Online Appendix for “Effects of R&D Policy on Income Inequality in A Growth Model with Heterogeneous Assets and Skills"

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Online Appendix A

A.1 Definition of equilibrium

This section defines the decentralized equilibrium in the economy. An equilibrium is represented as time paths of consumption levels, aggregate spending on intermediate goods, and aggregate R&D expenditure, $[C_t, X_t, Z_t]_{t=0}^{\infty}$; stochastic paths of prices and quantities for intermediate goods with the highest quality in their lines at that point, $[p_t(\epsilon|q), x_t(\epsilon|q)]_{\epsilon\in[0,1],t=0}^{\infty}$; and time paths of aggregate quality, $[Q_t]_{t=0}^{\infty}$, interest rates, $[r_t]_{t=0}^{\infty}$, wage rates $[w_t]_{t=0}^{\infty}$, and value functions, $[V_t(\epsilon|q)]_{\epsilon\in[0,1],t=0}^{\infty}$ such that heterogeneous households $s \in [0,1]$ maximize their utility taking $\{r_t, w_t\}$ as given; competitive final-good firms produce y_t to maximize profits taking $\{w_t, p_t(\epsilon|q)\}$ as given; monopolistic intermediate-goods firms produce $x_t(\epsilon|q)$ to maximize profits taking the price of final good $\{P_t\}$ as given; competitive R&D firms choose $\{Z_t(\epsilon|q)\}$ to maximize their profits taking $\{r_t, P_t\}$ as given; the market-clearing condition for labor holds; the market-clearing condition for final good holds:

$$C_t + X_t + Z_t + G_t = Y_t, \tag{A.1}$$

and the market-clearing condition for financial assets holds:

$$A_t \equiv \int_0^1 A_t(s)ds = V_t. \tag{A.2}$$

A.2 Proof of stability

In this proof, we examine the stability of this model for a given level of μ and β. First, define the transformed variable $\Psi_t \equiv C_t/Y_t$. Then, taking the log of Ψ_t and differentiating it with respect to time yields

$$\frac{\dot{\Psi}_t}{\Psi_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t}. \tag{A.3}$$

Using (14) and (19), we obtain

$$Y_t = (\mu\eta)^{\frac{-\alpha}{1-\alpha}} \left[\frac{\varphi\lambda}{\alpha(1-\beta)}\right] V_t. \tag{A.4}$$

Hence, (A.4) implies

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{V}_t}{V_t}. \tag{A.5}$$

In addition, aggregating (11) for all ϵ yields

$$\begin{aligned}
&\int_0^1 r_t V_t(\epsilon|q)d\epsilon = \int_0^1 \Pi_t(\epsilon|q)d\epsilon + \int_0^1 \dot{V}_t(\epsilon|q)d\epsilon - \int_0^1 I_t(\epsilon|q)V_t(\epsilon|q)d\epsilon \\
\Leftrightarrow & r_t V_t = \Pi_t + \dot{V}_t - \int_0^1 \frac{\varphi Z_t(\epsilon|q)}{Hq_t(\epsilon)} \cdot \frac{(1-\beta)Hq_t(\epsilon)}{\varphi\lambda} d\epsilon \\
\Leftrightarrow & r_t V_t = \Pi_t + \dot{V}_t - (1-\beta)Z_t/\lambda \\
\Leftrightarrow & \dot{V}_t/V_t = r_t - \Pi_t/V_t + (1-\beta)Z_t/(\lambda V_t) \\
\Leftrightarrow & \dot{V}_t/V_t = r_t - \Pi_t/V_t + (1-\beta)[(1-\gamma)Y_t - C_t - X_t]/(\lambda V_t)
\end{aligned} \tag{A.6}$$

where the second equality applies (12) and (13), and the last equality uses (A.1) and $G_t = \gamma Y_t$. Inserting (14), (16), (18), and (19) into (A.6) yields

$$\begin{aligned}
\frac{\dot{V}_t}{V_t} &= r_t - \frac{\left(\frac{\mu-1}{\mu}\right)(\mu\eta)^{\frac{-\alpha}{1-\alpha}} Q_t H}{(1-\beta)Q_t H/\varphi\lambda} + \left(\frac{1-\beta}{\lambda}\right)\left\{\frac{\varphi\lambda(1-\gamma)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha(1-\beta)} - \frac{C_t}{Y_t}\frac{\varphi\lambda(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha(1-\beta)} - \frac{\eta(\mu\eta)^{\frac{-1}{1-\alpha}} Q_t H}{(1-\beta)Q_t H/\varphi\lambda}\right\} \\
&= r_t - \frac{(\mu-1)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}\varphi\lambda}{\mu(1-\beta)} + \frac{\varphi(1-\gamma)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha} - \Psi_t \frac{\varphi(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha} - \eta\varphi(\mu\eta)^{\frac{-1}{1-\alpha}}.
\end{aligned} \tag{A.7}$$

Then, substituting (A.5) and (A.7) into (A.3), together with the Euler equation (3), yields a one-dimensional differential equation for Ψ_t such that

$$\begin{aligned}
\frac{\dot{\Psi}_t}{\Psi_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} &= -\rho + \frac{(\mu-1)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}\varphi\lambda}{\mu(1-\beta)} - \frac{\varphi(1-\gamma)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha} + \Psi_t \frac{\varphi(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha} + \eta\varphi(\mu\eta)^{\frac{-1}{1-\alpha}} \\
&= \frac{\varphi(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha}\Psi_t - \rho + \frac{(\mu-1)\varphi\lambda(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\mu(1-\beta)} - \frac{\varphi(1-\gamma)(\mu\eta)^{\frac{-\alpha}{1-\alpha}}}{\alpha} + \eta\varphi(\mu\eta)^{\frac{1}{1-\alpha}}
\end{aligned} \tag{A.8}$$

Therefore, given that Ψ_t is a control variable and that its coefficient in (A.8) is positive, the dynamics of Ψ_t is characterized by saddle-point stability such that Ψ_t jumps immediately to its steady-state value such that

$$\Psi = 1 - \gamma - \frac{\alpha}{\mu} - \frac{\alpha\lambda(\mu - 1)}{\mu(1-\beta)} + \frac{\alpha\rho}{\varphi}(\mu\eta)^{\frac{\alpha}{1-\alpha}}. \tag{A.9}$$

where the parameter space is restricted to ensure $\Psi > 0$. Given the stationarity of Ψ, (A.3) and (A.5) immediately follow that $\dot{C}_t/C_t = \dot{Y}_t/Y_t = \dot{V}_t/V_t$. In addition, (14) and (16) imply that $\dot{Y}_t/Y_t = \dot{X}_t/X_t$. Therefore, Z_t grows at the same rate as $\{Y_t, C_t, X_t\}$ according to (A.1) and the relation $G_t = \gamma Y_t$. Moreover, since the aggregate effective labor supply H is always stationary, we have $\dot{Y}_t/Y_t = \dot{Q}_t/Q_t = \dot{w}_t/w_t$ according to (14). Finally, we formally have

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{V}_t}{V_t} = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{X}_t}{X_t} = \frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_t}{w_t}. \tag{A.10}$$

A.3 Skill distribution and asset distribution

Denote by $H \equiv \int_0^1 H(s)ds$ and $\theta_{H,0}(s) \equiv H(s)/H$ the aggregate effective labor supply and the skill share of household s at time 0, respectively. Because all households' skills are exogenously given, the skill share of household s must be stationary such that $\theta_{H,t}(s) = \theta_{H,0}(s)$. Consider a general distribution function of skill share with a mean of one and a standard deviation of $\sigma_H > 0$. Then this distribution must be stationary over time.

Similarly, define $\theta_{A,t}(s) \equiv A_t(s)/A_t$ and $\theta_{A,0}(s) \equiv A_0(s)/A_0$ the asset share of household s at time t and 0, respectively. We also consider a general distribution function of initial asset share with a mean of one and a standard deviation of $\sigma_A > 0$. The lemma below shows that the asset share is constant over time and exogenously determined at time 0.

Lemma 0. *For household s, the asset share is constant over time and exogenously determined at time 0 such that $\theta_{A,t}(s) = \theta_{A,0}(s)$ for $t > 0$.*

Proof. Aggregating (2) for all s yields

$$\dot{A}_t = r_t A_t + w_t H - C_t. \tag{A.11}$$

Combining (2) with (A.11), we can derive the motion of $\theta_{At}(s)$ such that

$$\frac{\dot{\theta}_{A,t}(s)}{\theta_{A,t}(s)} = \frac{\dot{A}_t(s)}{A_t(s)} - \frac{\dot{A}_t}{A_t} = \frac{C_t - w_t H}{A_t} - \frac{C_t\theta_{C,t}(s) - w_t H\theta_{H,0}(s)}{A_t(s)},$$

which can be rewritten as

$$\dot{\theta}_{A,t}(s) = \frac{C_t - w_t H}{A_t}\theta_{A,t}(s) - \frac{C_t\theta_{C,t}(s) - w_t H\theta_{H,0}(s)}{A_t}, \tag{A.12}$$

where $\theta_{C,t}(s) \equiv C_t(s)/C_t$ is the share of consumption by household s at time t. Taking the log of $\theta_{C,t}(s)$ and differentiating the resulting equation with respect to time yields

$$\frac{\dot{\theta}_{C,t}(s)}{\theta_{C,t}(s)} = \frac{\dot{C}_t(s)}{C_t(s)} - \frac{\dot{C}_t}{C_t} = 0, \tag{A.13}$$

according to the Euler equation (3). Equation (A.13) then implies $\theta_{C,t}(s) = \theta_{C,0}(s)$ for all $t > 0$.

Since the economy is always on the balanced growth path, variables $\{C_t, A_t, w_t\}$ must grow at the same rate of g. Using (3) and the aggregate budget constraint $\dot{A}_t = r_t A_t + w_t H - C_t$, we have

$$\frac{C_t - w_t H}{A_t} = r_t - \dot{A}_t/A_t = \rho > 0. \tag{A.14}$$

This implies that the coefficient on $\theta_{A,t}(s)$ in (A.12) equals to $\rho > 0$. Since $\theta_{A,t}(s)$ is a state variable and the coefficient of $\theta_{A,t}(s)$ is positive, the only solution for the one-dimensional differential equation that describes the potential evolution of $\theta_{A,t}(s)$ given an initial $\theta_{A0}(s)$, as presented in (A.12), is $\dot{\theta}_{A,t}(s) = 0$ for all $t > 0$. This can be achieved by letting consumption share $\theta_{C,t}(s)$ jump to its steady-state value $\theta_{C,0}(s)$. Imposing $\dot{\theta}_{A,t}(s) = 0$ on (A.12) yields

$$\theta_{C,0}(s) = \frac{\rho A_t}{C_t}\theta_{A,0}(s) + \frac{w_t H}{C_t}\theta_{H,0}(s) + \frac{\tau_t}{C_t}. \tag{A.15}$$

□

Online Appendix B Quantitative analysis

In this Appendix, we calibrate the model using the US data and provide several numerical exercises to investigate further the impacts of patent protection and R&D subsidies on income inequality.

The parameters to be calibrated include $\{\rho, \lambda, \beta, \alpha, \mu, \eta, \varphi, \gamma\}$. We apply a similar strategy for calibrating the parameters as Chu and Cozzi (2018). To begin with, we follow Acemoglu and Akcigit (2012) to set the subjective discount rate ρ to 0.05 and the quality step size of innovation λ to 1.05. Following Belo *et al.* (2013), the government spending to GDP ratio γ is set to be 0.2, which coincides with the US data. For the R&D subsidy rate β, we follow Chu and Cozzi (2018) to set it to 0.188, which is the US subsidy rate calculated by Impullitti (2010). This value implies that approximately 19% of the total R&D spending is subsidized by the government. The elasticity of substitution between any two goods is assumed to be 2.9, in accordance with Belo *et al.* (2013) and Acemoglu *et al.* (2018). This gives $\alpha = 1 - 1/2.9 \approx 0.655$. Without loss of generality, we follow Acemoglu (2009) (Chapter 14) to normalize $\eta = \alpha$. Following Chu and Cozzi (2018), the arrival rate of innovation I in equation (25) is set to 12.5%. In addition, since all investments in our model are made for R&D, we follow Impullitti (2010) to proxy the R&D to GDP ratio $Z_t/(Y_t - X_t)$ as the share of intangible capital investment to GDP, which is 13.5%.

Table 1: The calibrated parameter values

ρ	λ	γ	β	φ	α	μ	η
0.05	1.05	0.2	0.188	0.901	0.655	1.3	0.655

Matching this target results in R&D productivity φ of 0.901. Regarding the limit of the price to marginal cost rate, we follow Yang (2021) and set the benchmark value of 1.3, which is also the estimate in Jones and Williams (2000). The value of μ will be varied in our analysis below. The values of the calibrated parameters are reported in Table 1.

We utilize the calibrated parameters to numerically solve for the model and examine the impacts of patent policy and subsidy policy on income inequality. Figure 1a and 1b plot, respectively, the income variance σ_D^2, defined in equation (28), against patent breadth μ and R&D subsidy β. As a first experiment, we take the ratio of the variance of skill to the variance of assets σ_H^2/σ_A^2 to be 0.08/1. As shown, σ_D^2 exhibits a U-shape with both μ and β, consistent with Proposition 1. In particular, on the left panel, the income dispersion decreases monotonically as the level of patent breadth μ increases from 1. Then, when $\mu = 1.3$, it reaches its minimum, and gradually rises afterward. In other words, a slight increase in patent breadth from a no-market-power scenario rewards patentees with more market power to raise their prices, resulting in a reduction in income inequality. This inequality-patents relation reverses, however, when the level of patent protection gets closer to "complete". Similarly, on the right panel, the U-shaped inequality-subsidy relation shows that only a modest increase in R&D subsidies is effective in reducing income inequality. The relation starts to reverse when the subsidy rate exceeds 16%.

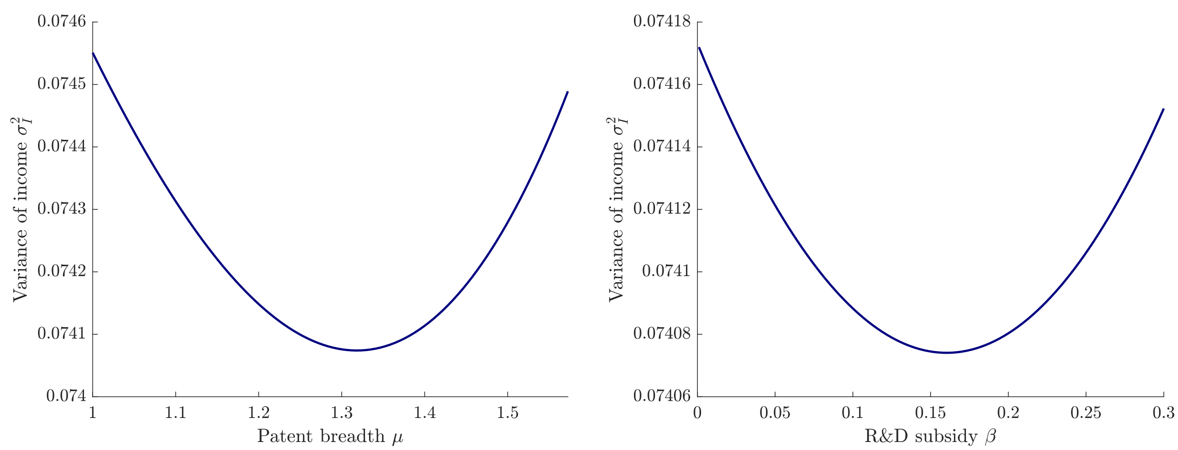

Figure 1: (a) Patent breadth and income inequality (b) R&D subsidy and income inequality

As explained by Proposition 1, the effects of patent policy on income inequality will be monotonic if the ratio of skill variance to asset variance σ_H^2/σ_A^2 falls below certain thresholds. To illustrate, in the second experiment we decrease the variance of skill distribution such that

$\sigma_H^2 = (0.08/2)\sigma_A^2$. The resulting curves are shown in Figure 2a and 2b. Based on the figures, it is evident that if the variance ratio is sufficiently low, implementing broader patent breadth may exacerbate income disparity. In contrast, one can mitigate income dispersion by increasing the R&D subsidy rate. In particular, from the right panel, when the R&D subsidy rate increases from 0 to 20%, the income variance is reduced by approximately 2.5% ($(0.0396/0.0406 - 1) \times 100$). In summary, we have demonstrated the significance of the dispersion in asset holding and human capital in assessing the effectiveness of patent and subsidy policies.

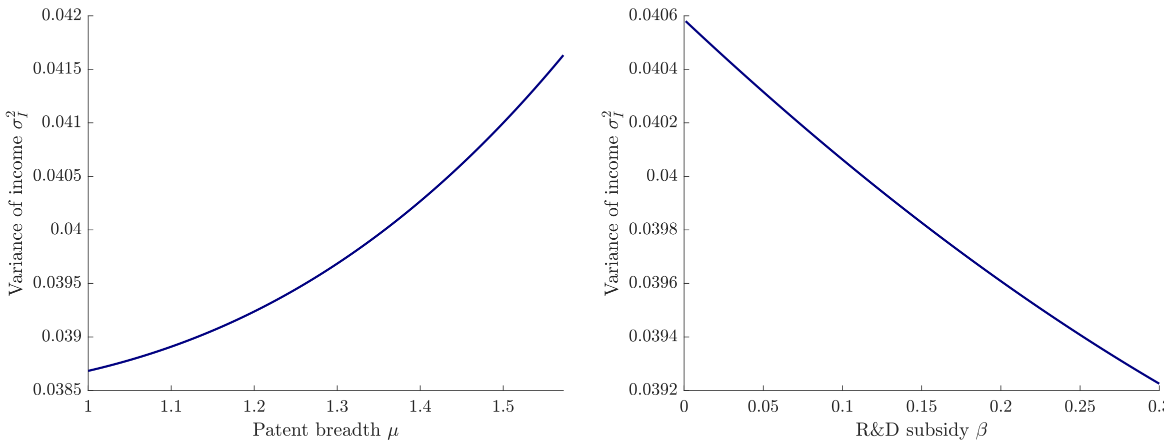

Figure 2: (a) Patent breadth and income inequality (b) R&D subsidy and income inequality

As stated in the main text, we assume that the covariance between wealth and skill distributions $\sigma_{H,A}$ is zero. Here, we extend our model by allowing the two distributions to be correlated. Based on the US household data, Pfeffer (2011) find that family wealth and child education attainment exhibit a positive correlation, with a correlation coefficient ranging from 0.288 to 0.376. To this end, we plot similar curves in Figure 3a and 3b by setting $\sigma_{H,A}$ equal 0.288 and 0.376, respectively, in addition to $\sigma_H^2 = (0.08/2)\sigma_A^2$. As shown, the monotonic relationship between the income variance and R&D policy parameters μ and β is preserved even when the correlation is not zero.

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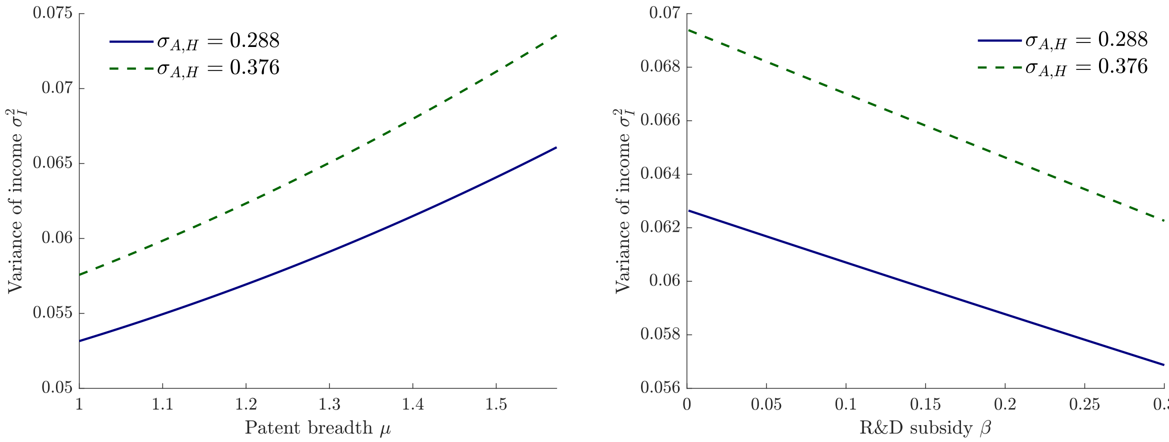

Figure 3: (a) Patent breadth and income inequality; (b) R&D subsidy and income inequality.

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