partial P_t(j)} = 0 \Rightarrow \frac{\partial \Pi_t(j)}{\partial P_t(j)} = \omega_t(j). \tag{B.2}$$

If $P_t(j) < \mu MC_t(j)$, then $\omega_t(j) = 0$. We then have $\partial\Pi_t(j)/\partial P_t(j) = 0$, implying $P_t(j) = [\sigma/(\sigma-1)]MC_t(j)$. A binding price constraint on $P_t(j)$ indicates $\omega_t(j) > 0$. Hence, we obtain $\partial\Pi_t(j)/\partial P_t(j) > 0$, implying $P_t(j) = \mu MC_t(j)$. Since we assume $\mu \leq \sigma/(1-\sigma)$, $P_t(j) = \mu MC_t(j)$ always holds.

### B.2 Proof of Lemma 1

First, define the transformed variables $\Phi_{n,t} \equiv Y_t/(V_{n,t}N_t)$ and $\Phi_{k,t} \equiv Y_t/(V_{k,t}K_t)$. Then, differentiating $\Phi_{n,t}$ with respect to time yields

$$\frac{\dot{\Phi}_{n,t}}{\Phi_{n,t}} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{V}_{n,t}}{V_{n,t}} - \frac{\dot{N}_t}{N_t}. \tag{B.3}$$

From the final-good resource constraint $Y_t = C_t$, the law of motion for $Y_t$ is

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = R_t - \rho, \tag{B.4}$$

where the second equality stems from the Euler equation in (4). From equation (12), the law of motion for $V_{n,t}$ is

$$\frac{\dot{V}_{n,t}}{V_{n,t}} = R_t - \frac{\Pi_{x,t}}{V_{n,t}}, \tag{B.5}$$

where $\Pi_{x,t} = \alpha(\mu-1)Y_t/(\mu N_t)$, which is obtained by applying symmetry across varieties in equation (5) to rewrite equation (6) as $P_t(j)X_t(j) = \alpha Y_t/N_t$ and substituting it into equation (9). Combining equations (B.3)–(B.5) yields

$$\frac{\dot{\Phi}_{n,t}}{\Phi_{n,t}} = \alpha\left(\frac{\mu-1}{\mu}\right)\Phi_{n,t} - \varphi L_{r,t} - \rho, \tag{B.6}$$

where we use the fact that $\dot{N}_t/N_t = \varphi L_{r,t}$. Using the same logic, differentiating $\Phi_{k,t}$ with respect to time yields

$$\frac{\dot{\Phi}_{k,t}}{\Phi_{k,t}} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{V}_{k,t}}{V_{k,t}} - \frac{\dot{K}_t}{K_t}. \tag{B.7}$$

From equation (14), the law of motion for $V_{k,t}$ is

$$\frac{\dot{V}_{k,t}}{V_{k,t}} = R_t - \frac{Q_t}{V_{k,t}}, \tag{B.8}$$

where $Q_t = \alpha\gamma Y_t/(\mu K_t)$, which is obtained by applying symmetry across varieties in equation (5) to rewrite equation (6) as $P_t(j)X_t(j) = \alpha Y_t/N_t$ and substituting it into equation (10). Plugging equations (B.4) and (B.8) into equation (B.7) yields

$$\frac{\dot{\Phi}_{k,t}}{\Phi_{k,t}} = \frac{\alpha\gamma}{\mu}\Phi_{k,t} - \phi L_{k,t} - \rho, \tag{B.9}$$

where we use the fact that $\dot{K}_t/K_t = \varphi L_{k,t}$.

Furthermore, combining equation (13) and equation (15) yields

$$\frac{\Phi_{n,t}}{\varphi} = \frac{\Phi_{k,t}}{\phi}, \tag{B.10}$$

which implies

$$\frac{\dot{\Phi}_{n,t}}{\Phi_{n,t}} = \frac{\dot{\Phi}_{k,t}}{\Phi_{k,t}}.$$

Using this result and equation (B.10), we rewrite equation (B.9) to express $L_{k,t}$ as a function of $\dot{\Phi}_{n,t}/\Phi_{n,t}$ and $\Phi_{n,t}$ such that

$$L_{k,t} = -\frac{1}{\phi}\left(\frac{\dot{\Phi}_{n,t}}{\Phi_{n,t}} - \frac{\alpha\gamma}{\mu}\frac{\phi}{\varphi}\Phi_{n,t} + \rho\right). \tag{B.11}$$

Then, we use equation (10) to derive

$$L_{x,t} = \int_0^{N_t} L_{x,t}(j)dj = \frac{(1-\gamma)\int_0^{N_t} P_t(j)X_t(j)dj/\mu}{W_t} = \frac{\alpha(1-\gamma)Y_t/\mu}{W_t} = \frac{\alpha(1-\gamma)}{\mu\varphi}\Phi_{n,t}, \tag{B.12}$$

where we use equations (5) and (6) in the third equality and equation (13) in the fourth equality.

Finally, substituting equations (B.11)–(B.12), and the labor-market-clearing condition $L_{x,t} + L_{r,t} + L_{k,t} = 1$ into equation (B.6), a few steps of manipulation yield a one-dimensional differential equation in $\Phi_{n,t}$ such that

$$\frac{\dot{\Phi}_{n,t}}{\Phi_{n,t}} = \left(1+\frac{\varphi}{\phi}\right)^{-1}\left\{\Phi_{n,t}\left[\frac{\alpha(\mu-1)}{\mu} + \frac{\alpha(1-\gamma)}{\mu} + \frac{\alpha\gamma}{\mu}\right] - \varphi - \rho - \frac{\varphi\rho}{\phi}\right\}. \tag{B.13}$$

Therefore, given that $\Phi_{n,t}$ is a control variable, the dynamics of $\Phi_{n,t}$ is characterized by saddle-point stability such that $\Phi_{n,t}$ jumps immediately to its interior steady-state value given by

$$\Phi_n = \frac{\varphi(1+\rho/\varphi+\rho/\phi)}{\alpha} \tag{B.14}$$

Then, equations (B.6), (B.9), and (B.12) imply that when $\Phi_n$ and $\mu$ are stationary, $L_r$, $L_k$, and $L_x$ must also be stationary. By imposing $\dot{\Phi}_{n,t}/\Phi_{n,t} = 0$ on equations (B.6), (B.11) and (B.12), respectively, we can derive the equilibrium labor allocations in equations (18)–(20).

### B.3 Proof of Lemma 2

First, it is obvious that $\theta_{C,t}(h)$ is stationary and equals $\theta_{C,0}(h)$ for all $t$, because

$$\frac{\dot{\theta}_{C,t}(h)}{\theta_{C,t}(h)} = \frac{\dot{C}_t(h)}{C_t(h)} - \frac{\dot{C}_t}{C_t} = 0, \tag{B.15}$$

according to the Euler equation (4). Thus, for a given patent policy $\mu$, the variables $\{C_t, W_t, A_t\}$ all grow at the rate of $g$ on the balanced growth path according to Proposition 2. As a result, equation (3) implies

$$C_t - W_t = \rho A_t, \tag{B.16}$$

and the coefficient associated with $\theta_{A,t}(h)$ in equation (24) becomes $\rho > 0$. Therefore, given that equation (24) is a one-dimensional differential equation that describes the potential evolution of the state variable $\theta_{A,t}(h)$, the only solution of equation (24) consistent with long-run stability is $\dot{\theta}_{A,t}(h) = 0$, so that $\theta_{A,t}(h) = \theta_{A,0}(h)$ for all $t$; this is achieved by $\theta_{C,t}(h)$ jumping to its steady-state value $\theta_{C,0}(h)$. Applying $\dot{\theta}_{A,t}(h) = 0$ into equation (24) yields the value of $\theta_{C,0}(h)$ such that

$$\rho\theta_{A,t}(h) = \frac{C_t}{A_t}\theta_{c,t}(h) - \frac{W_t}{A_t} \Leftrightarrow \theta_{C,t}(h) - 1 = \frac{\rho A_t}{C_t}[\theta_{A,t}(h) - 1], \tag{B.17}$$

where we have applied $C_t - W_t = \rho A_t$ in equation (B.16). By substituting equations (13) and (15) into equation (16), we obtain

$$\begin{aligned} \frac{A_t}{C_t} &= \frac{A_t}{W_t + \rho A_t} = \frac{1}{W_t/A_t + \rho} = \frac{1}{V_{n,t}N_t/W_t + V_{k,t}K_t/W_t + \rho} \\ &= \frac{1}{\frac{1}{1/\varphi + 1/\phi} + \rho} = \frac{1/\varphi + 1/\phi}{1 + \rho(1/\varphi + 1/\phi)}. \end{aligned} \tag{B.18}$$

Finally, combining equations (B.17) and (B.18) yields equation (25).