Online appendix for "Inflation and income inequality in a variety-expansion growth model with menu costs"

Zhijie Zheng*
Beijing Normal University at Zhuhai

Tapas Mishra
University of Southampton

Yibai Yang
University of Macau

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Appendix A

A.1 Proof of Lemma 1

We follow the appendix in Arawatari, Hori and Mino (2018) to prove Lemma 1. Recall that the wage rate is

$$w_t = \alpha \int_0^{N_t} \left(\frac{1}{x_t(j)} \right)^{\alpha-1} dj = \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \int_0^{N_t} [\xi_t(j)]^{-\frac{1-\alpha}{\alpha}} dj, \tag{A.1}$$

where (5) and $\xi_t(j) = p_t(j)/P_t$ have been used. Consider firms whose age is $\gamma = a + s\Delta$, where $a \in [0, \Delta)$. The number of such firms is given by

$$\dot{N}_{t-\gamma} = gN_{t-\gamma} = gN_{t-(a+s\Delta)} = gN_{t-s\Delta}e^{-ga}. \tag{A.2}$$

*Center for Innovation and Development Studies, Beijing Normal University at Zhuhai, 519087, China. *Email* address: zhengzhijie1919@gmail.com.

Remember that each monopolist follows the same pricing rule. Thus, we have

$$\begin{aligned}\int_0^{N_t} [\xi_t(j)]^{-\frac{1-\alpha}{\alpha}} dj = & \int_0^{\Delta} (\xi_{t-\gamma})^{-\frac{1-\alpha}{\alpha}} g N_{t-\gamma} d\gamma \\ & + \int_{\Delta}^{2\Delta} (\xi_{t-\gamma})^{-\frac{1-\alpha}{\alpha}} g N_{t-\gamma} d\gamma + \cdots \\ & + \int_{s\Delta}^{(s+1)\Delta} (\xi_{t-\gamma})^{-\frac{1-\alpha}{\alpha}} g N_{t-\gamma} d\gamma + \cdots \\ = & \sum_{s=0}^{\infty} \int_{s\Delta}^{(s+1)\Delta} \left(\xi_{t-(\gamma-s\Delta)}\right)^{-\frac{1-\alpha}{\alpha}} g N_{t-\gamma} d\gamma,\end{aligned} \tag{A.3}$$

where

$$\begin{aligned}\int_{s\Delta}^{(s+1)\Delta} \left(\xi_{t-(\gamma-s\Delta)}\right)^{-\frac{1-\alpha}{\alpha}} g N_{t-\gamma} d\gamma &= \int_0^{\Delta} (\xi_{t-a})^{-\frac{1-\alpha}{\alpha}} g N_{t-(a+s\Delta)} da \\ &= g N_{t-s\Delta} \int_0^{\Delta} (\xi_{t-a})^{-\frac{1-\alpha}{\alpha}} e^{-ga} da \\ &= g N_t e^{-gs\Delta} \int_0^{\Delta} (\xi_{t-a})^{-\frac{1-\alpha}{\alpha}} e^{-ga} da \\ &= g N_t e^{-gs\Delta} \int_0^{\Delta} \mu^{-\frac{1-\alpha}{\alpha}} e^{\frac{1-\alpha}{\alpha}\pi a} e^{-ga} da \\ &= g N_t e^{-gs\Delta} \underbrace{\frac{\left[e^{(\frac{1-\alpha}{\alpha}\pi - g)\Delta} - 1\right] \mu^{-\frac{1-\alpha}{\alpha}}}{(1-\alpha)\pi/\alpha - g}}_{\Omega(\Delta)},\end{aligned} \tag{A.4}$$

where we applied $\xi_{t-a} = \mu e^{-\pi a}$ in the forth equation of (A.4). Inserting (A.4) into (A.3) and plugging the resulting equation into (A.1) yield the steady state equilibrium wage rate in Lemma 1 such that

$$w_t = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{g N_t}{1-e^{-g\Delta}} \Omega(\Delta). \tag{A.5}$$

Therefore, the aggregate output is expressed as

$$y_t = \frac{w_t l_t}{\alpha} = \frac{w_t}{\alpha} = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{g N_t}{1-e^{-g\Delta}} \Omega(\Delta). \tag{A.6}$$

Similarly, the aggregate amount of intermediate goods is derived as

$$\begin{aligned} x_t &= \int_0^{N_t} x_t(j)dj = (1-\alpha)^{\frac{1}{\alpha}} \int_0^{N_t} [\xi_t(j)]^{-\frac{1}{\alpha}} dj \\ &= (1-\alpha)^{\frac{1}{\alpha}} \sum_{s=0}^{\infty} \int_{s\Delta}^{(s+1)\Delta} \left(\xi_{t-(\gamma-s\Delta)}\right)^{-\frac{1}{\alpha}} gN_{t-\gamma} d\gamma \\ &= (1-\alpha)^{\frac{1}{\alpha}} \sum_{s=0}^{\infty} gN_t e^{-gs\Delta} \int_0^{\Delta} \left(\xi_{t-a}\right)^{-\frac{1}{\alpha}} e^{-ga} da \\ &= (1-\alpha)^{\frac{1}{\alpha}} \sum_{s=0}^{\infty} gN_t e^{-gs\Delta} \int_0^{\Delta} \mu^{-\frac{1}{\alpha}} e^{\frac{\pi}{\alpha}a} e^{-ga} da \\ &= (1-\alpha)^{\frac{1}{\alpha}} \frac{gN_t}{1-e^{-g\Delta}} \underbrace{\frac{\left[e^{(\pi/\alpha-g)\Delta}-1\right]\mu^{-\frac{1}{\alpha}}}{\pi/\alpha-g}}_{\Phi(\Delta)}. \end{aligned} \tag{A.7}$$

Moreover, at time t, new intermediate-goods firms, with an amount of $\dot{N}_t = gN_t$, incur menu costs of κ to set their prices at entry. In addition, firms whose age is $s\Delta$ also pay menu costs to adjust their prices, where $s = 0, 1, 2, ...$ Given that the number of these firms is $gN_{t-s\Delta} = gN_t e^{-gs\Delta}$, the total menu costs are given by

$$z_t = \sum_{s=0}^{\infty} gN_t e^{-gs\Delta}\kappa = \frac{\kappa g N_t}{1-e^{-g\Delta}}. \tag{A.8}$$

Substituting $R_t = gN_t/\beta$, (A.6), (A.7) and (A.8) into the final-goods market clearing condition (13) yields the aggregate level of consumption such that

$$c_t = y_t - x_t - z_t - R_t = \frac{gN_t}{1-e^{-g\Delta}} \left[(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Omega(\Delta) - (1-\alpha)^{\frac{1}{\alpha}}\Phi(\Delta) - \frac{1-e^{-g\Delta}}{\beta} - \kappa\right]. \tag{A.9}$$

A.2 Additional numerical results

In this subsection, we show numerically that both the asset-value and inequality effect can be positive. When adjusting the markup value from the benchmark value to 1.1 and menu cost parameter κ to 0.6, there arises a positive effect of inflation on the asset value (relative to wage rate) as in Figure 1. Yet, the nexus between inflation and income inequality is still negative, which indicates that the negative interest-rate effect dominates the positive asset-value effect. In this case, the economic growth rate decreases very fast to zero. The intuition is straightforward. When the price-adjusting cost is sufficiently high, represented by κ, even for a small rise in the inflation rate, the cost for firms to change their prices is very high. As a result, firms' profits could decrease to zero, making R&D unattractive and leading to no growth.

In this model, for a very large wide and reasonable range of parameter values, the relationship

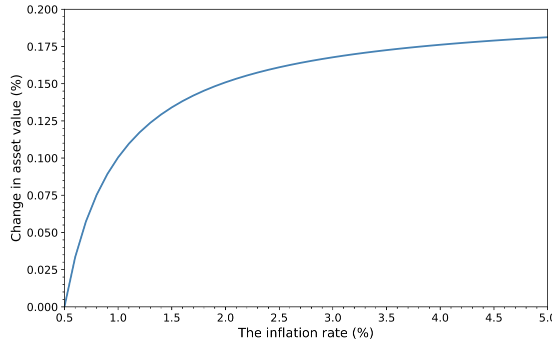


Figure 1: Inflation and asset value.

of inflation and inequality is negative. In order to show the mathematical possibility of a positive linkage, we fix κ to 0.6 and set the markup to 0.2, which is apparently implausible. Figure 2a and 2b then show that the impact of inflation on the asset-value effect and income inequality become positive. Again, since this case is unrealistic, we intend to keep it only as a mathematical experiment.

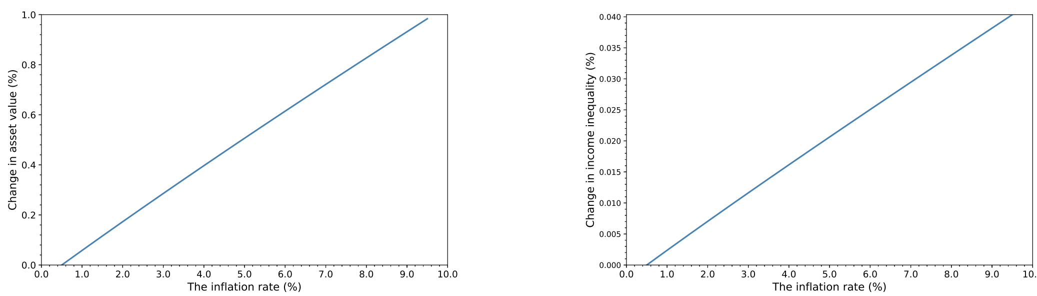


Figure 2: (a) Inflation and asset value; (b) Inflation and income inequality.

Appendix B Extension of elastic labor supply

In this appendix, we consider elastic labor supply by households. In this case, household h's utility function is given by

$$U_t = \int_t^{\infty} e^{-\rho(t'-t)} \left\{ \ln c_{t'}(h) + \varphi \ln[1 - l_{t'}(h)] \right\} dt', \tag{B.1}$$

where $\varphi > 0$ measures household h's preference of leisure. The asset-accumulation function is now given by $\dot{a}_t(h) = r_t a_h + w_t l_t(h) - c_t(h)$. Solving household h's utility-maximizing problem yields

$$w_t[1 - l_t(h)] = \varphi c_t(h) \tag{B.2}$$

and the Euler equation as in (3). The demand function of $x_t(j)$ in (5) and the wage rate in (6) are now given by, respectively,

$$x_t(j) = (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{P_t}{p_t(j)}\right)^{\frac{1}{\alpha}} l_t, \tag{B.3}$$

$$w_t = \alpha \int_0^{N_t} \left(\frac{l_t}{x_t(j)}\right)^{\alpha-1} dj. \tag{B.4}$$

The real-period profit function of incumbents is

$$\Pi_t(p_t(j)) = \frac{\xi_t(j) - 1}{[\xi_t(j)]^{1/\alpha}}(1-\alpha)^{1/\alpha} l_t. \tag{B.5}$$

Since $l_t = l$ is constant on the steady state equilibrium, the firm value in (10) and the optimal time interval determined in (11) are, respectively, given by

$$V_t(j) = \frac{l[(1-\alpha)/\mu]^{1/\alpha}}{(1-e^{-r\Delta})} \left\{ \frac{\mu\left[1 - e^{-\left(r - \frac{1-\alpha}{\alpha}\pi\right)\Delta}\right]}{r - \frac{1-\alpha}{\alpha}\pi} - \frac{1 - e^{-\left(r - \frac{\pi}{\alpha}\right)\Delta}}{r - \frac{\pi}{\alpha}} \right\} - \frac{\kappa}{1 - e^{-r\Delta}}, \tag{B.6}$$

$$rV_t(j) = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\alpha}} e^{\frac{\pi}{\alpha}\Delta}(\mu e^{-\pi\Delta} - 1)l. \tag{B.7}$$

Moreover, following the derivations in Appendix A, we obtain

$$w_t = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{gN_t}{1 - e^{-g\Delta}} \Omega(\Delta), \tag{B.8}$$

$$y_t = \frac{w_t l_t}{\alpha} = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{gN_t}{1 - e^{-g\Delta}} \Omega(\Delta) l, \tag{B.9}$$

$$x_t = (1-\alpha)^{\frac{1}{\alpha}} \frac{gN_t}{1 - e^{-g\Delta}} \frac{\left[e^{(\pi/\alpha - g)\Delta} - 1\right] \mu^{-\frac{1}{\alpha}}}{\pi/\alpha - g}, \tag{B.10}$$

and the expressions of z_t and R_t remain unchanged. Substituting these equations into the aggregate consumption yields

$$c_t = \frac{gN_t}{1 - e^{-g\Delta}} \left\{ (1-\alpha)^{\frac{1-\alpha}{\alpha}} \Omega(\Delta) l - (1-\alpha)^{\frac{1}{\alpha}} \Phi(\Delta) l - \frac{1 - e^{-g\Delta}}{\beta} - \kappa \right\} \tag{B.11}$$

The model is eventually reduced to a three-equations system of endogenous variables $\{l, \Delta, g\}$.

The first equation is obtained by combining (B.2) with (B.11) such that

$$\alpha\Omega(\Delta) - \varphi\left(\frac{1-e^{-g\Delta}}{\beta}+\kappa\right)(1-\alpha)^{\frac{1-\alpha}{\alpha}} = l\left[(\varphi+\alpha)\Omega(\Delta) - \varphi(1-\alpha)\Phi(\Delta)\right] \tag{B.12}$$

Combining (12) with (B.7) yields the second equation such that

$$\frac{\rho+g}{\beta} = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\alpha}} e^{\frac{\pi}{\alpha}\Delta}(\mu e^{-\pi\Delta}-1)l. \tag{B.13}$$

Combining (12) with (B.6) yields the third equation such that

$$\frac{(1-e^{-r\Delta})}{\beta} = l[(1-\alpha)/\mu]^{1/\alpha}\left\{\frac{\mu\left[1-e^{-\left(r-\frac{1-\alpha}{\alpha}\pi\right)\Delta}\right]}{r-\frac{1-\alpha}{\alpha}\pi} - \frac{1-e^{-\left(r-\frac{\pi}{\alpha}\right)\Delta}}{r-\frac{\pi}{\alpha}}\right\} - \kappa. \tag{B.14}$$

As for the effect of inflation on income inequality, income inequality is still an increasing function of $r_t a_t/w_t$ as in (23), which can be further decomposed into the interest-rate effect (i.e., r) and the asset-value effect (i.e., a_t/w_t). The expression of a_t/w_t is same as (24). The expression of income deviation is now given by

$$\sigma_I = \frac{(\rho+g+\varphi g)a_t/w_t}{(\rho+g+\varphi g)a_t/w_t+1}\sigma_a, \tag{B.15}$$

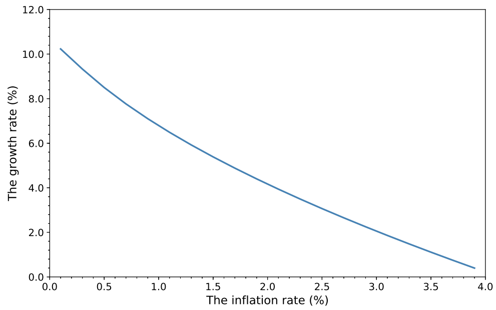
where φ captures the effect of elastic labor supply.

Again, we numerically evaluate the effects of inflation on economic growth and income inequality in this extension. We find that the result is robust to the counterpart in the benchmark model. We use the standard moment of labor supply (i.e, $l = 1/3$) to calibrate the newly added parameter φ. Other parameters (except β) remain unchanged as in the benchmark parametrization. Then, the calibrated values of parameters are given by $\varphi = 1.5523$ and $\beta = 14.3680$. Figure 3a and 3b show that the rate of economic growth and the degree of income inequality are monotonically deceasing in the rate of inflation.[1]

Appendix C Knowledge-based specification

In this section, we consider a knowledge-based version of the benchmark model. The main modification is that entrepreneurs employ labors, instead of final goods, for performing R&D. The discovery rate of new innovations is given by $\dot{N}_t = \beta N_t l_{r,t}$, where $l_{r,t}$ denotes the level of

[1]Given the calibrated parameters, the economic growth rate decreases to zero at the inflation rate of 4%. Thus, the range of the inflation rate in this numerical analysis is restricted within $[0, 0.04]$.



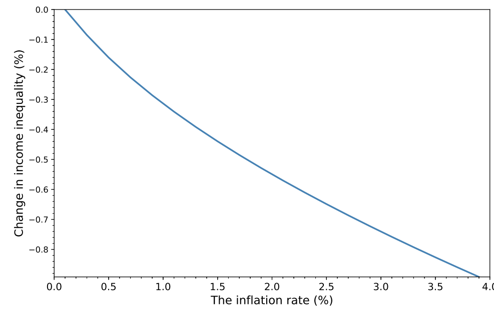


Figure 3: (a) Inflation and economic growth; (b) Inflation and income inequality.

labor employment hired for creating inventions. The free-entry condition to R&D sector is

$$\dot{N}_t V_t(j) = w_t l_{rt} \Leftrightarrow \beta V_t(j) N_t = w_t \Leftrightarrow V_t = w_t/\beta, \tag{C.1}$$

where $V_t = \int_0^{N_t} V_t(j)dj$ is the aggregate firm value. For simplicity, we assume inelastic labor supply as in the benchmark framework. We denote by $l_{y,t}$ the level of labor used in final-goods production and thus obtain the labor-market clearing condition such that $l_{y,t} + l_{r,t} = 1$. The real-period profit of the monopolistic intermediate-goods producers is given by

$$\Pi_t(p_t(j)) = \frac{\xi_t(j) - 1}{[\xi_t(j)]^{1/\alpha}} (1-\alpha)^{1/\alpha} l_{y,t}. \tag{C.2}$$

The firm value in (10) and the optimal time interval determined in (11) are now given by

$$V_t(j) = \frac{l_{yt}[(1-\alpha)/\mu]^{1/\alpha}}{(1-e^{-r\Delta})} \left\{ \frac{\mu \left[1 - e^{-\left(r - \frac{1-\alpha}{\alpha}\pi\right)\Delta}\right]}{r - \frac{1-\alpha}{\alpha}\pi} - \frac{1 - e^{-\left(r - \frac{\pi}{\alpha}\right)\Delta}}{r - \frac{\pi}{\alpha}} \right\} - \frac{\kappa}{1 - e^{-r\Delta}}, \tag{C.3}$$

and

$$rV_t(j) = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\alpha}} e^{\frac{\pi}{\alpha}\Delta} (\mu e^{-\pi\Delta} - 1) l_{yt}, \tag{C.4}$$

respectively. The final-goods market clearing condition is $c_t + z_t + x_t = y_t$, where

$$z_t = \kappa \sum_{s=0}^{\infty} g N_t e^{-gs\Delta} = \frac{g\kappa}{1 - e^{-g\Delta}} N_t, \tag{C.5}$$

$$x_t = (1-\alpha)^{\frac{1}{\alpha}} l_y \frac{g N_t}{1 - e^{-g\Delta}} \Phi(\Delta), \tag{C.6}$$

$$y_t = \frac{w_t l_{yt}}{\alpha} = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{gN_t}{1-e^{-g\Delta}} \Omega(\Delta) l_{yt}, \tag{C.7}$$

where $\Omega(\Delta)$ and $\Phi(\Delta)$ are denoted in the benchmark model. The model is reduced to a system of two equations and endogenous variables Δ and g. Combining (C.1) with (C.4), together with the expression of wage rate in (C.7), yields the first equation such that

$$\frac{\alpha\mu g}{1-\alpha} \frac{e^{(\frac{1-\alpha}{\alpha}\pi - g)\Delta} - 1}{\frac{1-\alpha}{\alpha}\pi - g} = \frac{1-e^{-g\Delta}}{\rho+g}(\beta-g)e^{\frac{\pi}{\alpha}\Delta}(\mu e^{-\pi\Delta} - 1). \tag{C.8}$$

Inserting (C.3) into (C.4) yields the second equation such that

$$\frac{\mu\left(1-e^{-(\rho+g-\frac{1-\alpha}{\alpha}\pi)\Delta}\right)}{\rho+g-(1-\alpha)\pi/\alpha} - \frac{1-e^{-(\rho+g-\frac{\pi}{\alpha})\Delta}}{\rho+g-\pi/\alpha} - \frac{(1-e^{-(\rho+g)\Delta})e^{\frac{\pi\Delta}{\alpha}}(\mu e^{-\pi\Delta}-1)}{\rho+g} = \left(\frac{\mu}{1-\alpha}\right)^{\frac{1}{\alpha}} \frac{\kappa}{1-g/\beta}, \tag{C.9}$$

where the steady-state conditions $g = \dot{N}_t/N_t = \beta l_r$ and $l_r + l_y = 1$ have been applied.

In this knowledge-based framework, inflation affects income inequality only in the way of changing the economic growth rate and thus the real interest rate (i.e., r_t), whereas leaving $a_t/w_t = V_t/w_t = 1/\beta$ unchanged. That is, the channel via the asset-value effect is silent. This feature is similar to that in Zheng (2020). Moreover, in this case, higher inflation can either stimulate or depress economic growth, which depends on two opposing forces. On the one hand, higher inflation induces firms to change prices more frequently and pay more menu costs, which in turn reduces the firm value and discourages innovation and growth. On the other hand, higher inflation decreases the demand of production labor and leads to a reallocation of labor to the R&D sector. As a result, the equilibrium (productivity-adjusted) wage rate (i.e., w_t/N_t) tends to rise and the free-entry condition implies an increase in the firm value, which encourages innovation and growth. Therefore, the effect of inflation on income inequality, which is completely determined by the growth effect of inflation, can be negative (positive) if the former force dominates (is dominated by) the latter force.

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