Effects of R&D Policy on Income Inequality in a Growth Model with Heterogeneous Assets and Skills*

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February 3, 2022

Abstract

This study examines how two R&D policy instruments: patent protection and research subsidies, affect income inequality in an endogenous growth model with households who possess heterogeneity in assets and skills. We find that the effect of strengthening patent protection on income inequality can be positive or U-shaped, whereas the effect of increasing research subsidies can be positive, negative, or U-shaped; these effects are disambiguated by the comparison between asset heterogeneity and skill heterogeneity.

JEL classification: D30; O31; O34; O40.
Keywords: Economic growth; Income inequality; Patent protection; R&D subsidies.

*The authors would like to thank Eric Young (the Editor) and an anonymous Referee for helpful comments. Ying Tung Chan: Beijing Normal University at Zhuhai, 18 Jinfeng Road, Xiangzhou District, Zhuhai, Guangdong, 519087, China. Yibai Yang: Department of Economics, University of Macau, Taipa, Macao, China. Zhijie Zheng: Center for Innovation and Development Studies, Beijing Normal University at Zhuhai, 519087, China. Tel: +86-15985862303. (E-mail: zhengzhijie1919@gmail.com). Yang gratefully acknowledges financial support from the Asia Pacific Academy of Economics and Management at the University of Macau. Zheng also gratefully acknowledges the support given by the Start-up Research Grant of Beijing Normal University at Zhuhai.

1 Introduction

An increasingly important phenomenon in today's world is the rising degree of income inequality (See, for example, Piketty 2014 and Saez and Zucman 2016). This issue motivates researchers to study distributional implications of various policy regimes, such as monetary policy (e.g., Kaplan *et al.* 2018 and Rocheteau *et al.* 2018) and patent policy (e.g., Kiedaisch 2020 and Chu *et al.* 2021), in addition to the macroeconomic implications. Given that patent protection and R&D subsidies are two frequently used policy instruments that stimulate technological progress and economic growth, this study revisits their effects on income inequality in a lab-equipment specification of quality-ladder growth model with heterogeneous households. Differing from the existing literature such as Chu (2010) and Chu and Cozzi (2018) who consider wealth heterogeneity in terms of asset holdings, this study also takes into account worker heterogeneity in terms of skill endowments; the former captures households' asset income inequality whereas the latter captures their wage income inequality.[1] We find that the relation between the R&D policy levers and income inequality is contingent on the relative importance of the two types of heterogeneity.

As in the existing studies, strengthening patent protection (by means of patent breadth) and raising R&D subsidies both increase the rates of innovation and economic growth; this positive growth effect increases the ratio of asset income to wage income. However, strengthening patent protection increases the value of financial assets whereas raising R&D subsidies decreases it. Overall, the ratio of asset income to wage income is increasing in patent protection and decreasing in R&D subsidies. Since skill heterogeneity leads to wage income inequality, income inequality is increasing (decreasing) in the ratio of asset income to wage income only if asset heterogeneity is larger (smaller) than skill heterogeneity. Therefore, the effects of patent protection versus R&D subsidies on income inequality also depend on the comparison between asset heterogeneity and skill heterogeneity.

Specifically, if asset heterogeneity dominates skill heterogeneity, a larger patent breadth increases the ratio of asset income to wage income, thereby enlarging income inequality. Nevertheless, a higher R&D subsidy rate decreases the ratio of asset income to wage income, so it would have a decreasing or even a U-shaped effect on income inequality.[2] By contrast, if asset heterogeneity is dominated by skill heterogeneity, a larger patent breadth first mitigates income inequality and finally enlarges it, giving rise to a U-shaped pattern.[3] Nevertheless, a higher R&D

[1]When considering elastic labor supply, this strand of literature (e.g., Chu 2010) can also produce an unequal distribution of wage income. However, this wage income distribution is essentially determined by the asset distribution, because the amount of working time by each household is related to her asset endowments. Our model adds a skill dimension to break this strong link between labor income heterogeneity and asset heterogeneity.

[2]As shown in Section 3, the U-shaped relation between R&D subsidies and income inequality arises because R&D subsidies alter the relative heterogeneity between assets and skills. Therefore, there exists a threshold for the asset-skill relative heterogeneity below (above) which a higher R&D subsidy rate mitigates (enlarges) income inequality.

[3]Similarly, Section 3 shows that the U-shaped relation between patent protection and income inequality arises because patent breadth also alters the relative heterogeneity between assets and skills. Therefore, there exists a threshold for the asset-skill relative heterogeneity below (above) which stronger patent protection mitigates (enlarges) income inequality.

subsidy rate strictly enlarges income inequality.

By considering these two dimensions of heterogeneity, this model provides a potential mechanism in reconciling the empirical inconsistency on the connection between patent protection and income inequality.[4] In addition, our theoretical analysis shows that the connection between R&D subsidies and income inequality may have even more variations.

2 The model

In this section, we extend the quality-ladder growth model in Acemoglu (2009) (Chapter 14) by introducing into households asset heterogeneity as in Chu (2010) and Chu *et al.* (2021) and skill heterogeneity as in García-Peñalosa and Turnovsky (2015). Moreover, we consider patent breadth and R&D subsidies as policy instruments as in Yang (2018) and Chu and Cozzi (2018).

2.1 Households

The economy admits a unit continuum of households indexed by $s \in [0,1]$, who have the same preference over consumption $C_t(s)$ but possess different levels of initial assets and skills. The lifetime utility function for household s is

$$U(s) = \int_0^\infty e^{-\rho t} \ln C_t(s) dt, \tag{1}$$

where $\rho > 0$ represents the discount rate. Households maximize their lifetime utility subject to the following budget constraint:

$$\dot{A}_t(s) = r_t A_t(s) + w_t H(s) - C_t(s) - \tau_t, \tag{2}$$

where $A_t(s)$ is the value of financial assets owned by household s and r_t is the real interest rate. Household $s \in [0,1]$ is endowed with $H(s)$ units of skills and does not accumulate skills.[5] Each household inelastically supplies one unit of labor, providing $H(s)$ units of skill augmented labor (or effective labor). Moreover, w_t is the wage rate and τ_t is the lump-sum tax.

From standard dynamic optimization, we derive the familiar Euler equation

$$\frac{\dot{C}_t(s)}{C_t(s)} = r_t - \rho. \tag{3}$$

implying that all households have the same growth rate of consumption such that $\dot{C}_t(s)/C_t(s) = \dot{C}_t/C_t$, where $C_t \equiv \int_0^1 C_t(s)ds$ is the aggregate consumption of final good.

[4]For instance, empirical findings in Adams (2008) indicate a positive correlation between intellectual property rights and income inequality, whereas Chu *et al.* (2021) find a negative long-run relation between patent strength and income inequality.

[5]To ensure the tractability of the model, we assume that households' skills are exogenously given. See Turnovsky and Mitra (2013) for a generalized model that allows both endogenous physical and human capital accumulation.

2.2 Final good

Final good (numeraire) is produced by a mass of perfectly competitive firms according to

$$Y_t = \frac{H^{1-\alpha}}{\alpha} \int_0^1 q_t(\epsilon) x_t(\epsilon|q)^{\alpha} d\epsilon, \quad \alpha \in (0,1) \tag{4}$$

where H is the efficient labor and $x_t(\epsilon|q)$ is the quantity of intermediate goods in industry $\epsilon \in [0,1]$, whose quality at time t is $q_t(\epsilon)$. The quality evolves as follows:

$$q_t(\epsilon) = q_0(\epsilon)\lambda^{n_t(\epsilon)}, \tag{5}$$

where q_0 is the quality level at time 0, $\lambda > 1$ measures the quality step size of each innovation, and $n_t(\epsilon)$ denotes the number of innovations on this product line between time 0 and t. From profit maximization, we obtain the conditional demand functions for H and $x_t(\epsilon|q)$, respectively,

$$H = (1-\alpha)Y_t/w_t, \tag{6}$$

and

$$x_t(\epsilon|q) = \left(\frac{q_t(\epsilon)}{p_t(\epsilon|q)}\right)^{\frac{1}{1-\alpha}} H, \tag{7}$$

where $p_t(\epsilon|q)$ is the price of $x_t(\epsilon|q)$.

2.3 Intermediate goods

Differentiated intermediate goods in industry ϵ are produced by a monopolistic leader who holds a patent on the latest innovation. The leader's products will be replaced when a new entrant, who has a more advanced innovation, enters the market. The marginal cost of producing a unit of intermediate good is $\eta q_t(\epsilon)$ units of final good, where $\eta \in (0,1)$. The ϵ-th intermediate-good producer maximizes her profits $\Pi_t(\epsilon) = [p_t(\epsilon|q) - \eta q_t(\epsilon)]x_t(\epsilon|q)$, subject to (7). Following Yang (2018), we assume incomplete patent protection in the sense that the current leader's markup μ is a policy instrument that reflects the level of patent breadth set by the policymaker, and the range of the markup is given by $\mu \in (1, 1/\alpha]$, where $1/\alpha$ is the unconstrained markup under perfect patent protection. Therefore, the profit-maximizing price is

$$p_t(\epsilon|q) = \mu\eta q_t(\epsilon). \tag{8}$$

Combining (7) and (8) yields the quantity of intermediate goods in industry ϵ:

$$x_t(\epsilon|q) = (\mu\eta)^{\frac{-1}{1-\alpha}} H. \tag{9}$$

Consequently, the monopoly flow profit is

$$\Pi_t(\epsilon|q) = \left(\frac{\mu - 1}{\mu}\right)(\mu\eta)^{\frac{-\alpha}{1-\alpha}} q_t(\epsilon)H. \tag{10}$$

2.4 Innovations and R&D

Denote by $V_t(\epsilon|q)$ the value of a firm who holds the most recent innovation in line ϵ. Accordingly, the familiar Hamilton-Jacobi-Bellman (HJB) equation for $V_t(\epsilon|q)$ is

$$r_t V_t(\epsilon|q) = \Pi_t(\epsilon|q) + \dot{V}_t(\epsilon|q) - I_t(\epsilon|q)V_t(\epsilon|q). \tag{11}$$

If a firm spends $Z_t(\epsilon|q)$ units of final good for research in product line ϵ when quality is at level q, it then generates a flow rate

$$I_t(\epsilon|q) = \frac{\varphi Z_t(\epsilon|q)}{Hq_t(\epsilon)}, \quad \varphi > 0 \tag{12}$$

of innovation. This innovation improves the product line ϵ to a more advanced quality $\lambda q_t(\epsilon)$. Equation (12) implies that the probability of the next successful innovation is decreasing in quality $q_t(\epsilon)$, capturing the increasing research complexity.[6]

Free entry into research implies that the expected profit on R&D investment $Z_t(\epsilon|q)$ must be zero such that $I_t(\epsilon|q\lambda^{-1})V_t(\epsilon|q) - (1-\beta)Z_t(\epsilon|q\lambda^{-1}) = 0$, where the policy parameter $\beta \in (0,1)$ is the R&D subsidy rate. Combining this equation with (12) yields

$$V_t(\epsilon|q) = \frac{(1-\beta)Hq_t(\epsilon)}{\varphi\lambda}. \tag{13}$$

2.5 Aggregation

Substituting (9) into (4) yields the total output:

$$Y_t = (\mu\eta)^{\frac{-\alpha}{1-\alpha}} Q_t H/\alpha, \tag{14}$$

where

$$Q_t \equiv \int_0^1 q_t(\epsilon)d\epsilon \tag{15}$$

is the average total quality of intermediate goods. Using (8) and (9), the aggregate expenditure on final good used to produce intermediate goods is

$$X_t = \int_0^1 \eta q_t(\epsilon)x_t(\epsilon|q)d\epsilon = \eta(\mu\eta)^{\frac{-1}{1-\alpha}} Q_t H. \tag{16}$$

[6]See Annicchiarico *et al.* (2022) for a similar R&D specification and a detailed discussion for this setup.

Substituting (14) into (6) yields the equilibrium wage rate:

$$w_t = \left(\frac{1-\alpha}{\alpha}\right)(\mu\eta)^{\frac{-\alpha}{1-\alpha}} Q_t. \tag{17}$$

From (10) and (15), the total profit of the intermediate-good sector is

$$\Pi_t \equiv \int_0^1 \Pi_t(\epsilon|q)d\epsilon = \left(\frac{\mu-1}{\mu}\right)(\mu\eta)^{\frac{-\alpha}{1-\alpha}} Q_t H. \tag{18}$$

Finally, denote by V_t the aggregate market value of firms in the intermediate-good sector. Using (15), V_t is expressed as

$$V_t = \int_0^1 V_t(\epsilon|q)d\epsilon = \frac{(1-\beta)Q_t H}{\varphi\lambda}. \tag{19}$$

2.6 Government

The government decides on the level of patent protection μ. In addition, it collects tax revenues to finance R&D subsidies and non-productive government expenditure G_t subject to the following balanced-budget condition:

$$\tau_t = \beta Z_t + G_t, \tag{20}$$

where $Z_t = \int_0^1 Z(\epsilon|q)d\epsilon$ is the aggregate R&D expenditure and $G_t = \gamma Y_t$ is assumed to be proportional to output. The parameter $\gamma \equiv G_t/Y_t$ is the ratio of government expenditure to output.

2.7 Decentralized equilibrium

We define the decentralized equilibrium in Appendix A.

2.8 Effects on innovation and economic growth

When policy tools, i.e., μ and β, change to their new and permanent levels, the economy immediately jumps to a unique and stable balanced growth path (BGP) along which variables $\{Y_t, C_t, X_t, Z_t, Q_t, V_t, w_t\}$ grow at the same and constant rate.[7] Therefore, for a given level of quality $q_t(\epsilon)$, which is constant between time t and $t + \Delta t$ (until a new innovation arrivies in this line), the value of a firm in line ϵ (i.e., $V_t(\epsilon|q)$) is also constant, namely $\dot{V}_t(\epsilon|q) = 0$. Thus, from (11), we have

$$V(\epsilon|q) = \frac{\Pi(\epsilon|q)}{r + I(\epsilon|q)} \Leftrightarrow r^* + I^* = \frac{\varphi\lambda(\mu-1)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\mu(1-\beta)}, \tag{21}$$

[7]See Appendix A for the proof.

where the second equality applies (10) and (13). Notice that r^* and I^* are the steady-state levels of the interest rate and the innovation arrival rate, implying that I^* is identical across product lines. Substituting (21) into (3) yields the equation solving for the steady-state growth rate of output (and technology):

$$g^* = r^* - \rho = \frac{\varphi\lambda(\mu - 1)\,(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\mu(1-\beta)} - I^* - \rho. \tag{22}$$

Moreover, by definition, in a time interval Δt, there are $I_t\Delta t$ sectors that experience one innovation, which increases the productivity by λ. Therefore, the dynamics of Q_t is

$$Q_{t+\Delta t} = I_t\Delta t \int_0^1 \lambda q_t(\epsilon)d\epsilon + (1 - I_t\Delta t)\int_0^1 q_t(\epsilon)d\epsilon = Q_t[1 + I_t\Delta t(\lambda - 1)]. \tag{23}$$

Subtracting Q_t from both sides, dividing by Δt, and taking the limit as $\Delta t \to 0$ yields

$$g^* = \frac{\dot{Q}_t}{Q_t} = I^*(\lambda - 1), \tag{24}$$

where $\dot{Q}_t = \lim_{\Delta t \to 0}(Q_{t+\Delta t} - Q_t)/\Delta t$. Then combining (22) and (24) yields

$$I^* = \frac{\varphi(\mu - 1)\,(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\mu(1-\beta)} - \frac{\rho}{\lambda}, \tag{25}$$

and substituting (25) into (24) yields the steady-state output growth rate

$$g^* = \frac{\varphi(\lambda - 1)(\mu - 1)\,(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\mu(1-\beta)} - \frac{\rho(\lambda - 1)}{\lambda}. \tag{26}$$

It can be verified that both (25) and (26) are increasing in μ and β; these are the traditional macroeconomic effects of patent protection and R&D subsidies as in Yang (2018) and Chu and Cozzi (2018).

3 R&D policy and income inequality

Denote by $H \equiv \int_0^1 H(s)ds$ and $\theta_{H,0}(s) \equiv H(s)/H$ the aggregate effective labor supply and the skill share of household s at time 0, respectively. Because all households' skills are exogenously given, the skill share of household s must be stationary such that $\theta_{H,t}(s) = \theta_{H,0}(s)$. Consider a general distribution function of skill share with a mean of one and a standard deviation of $\sigma_H > 0$. Then this distribution must be stationary over time.

Similarly, denote by $\theta_{A,t}(s) \equiv A_t(s)/A_t$ and $\theta_{A,0}(s) \equiv A_0(s)/A_0$ the asset share of household s at time t and 0, respectively. We also consider a general distribution function of initial asset share with a mean of one and a standard deviation of $\sigma_A > 0$. Appendix A shows that the asset share is constant over time and exogenously determined at time 0.

The amount of income earned by household s and all households are $D_t(s) = r_t A_t(s) + w_t H(s)$ and $D_t = r_t A_t + w_t H$, respectively. Combining these equations yields the share of income earned by household s:

$$\theta_{D,t}(s) \equiv \frac{D_t(s)}{D_t} = \frac{r_t A_t \theta_{A,0}(s) + w_t H \theta_{H,0}(s)}{r_t A_t + w_t H}, \tag{27}$$

where the second equality applies $\theta_{A,t}(s) = \theta_{A,0}(s)$. Therefore, the distribution of income at time t has a mean of one and the following variance:

$$\begin{aligned} \sigma_{D,t}^2 \equiv \int_0^1 [\theta_{D,t}(s) - 1]^2 ds &= \left(\frac{r_t A_t}{r_t A_t + w_t H}\right)^2 \sigma_A^2 + \left(\frac{w_t H}{r_t A_t + w_t H}\right)^2 \sigma_H^2 \\ &= \left(\frac{\Phi_t}{1+\Phi_t}\right)^2 \sigma_A^2 + \left(\frac{1}{1+\Phi_t}\right)^2 \sigma_H^2, \end{aligned} \tag{28}$$

where $\Phi_t \equiv r_t A_t / w_t H$ is the ratio of asset income to wage income. For simplicity, we follow Jin (2009) to assume a zero covariance $\sigma_{A,H} = 0$.[8]

Equation (28) shows that the degree of income inequality, measured by the variance of income distribution σ_D^2, is a weighted average of the variances of asset distribution σ_A^2 and skill distribution σ_H^2. Since both distributions are stationary and independent of the policy tools, a change in μ or β affects σ_D^2 by altering the relative contribution of asset heterogeneity to skill heterogeneity, which is governed by the ratio of asset income to wage income, Φ_t. In the existing literature in which wage income inequality is absent (e.g., Chu and Cozzi 2018), income inequality is monotonically increasing in Φ_t, because a higher Φ_t implies that asset heterogeneity has more weights on the income distribution. Nevertheless, the introduction of skill heterogeneity in our model alters this outcome. A rise in Φ_t increases the degree of income inequality only if asset heterogeneity is larger than skill heterogeneity (i.e., a lower σ_H^2/σ_A^2); otherwise, a rise in Φ_t decreases income inequality.

Lemma 1. *The degree of income inequality is increasing (decreasing) in the ratio of asset income to wage income Φ_t if $\Phi_t > (<) \sigma_H^2/\sigma_A^2$.*

Proof. Differentiating (28) with respect to Φ_t shows that $\partial \sigma_{D,t}^2 / \partial \Phi_t \gtrless 0 \Leftrightarrow \Phi_t \sigma_A^2 \gtrless \sigma_H^2$. □

Next, we explore the effects of a rise in μ and β on Φ_t, respectively. Combining (17) and (19) yields

$$\frac{A_t}{w_t H} = \frac{Q_t H / \varphi \lambda}{(1-\alpha) Q_t H (\mu\eta)^{\frac{-\alpha}{1-\alpha}} / \alpha} = \frac{\alpha (1-\beta)(\mu\eta)^{\frac{\alpha}{1-\alpha}}}{\lambda \varphi (1-\alpha)}, \tag{29}$$

which is stationary in equilibrium, increasing in μ, and decreasing in β. Furthermore, together

[8] In Appendix B, we will discuss numerically the case of non-zero covariance.

with the fact that $r^* = \rho + g^*$ in (3) is also stationary, we have

$$\Phi_t = \Phi = \frac{\alpha(\lambda - 1)(\mu - 1)}{\mu\lambda(1-\alpha)} + \frac{\alpha\rho(1-\beta)(\mu\eta)^{\frac{\alpha}{1-\alpha}}}{\varphi\lambda^2(1-\alpha)} \tag{30}$$

and $\sigma^2_{D,t} = \sigma^2_D$ are both stationary.

A change in μ or β then affects Φ through two channels: through the interest rate r^* and the ratio of asset to wage income A_t/w_tH. First, a larger μ or β increases Φ because it raises g^* in (26) and thus r^* in (3); this is identified as the *interest-rate effect* in Chu and Cozzi (2018). Second, increasing μ raises the ratio of assets to wage income A_t/w_tH, because it increases the asset value A_t by driving up the monopoly profits flow according to (10). However, a higher subsidy rate β reduces the asset income by decreasing the value of inventions in (13); this is identified as the *asset-value effect* in Chu and Cozzi (2018). Combining these effects leads to a positive impact of μ on Φ and a negative impact of β on Φ.

We now analyze how a rise in μ affects σ^2_D.[9] Differentiating σ^2_D in (28) with respect to μ shows

$$\frac{\partial\sigma^2_D}{\partial\mu} \gtrless 0 \Leftrightarrow \left(\Phi\sigma^2_A - \sigma^2_H\right)\underbrace{\frac{\partial\Phi}{\partial\mu}}_{+} \gtrless 0. \tag{31}$$

When asset heterogeneity is larger than skill heterogeneity in the no-market-power environment such that $\Phi_{\mu=\underline{\mu}\to 1^+} > \sigma^2_H/\sigma^2_A$, the positive effect of μ on Φ and Lemma 1 together imply that σ^2_D is monotonically increasing in μ. The reason is that as μ rises, the condition $\Phi_{\mu>\underline{\mu}} > \sigma^2_H/\sigma^2_A$ continues to hold because $\partial\Phi/\partial\mu > 0$. By contrast, when asset heterogeneity is smaller than skill heterogeneity such that $\Phi_{\mu=\underline{\mu}\to 1^+} < \sigma^2_H/\sigma^2_A$, then starting from a low level of patent breadth, increasing μ reduces σ^2_D. However, as μ continues to rise, the sign of the condition $\Phi_{\mu>\underline{\mu}} < \sigma^2_H/\sigma^2_A$ will eventually be reversed, so that income inequality becomes increasing in μ. Therefore, the overall impact of a rise in μ on income inequality is U-shaped.

Similarly, to see how a rise in β affects σ^2_D, differentiating σ^2_D with respect to β yields

$$\frac{\partial\sigma^2_D}{\partial\beta} \gtrless 0 \Leftrightarrow \left(\Phi\sigma^2_A - \sigma^2_H\right)\underbrace{\frac{\partial\Phi}{\partial\beta}}_{-} \gtrless 0. \tag{32}$$

When asset heterogeneity is larger than skill heterogeneity in the no-research-subsidy environment such that $\Phi_{\beta\to 0^+} > \sigma^2_H/\sigma^2_A$, the negative effect of β on Φ and Lemma 1 together imply two cases. First, as β increases to unity and if $\Phi_{\beta\to 1^-} > \sigma^2_H/\sigma^2_A$ still holds, σ^2_D is a monotonically decreasing function of β. Second, as β increases to unity and if $\Phi_{\beta\to 1^-} < \sigma^2_H/\sigma^2_A$ holds instead, σ^2_D is a U-shaped function of β. By contrast, when asset heterogeneity is smaller than skill heterogeneity such that $\Phi_{\beta\to 0^+} < \sigma^2_H/\sigma^2_A$, σ^2_D is a monotonically increasing function of β because

[9] In Appendix B, we calibrate our model to the US data and perform numerical exercises to examine the impacts of patent protection and R&D subsidies on income inequality.

$\Phi_{\beta>1^-} < \sigma_H^2/\sigma_A^2$ continues to hold as $\partial\Phi/\partial\beta < 0$. The following proposition summarizes the above results.

Proposition 1. *The degree of income inequality can be a monotonically increasing function of patent breadth* μ *if* $\Phi_{\mu=\underline{\mu}\to 1^+} > \sigma_H^2/\sigma_A^2$*, and a U-shaped function if* $\Phi_{\mu=\underline{\mu}\to 1^+} < \sigma_H^2/\sigma_A^2$*. In addition, the degree of income inequality can be a monotonically increasing function of the R&D subsidy rate* β *if* $\Phi_{\beta\to 0^+} < \sigma_H^2/\sigma_A^2$*, and a monotonically decreasing or U-shaped function if* $\Phi_{\beta\to 0^+} > \sigma_H^2/\sigma_A^2$*.*

Proof. Proven in text. □

4 Conclusion

This study analyzes the effects of patent protection versus R&D subsidies on innovation and income inequality in a Schumpeterian growth model with households who are heterogeneous in both asset holdings and skill endowments. Although raising the level of patent protection and the rate of R&D subsidy leads to a symmetric, positive effect on innovation and economic growth, they generate asymmetric distributional effects on income inequality, depending on the relative importance of asset heterogeneity and skill heterogeneity.

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