Inflation and Growth: A Non-Monotonic Relationship in an Innovation-Driven Economy*

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Abstract

This paper investigates the effects of monetary policy on long-run economic growth via different cash-in-advance (CIA) constraints on R&D in a Schumpeterian growth model with vertical and horizontal innovations. The relationship between inflation and growth is contingent on the relative extents of CIA constraints and diminishing returns to two types of innovation. This model can generate a mixed (monotonic or non-monotonic) relationship between inflation and growth, given that the relative strength of monetary effects on growth between different CIA constraints and that of R&D-labor-reallocation effects between different diminishing returns vary with the nominal interest rate. In the empirically relevant case where horizontal R&D is subject to larger diminishing returns than vertical R&D, inflation and growth can exhibit an inverted-U relationship when the CIA constraint on horizontal R&D is sufficiently larger than that on vertical R&D. Finally, we calibrate the model to the US economy and find that the growth-maximizing rate of inflation is around 2.4%, which is consistent with recent empirical estimates.

JEL classification: O30; O40; E41.
Keywords: Inflation; Endogenous growth; CIA constraint on R&D

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1 Introduction

The relationship between inflation and growth has long been debated among monetary economists. Is inflation negatively related to long-run economic growth conclusively? And do they maintain a steadily monotonic relationship regardless of the inflation level? Earlier studies indeed find a negative relationship between steady inflation and output/growth across countries (such as Fischer (1983) and Cooley and Hansen (1989)), whereas later works by Bruno and Easterly (1998) and Ahmed and Rogers (2000) seemingly find no robust relationship or even a positive correlation in low-inflation industrialized economies.

Recent empirical works challenge most previous studies that document only monotonic relationships between inflation and growth. They suggest a non-monotonic relationship in which the real growth effect of inflation could be either positive or negative, depending on the status quo inflation rate. This series of studies can be traced back to Sarel (1996), who identifies a structural break in the function that relates growth rates to inflation. His analysis shows that when inflation is low (i.e., 8% annually), there is no significant negative effect (or even a slightly positive effect) on economic growth. When inflation is high, however, there exists a robust, statistically significant negative effect on growth. Subsequent studies (such as Burdekin *et al.* (2004) and Eggoh and Khan (2014)) find a nonlinear correlation.[1] In this study, our model is calibrated to the aggregate data of the US economy to provide a quantitative analysis. We find that the growth-maximizing inflation rate is within the range for industrialized economies, i.e., 1-8%. Moreover, we show that the fraction of the CIA constraint on consumption and/or R&D is crucial in determining the inflation threshold.

In the present study, we reconcile the theories and recent empirical evidence on inflation and growth in the context of an innovation-driven growth model characterized by both vertical and horizontal innovations such as Peretto (1996, 1998), Howitt (1999) and Segerstrom (2000). Furthermore, various CIA constraints on R&D are incorporated, which shed light on how monetary policy can generate a non-monotonic relationship between inflation and growth through these constraints. In particular, our analysis builds on the framework developed by Howitt (1999) and Segerstrom (2000).[2] Vertical innovation serves to improve the quality of existing products whereas horizontal innovation aims at expanding product varieties, both of which are conducted by forward-looking entrepreneurs. Monetary policy, which acts as nominal interest rate targeting, affects the long-run growth rate by affecting the two types of innovation through the relative extents of different CIA constraints and diminishing returns to two types of R&D.

Imposing CIA constraints on R&D is in line with the following empirical findings. First, monetary evidence (e.g., Hall (1992) and Himmelberg and Petersen (1994)) reports a strong R&D-cash flow sensitivity for firms. Hall and Lerner (2010) report that more than 50 percent of R&D spending consists of wages and salaries of R&D personnel. Since hiring scientists and engineers usually involves a very high adjustment cost,[3] R&D-intensive firms are required to hold cash in order to smooth their R&D spending over time. Brown and Petersen (2011) offer direct evidence that US

[1] The specific threshold remains inconclusive, varying from 1% to 15-18%. As is documented in López-Villavicencio and Mignon (2011), the reasons for controversies include the frequency of data, the considered framework and the methodologies applied, the countries under study, and the existence of high-inflation observations.

[2] This class of R&D-driven growth models with two dimensional innovations have received more strong empirical support in recent years, such as in studies by Madsen (2008) and Ang and Madsen (2011).

[3] Because their skills are highly specific and unique, their vacancy may make the whole R&D process fail and dramatically decrease firms' profits. See Hall and Lerner (2010) for more detailed discussions.

firms relied heavily on cash reserves to smooth R&D spending during the 1998-2002 boom. The above evidence suggests that relative to traditional physical investment, R&D activities exhibit a stronger investment-cash flow sensitivity.

In addition, several important empirical findings concerning firm characteristics motivate us to capture these insights through an endogenous growth model with two modes of innovation. First, bigger firms invest relatively larger amounts in process and incremental (vertical) R&D, while smaller firms are usually involved in more radical (horizontal) product innovation (e.g., Akcigit (2009)). Second, the requirements of cash holdings show distinct patterns of these two modes of innovation. Existing empirical evidence shows that there is a stronger impact from cash holdings on R&D in smaller firms, which are more likely to confront binding liquidity and financing constraints (see Brown *et al.* (2012) and Caggese (2015)). Together with the fact that radical and original R&D are more adequately represented by horizontal innovation (see Acemoglu *et al.* (2014)), it is reasonable to consider that horizontal R&D is subject to a severer CIA constraint than vertical R&D. Accordingly, vertical innovation gains a cost advantage relative to horizontal innovation. Finally, empirical evidence in management (e.g., McDermott and O'Connor (2002)) show that radical innovation relies on less standardized capital, is often involved in new facilities and equipment, and faces higher technological uncertainty compared to incremental innovation. These features are consistent with the findings in Audretsch *et al.* (2006) and captured by Howitt (1999) that radical innovation is prone to suffer greater diminishing returns than incremental innovation.

Taking into consideration various CIA constraints, monetary policy in this study can generate different impacts on economic growth subject to the relative extents of the CIA constraints and the different diminishing returns to two innovations. To be specific, with a change in the nominal interest rate, different CIA constraints imply a force that transmits different inflation costs, which distort the incentives and the use of economic resources in different sectors; at the same time, different diminishing returns to R&D imply another force that triggers a reallocation of resources between two types of R&D activities. Both forces jointly determine the long-run relationship between inflation and growth.

We first investigate the polar cases subject to a single type of CIA constraint. In the presence of a CIA constraint on consumption, raising the nominal interest rate increases (decreases) the economic growth rate if horizontal R&D exhibits greater (smaller) diminishing returns than vertical R&D. In this case, the degree of relative diminishing returns to R&D plays a crucial role in determining the allocation of R&D resources. Along with a rise in the nominal interest rate, larger (smaller) diminishing returns to horizontal R&D allow more R&D resources to be allocated to horizontal (vertical) innovation than to vertical (horizontal) innovation, increasing the growth of variety (quality) at the expense of the growth of quality (variety) and thus leading to a decrease (an increase) in the long-run economic growth. By contrast, in the presence of a CIA constraint on vertical (horizontal) R&D, increasing the nominal interest rate always decreases (increases) economic growth rate regardless of the relative diminishing returns to both types of R&D. The reason is that R&D resources will always be shifted away from the CIA-constrained sector to the non-constrained one regardless of which R&D sector exhibits larger diminishing returns. The diminishing returns to R&D in these cases only govern "the amount" but not "the direction" of the shift in R&D resources.

More interestingly, incorporating all the CIA constraints into the model yields a diverse relationship between inflation and growth. In particular, by focusing on the empirically relevant scenario where horizontal R&D exhibits greater diminishing returns, we find that increasing the nominal interest rate may induce a non-monotonic (inverted-U) relationship between inflation and growth, provided that the CIA constraint on horizontal R&D is sufficiently stronger than that on vertical R&D. In this case, increasing the nominal interest rate from a low level yields a strong positive growth effect from the CIA constraint on horizontal R&D, which dominates the negative growth effects from the CIA constraints on consumption and vertical R&D; thus a positive relationship between inflation and growth emerges. Nevertheless, as the nominal interest rate increases and exceeds a threshold, the positive growth effect is dampened and becomes overwhelmed by the negative growth effects, leading to a negative relationship between inflation and growth. Overall, a non-monotonic relationship (inverted-U shape) is formed in this situation.

By applying the US aggregate data, our quantitative analysis in the benchmark case generates an inverted-U relationship between inflation and growth, showing that the threshold value of the inflation rate is around 2.4%, which is closely in line with the existing empirical estimates in Ghosh and Phillips (1998) (i.e., 2.5%) and Kremer *et al.* (2013) (i.e., 2%). The welfare, however, is monotonically decreasing in inflation, implying that the Friedman rule (i.e., the zero nominal interest rate) is optimal. Interestingly, when the relative extent of the CIA constraint on horizontal to vertical R&D decreases, the inflation-growth relationship becomes negative, which conforms to our analytical finding. The welfare, instead, becomes an inverted-U shape in inflation, implying the suboptimality of the Friedman rule. Finally, a sensitivity analysis is performed with alternative calibrated values for several key parameters, and it shows that our quantitative results are robust.

The literature pertaining to the analysis of monetary policy and economic growth is both large and diverse thereby deserving a detailed review. Nevertheless, closely related works are those that use endogenous growth models with R&D to analyze the effects of monetary policy on long-run growth. The pioneering work is the study by Marquis and Reffett (1994), who explore the effects of monetary policy on growth via a CIA constraint on consumption in the framework of Romer (1990). Subsequent studies (e.g., Chu and Lai (2013) and Chu and Cozzi (2014)) analyze monetary policy in a Schumpeterian quality-ladder model. The present study differs from the above works by considering a scale-invariant Schumpetarian growth model that features two dimensions of innovations. Another strand of the literature such as the papers by Huang *et al.* (2015) and Chu and Ji (2016) also analyzes the growth and welfare effects of monetary policy in a scale-invariant fully endogenous growth model based on Peretto (1998) that features both horizontal and vertical innovations. Nonetheless, their models only predict a monotonic linkage between inflation and long-run growth, whereas our model can yield a non-monotonic relationship between them, depending on the status quo inflation. Finally, to characterize a nonlinear relationship between inflation and growth, Arawatari *et al.* (2017) use a variety expansion model with heterogeneous R&D abilities. They find a cutoff inflation level around which a negative nonlinear relationship between inflation and growth occurs, but their analysis does not imply an inverted-U relationship between them. One notable exception is Chu *et al.* (2017), who also find an inverted-U relationship between inflation and growth in a canonical Schumpeterian growth model featuring a random quality improvement. Our results complement their work in two respects. First, their framework only considers vertical innovation, whereas our model considers vertical innovation in addition to horizontal innovation, and these two types of innovation are shown to play very different roles in explaining the impact of monetary policy on economic growth. Second, the model in Chu *et al.* (2017) removes scale effects by normalizing the size of population, whereas our model is made to be scale invariant by taking into account product proliferation.

The remainder of this study proceeds as follows. The basic model is presented in Section 2. Section 3 analyzes the effects of monetary policy in different cases of CIA constraints. Section 4 provides the numerical analysis, and the final section concludes.

2 Model

We consider a monetary variant of Howitt (1999) and Segerstrom (2000) that features two-dimensional innovations. The model is extended to examine the effects of monetary policy by allowing for elastic labor supply and CIA constraints on consumption and R&D investments. The economy consists of households, firms (including incumbents for intermediate goods production and entrants for vertical and horizontal R&D), and a government that is solely represented by the monetary authority.

2.1 The Household

Consider a closed economy that admits a representative household that is populated by a mass of individuals L_t with the population size growing at an exponential rate g_L. Each individual supplies labor elastically and faces a life-time utility function given by

$$U = \int_0^\infty e^{-\rho t}[\ln c_t + \theta \ln(1 - l_t)]dt, \tag{1}$$

where ρ is the discount rate, c_t is the consumption of final goods per capita at time t, l_t is the supply of labor per person at time t, and θ determines the preference for leisure relative to consumption.

An individual maximizes (1) subject to the budget constraint and a CIA constraint, which are respectively given by:

$$\dot{a}_t + \dot{m}_t = (r_t - g_L)a_t + w_t l_t + i_t b_t + \zeta_t - (\pi_t + g_L)m_t - c_t + d_t, \tag{2}$$

and

$$\xi_c c_t + b_t \leq m_t. \tag{3}$$

a_t is the real assets owned by each person and r_t is the real interest rate. Each individual supplies labor to earn a real wage rate w_t and loans out an amount b_t of money to the entrepreneurs, with a return rate i_t (i.e., the nominal interest rate). Each individual receives a lump-sum transfer ζ_t from the government. d_t is the real value of R&D profit. m_t is the real money balances held by the individual, and π_t is the inflation rate. The CIA constraint in (3) states that the holding of real money balances m_t by each household is used not only to finance the R&D investments but also to partly purchase consumption c_t, where $\xi_c \in [0, 1]$ represents the share of consumption required to be purchased by cash/money.

From standard dynamic optimization, we derive the following optimality conditions. The standard Euler equation that governs the growth of consumption is given by

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - g_L. \tag{4}$$

The optimal condition that determines the consumption-leisure tradeoff is

$$w_t(1 - l_t) = \theta c_t(1 + \xi_c i_t), \tag{5}$$

and the no-arbitrage condition between all assets and money implies the Fisher equation given by

$$i_t = r_t + \pi_t. \tag{6}$$

2.2 Final Goods

Final goods are produced by a mass of identical perfectly competitive firms that employ labor and a continuum of intermediate inputs according to the same constant returns to scale production technology. The production function of a typical firm k at time t is:

$$Y_{kt} = L_{ykt}^{1-\alpha} \int_0^{N_t} A_{it} x_{ikt}^{\alpha} di, \tag{7}$$

where L_{ykt} is the amount of labor employed by final-good firm k. N_t is the number of input varieties (or industries). A_{it} is the productivity level attached to the latest version of intermediate product i. x_{ikt} is the i-th type of intermediate inputs employed by firm k, and $\alpha \in (0, 1)$ is the elasticity of demand for intermediate products.

Firm k, subject to (7), chooses the amount of labor L_{ykt} and intermediate input x_{ikt} to maximize its profit, taking as given the wage rate w_t and the price of intermediate input p_{it}. The first-order condition with respect to x_{ikt} leads to the inverse demand for x_{ikt} such that $p_{it} = \alpha A_{it}(L_{ykt}/x_{ikt})^{1-\alpha}$. Since all firms face the same price p_{it}, the input ratios must be identical across firms such that $L_{ykt}/x_{ikt} = L_{yt}/x_{it}$, where $L_{yt} = \int L_{ykt} dk$ and $x_{it} = \int x_{ikt} dk$. Therefore, the price p_{it} can be reduced to

$$p_{it} = \alpha A_{it}(L_{yt}/x_{it})^{1-\alpha}. \tag{8}$$

Similarly, the inverse demand for L_{ykt} is given by

$$w_t = (1 - \alpha) \int_0^{N_t} A_{it} \left(\frac{x_{it}}{L_{yt}} \right)^{\alpha} di. \tag{9}$$

2.3 Incumbents

There is a continuum of industries N_t producing differentiated intermediate goods. Each industry is occupied by an industry leader who holds a patent on the latest innovation and monopolizes the production of one differentiated intermediate good i. The monopolistic leader dominates the market temporarily until its replacement by the next innovation.

The production technology across all incumbent firms is assumed to be identical, in which each incumbent requires α^2 units of final goods to produce one unit of intermediate good. Accordingly, firm i faces the following profit-maximization problem such that $\max_{x_{it}} \pi_{it} = p_{it} x_{it} - \alpha^2 x_{it}$. Then the solution yields the optimal price $p_{it} = \alpha$, and then the quantity of intermediate product i:

$$x_{it} = L_{yt} A_{it}^{\frac{1}{1-\alpha}}. \tag{10}$$

Substituting these results into π_{it} yields the equilibrium profit:

$$\pi_{it} = \alpha(1-\alpha)x_{it} = \alpha(1-\alpha)L_{yt}A_{it}^{\frac{1}{1-\alpha}}. \tag{11}$$

The industry leader i possesses this profit flow in each period until the arrival of the next innovation.

2.4 Entrants

Following Howitt (1999) and Segerstrom (2000), a new firm (an entrant) can enter the market by either engaging in a vertical or a horizontal innovation. If an entrant engages in a vertical innovation, then she targets an existing industrial product line and devotes resources to improve the quality of that product. The product with the improved quality allows the innovator to replace the incumbent of the original product and then become the industry leader until the next innovation in this industry occurs. By contrast, if an entrant engages in a horizontal innovation, by devoting resources to create an entirely new industry, she then becomes a new industry leader with an exclusive patent right to produce a differentiated good until the arrival of the next vertical innovation targeted at this industry.

2.4.1 Vertical R&D

Consider that the entrant j engages in vertical R&D by targeting an existing industry i to improve product quality at time t with a successful rate of innovation ϕ_{ijt} that follows Poisson process, which is given by

$$\phi_{ijt} = \frac{\lambda_v (L_{v,ijt})^{\delta}(K_{ijt})^{1-\delta}}{A_t}; \quad 0 < \delta < 1. \tag{12}$$

$\lambda_v > 0$ is a parameter indicating the productivity of vertical R&D, $L_{v,ijt}$ is the level of firm j's R&D employment. K_{ijt} is the stock of firm-specific knowledge possessed by firm j. δ measures the degree of diminishing returns to vertical R&D expenditures. A_t is the leading-edge productivity parameter at time t defined as $A_t \equiv \max\{A_{it}; i \in [0, N_t]\}$, and is also interpreted as the force of increasing research complexity. The evolution of A_t will be discussed in detail in a later subsection.

To capture the monetary effect of the CIA constraint on vertical R&D, we assume that a fraction ξ_v of vertical R&D spending is constrained by cash/money. This cash constraint forces the R&D firm to borrow an amount $\xi_v w_t L_{v,ijt}$ of money at the nominal interest rate i from the household for financing the R&D expenditure. Accordingly, the profit-maximization problem for each potential entrant is

$$\max_{L_{v,ijt}} \quad \phi_{ijt}\Pi_{vt} - w_t L_{v,ijt}(1+\xi_v i_t),$$

where $\Pi_{vt} \equiv \int_t^{\infty} e^{-\int_t^{\tau}(r_s+\phi_s)ds}\hat{\pi}_{t\tau}d\tau$ is the expected present value of the innovative firm's profit flows before the replacement of the next successful innovation and $\hat{\pi}_{t\tau}$ is the monopoly profit flow at time τ of a firm whose technology is of vintage t. Each innovation at time t produces a new generation of products in that industry, which embodies the leading-edge productivity parameter A_t. This results in a continuous flow of the same monopoly profit $\hat{\pi}_{t\tau}$ across industries after time t and is given by $\hat{\pi}_{t\tau} = \alpha(1-\alpha)L_{y\tau}A_t^{1/(1-\alpha)}$. Moreover, r_s is the instantaneous interest rate, and ϕ_s is the rate of creative destruction, namely, the instantaneous flow probability of being replaced by an innovation. Along with the same instantaneous discount rate $r_s + \phi_s$ applying the same amount of profit flow $\hat{\pi}_{t\tau}$ earned by each industry leader, it is easy to deduce that the expected reward for vertical innovation Π_{vt} does not vary across industries.

At time t, a potential entrant j that targets the vertical R&D at industry i solves the above profit-maximization problem, yielding the first-order condition such that

$$\frac{\lambda_v \delta \Pi_{vt}}{A_t} \left(\frac{L_{v,ijt}}{K_{ijt}} \right)^{\delta - 1} = w_t(1 + \xi_v i_t), \tag{13}$$

which reveals that the marginal expected benefit of an extra unit of vertical R&D equals its marginal cost.[4] It is clear from (13) that the marginal cost is positively correlated with the parameter ξ_v, capturing the adverse effect of the nominal interest rate i on the firm's R&D decision $L_{v,ijt}$ through increasing the marginal cost of vertical innovation.

In addition, we assume symmetry across R&D firms such that K_{ijt} is the same and infinitesimally small for all j.[5] Given this assumption, (13) implies that $L_{v,ijt}/K_{ijt} = L_{v,it}/K_{it}$ for all j, where $L_{v,it} = \int L_{v,ijt} dj$ and $K_{it} = \int K_{ijt} dj$. Furthermore, we assume $K_{it} \equiv \int K_{ijt} dj = L_t/N_t$ for all i, which is in line with Ha and Howitt (2007).[6] Thus, (13) can be re-expressed as

$$\frac{\lambda_v \delta \Pi_{vt}}{A_t} (l_{vt})^{\delta - 1} = w_t(1 + \xi_v i_t), \tag{14}$$

where $l_{vt} \equiv L_{vt}/L_t$ ($L_{vt} \equiv \int L_{v,it} di$) is the fraction of total labor employment that is allocated to vertical R&D. We further assume that the returns on conducting vertical R&D are identical across firm j and across time. This assumption together with the fact that $l_{vt} \equiv L_{vt}/L_t$ and $K_{it} = L_t/N_t$ indicates that the Poisson arrival rate of vertical innovations in each industry becomes

$$\phi_t = \int \phi_{ijt} dj = \frac{\lambda_v (L_{vt}/N_t)^{\delta} (L_t/N_t)^{1-\delta}}{A_t} = \lambda_v l_{vt}^{\delta} \iota_t, \tag{15}$$

where $\iota_t \equiv L_t/(A_t N_t)$. The expression (15) shows that the arrival rate of vertical innovations is increasing in per industry vertical R&D expenditure L_{vt}/N_t and the knowledge spillover L_t/N_t but decreasing in the R&D difficulty term A_t.

2.4.2 Horizontal R&D

An entrant q that engages in horizontal innovations devotes resources to create a new variety (and thus an entirely new industry). She faces the following rate of discovering new innovations, denoted as $\dot{N}_{qt}$:

$$\dot{N}_{qt} = \frac{\lambda_h (L_{hqt})^{\gamma} (\mathcal{K}_{qt})^{1-\gamma}}{A_t}; \quad 0 < \gamma < 1. \tag{16}$$

[4](13) implies an amount of R&D profit $d_{v,ijt} = (1 - \delta)\phi_{ijt}\Pi_{vt}$ for the vertical innovation entrant j who targets industry i. The presence of a positive profit in the R&D sector can be justified by the requirement of a fixed entry cost, which can be the entrepreneurial talent of R&D entrepreneurs in the specific industry. Given that not everyone possesses this entrepreneurial talent, there is no free entry so that the entrepreneurs keep the monopolistic rent.

[5]An infinitesimally small K_{ijt} implies that the optimal amount of firms' R&D resources $L_{v,ijt}$ is also infinitesimally small, governed by (13). Hence, the likelihood of any one firm winning a vertical R&D race can be negligible.

[6]We follow Ha and Howitt (2007) to capture the insight that the total amount of firm-specific knowledge in each industry equals per industry labor, which grows over time in equilibrium.

$\lambda_h > 0$ is a parameter indicating the productivity of horizontal R&D, L_{hqt} is the level of firm q's R&D employment, $\mathcal{K}_{qt}$ is the firm-specific knowledge possessed by firm q that benefits horizontal innovations, and the exponent γ measures the degree of diminishing returns to horizontal R&D expenditures. A_t reflects the fact for increasing research complexity.

Moreover, we assume that each horizontal innovation at time t results in a new intermediate variety whose productivity parameter is drawn randomly from an invariant long-run distribution of the existing productivity parameters A_{it} across industries i. This assumption makes sure that the process of variety-expanding will not affect the convergence of the distribution of existing parameters A_{it} to an invariant distribution in the long run. See the detailed discussion in the next subsection.

Next, to capture the monetary effect of the CIA constraint on horizontal R&D, we assume that a fraction ξ_h of horizontal R&D expenditure is constrained by cash/money. This cash constraint forces the innovative firm to borrow an amount $\xi_h w_t L_{hqt}$ of money at the nominal interest rate i from the household for financing the R&D expenditure. In addition, throughout the rest of this study, the assumption that $\xi_h > \xi_v$ is imposed to capture the empirical evidence that the investment on radical innovations is more cash-constrained than that on incremental innovations (e.g., Akcigit (2009) and Caggese (2015)). Accordingly, the profit-maximization problem for horizontal R&D firm q is

$$\max_{L_{hqt}} \pi_{hqt} = \dot{N}_{qt}\Pi_{ht} - w_t L_{hqt}(1 + \xi_h i_t),$$

and

$$\Pi_{ht} = \Gamma^{-1}\Pi_{vt}, \tag{17}$$

where $\Gamma \equiv 1 + [\sigma/(1-\alpha)]$ and Π_{ht} is the expected value of a successful horizontal innovation. (17) reveals the relationship between Π_{ht} and Π_{vt} from the aforementioned assumption regarding the randomly drawn productivity. The derivation of (17) will be provided in the next subsection.

Then, the first-order condition for the horizontal R&D firms' profit maximization is given by

$$\frac{\lambda_h \gamma \Pi_{ht}}{A_t}\left(\frac{L_{hqt}}{\mathcal{K}_{qt}}\right)^{\gamma-1} = w_t(1 + \xi_h i_t). \tag{18}$$

This equation clearly shows that the marginal cost is positively related to the CIA parameter ξ_h, capturing the negative effect of the nominal interest rate i_t on the firm's R&D decision L_{hqt} through increasing the marginal cost of horizontal innovation $w_t(1 + \xi_h i_t)$.[7]

Furthermore, (18) states that Π_{ht} only scales Π_{vt} with a constant factor, implying that Π_{ht} is also identical across industries for all entrants q. Together with the same marginal cost faced by each entrant, the above first-order condition implies that $L_{hqt}/\mathcal{K}_{qt} = L_{ht}/\mathcal{K}_t$ for all q, where $L_{ht} \equiv \int L_{hqt} dq$ and $\mathcal{K}_t \equiv \int \mathcal{K}_{qt} dq$. As in the previous subsection, a similar assumption is made such that $\mathcal{K}_{qt} = L_t/N_t$ for all q. Substituting $\mathcal{K}_{qt} = L_t/N_t$ and $L_{hqt}/\mathcal{K}_{qt} = L_{ht}/\mathcal{K}_t$ into (18) yields:

$$\frac{\lambda_h \gamma \Pi_{ht}}{A_t}(l_{ht})^{\gamma-1} = w_t(1 + \xi_h i_t), \tag{19}$$

where $l_{ht} \equiv L_{ht}/L_t$ is the fraction of labor allocated to horizontal R&D. The growth rate of the measure of industries is the summation of the discovery rates across all individual firms that engage

[7] Again, (18) implies a positive amount of R&D profit captured by the horizontal innovation entrant q such that $d_{hqt} = (1-\gamma)\dot{N}_t \Pi_{ht}$.

in horizontal R&D, that is

$$g_{Nt} \equiv \frac{\dot{N}_t}{N_t} = \int \frac{\dot{N}_{qt}}{N_t} dq = \frac{\lambda_h (L_{ht}/N_t)^{\gamma} (L_t/N_t)^{1-\gamma}}{A_t} = \lambda_h l_{ht}^{\gamma} \iota_t. \tag{20}$$

2.4.3 Spillovers

As in Howitt (1999) and Segerstrom (2000), the leading-edge productivity parameter A_t grows over time as a result of knowledge spillovers produced by vertical innovations. The growth rate of A_t is proposed to take the following standard form

$$g_{At} \equiv \frac{\dot{A}_t}{A_t} = \left(\frac{\sigma}{N_t}\right)(\phi_t N_t) = \sigma\phi_t = \sigma\lambda_v l_{vt}^{\delta}\iota_t, \tag{21}$$

where $\sigma > 0$ measures the R&D spillover effect and $\phi_t = \int \phi_{ijt} dj$ is the Poisson arrival rate of vertical innovations in each industry $i \in [0, N_t]$.

As shown in (21), g_{A_t} can essentially be decomposed as a product of two factors σ/N_t and $\phi_t N_t$, where $\phi_t N_t$ is the aggregate flow of vertical innovations in this economy. (21) states that the growth of knowledge spillovers is assumed to be proportional to the aggregate flow of vertical innovations $\phi_t N_t$. The factor of proportionality σ/N_t measures the marginal effect of each vertical innovation on the stock of public knowledge. The divisor N_t captures the fact that each vertical innovation has a smaller impact on the aggregate economy as the number of specialized products expands with the development of the economy.

Since the distribution of productivity parameter among new products at any time is identical to the distribution across existing products at that time, one can show that the distribution of the relative productivity parameter, which is defined as $z_{it} \equiv A_{it}/A_t$, converges monotonically to the invariant distribution $Pr\{z_{it} \leq z\} \equiv F(z) = z^{1/\sigma}$, wherein $0 < z \leq 1$. It follows that in the long run:[8]

$$E\left[(A_{it}/A_t)^{1/(1-\alpha)}\right] = \Gamma^{-1}. \tag{22}$$

Recall that the productivity parameter of each new innovative variety is drawn randomly from the above distribution. This implies that the realized monopoly profit flow for each horizontal R&D firm at date τ and its realized present value at time t are $\pi_{i\tau} = \alpha(1-\alpha)L_{y\tau}A_{it}^{1/(1-\alpha)}$ and $\Pi_{ht} = \int_t^{\infty} e^{-\int_t^{\tau}(r_s+\phi_s)ds}\pi_{i\tau}d\tau$, respectively. Along with the fact that a successful vertical innovation gains the profit flow $\hat{\pi}_{t\tau} = \alpha(1-\alpha)L_{y\tau}A_t^{1/(1-\alpha)}$ with the leading-edge productivity parameter A_t, it is easy to deduce $\Pi_{ht} = (A_{it}/A_t)^{1/(1-\alpha)}\Pi_{vt}$. Taking expectations on both sides of this equation yields (17).

2.5 Monetary Authority

The monetary authority implements its monetary policy by targeting a long-run nominal interest rate i_t. Denote the nominal money supply by M_t. Thus, the growth rate of nominal money supply is $\dot{M}_t/M_t = \mu_t$. Recall that m_t is real money balances per capita and is given by $m_t = M_t/(L_t P_t)$, where P_t denotes the nominal price of final goods (i.e., the deflator). The growth rate of real money

[8] See Howitt (1999) and Segerstrom (2000) for the detailed proof.

balances per capita is $g_{mt} \equiv \dot{m}_t/m_t = \mu_t - \pi_t - g_L$, where $\pi_t \equiv \dot{P}_t/P_t$ is the inflation rate of the price of final goods. Substituting this expression and the Euler equation (4) into the Fisher equation (6), along with the fact that $g_{mt} = g_{ct}$ in the steady state,[9] we obtain

$$i_t = r_t + \pi_t = (\rho + g_{ct} + g_L) + (\mu_t - g_{mt} - g_L) = \rho + \mu_t. \tag{23}$$

This equation illustrates a one-by-one monotonic relationship between the nominal interest rate i_t and the growth rate of nominal money supply μ_t, which indicates an isomorphic choice of monetary instruments between i_t and μ_t. Specifically, an exogenous increase in i_t corresponds to an endogenous increase in μ_t.

Upon increasing the nominal interest rate i_t, the government earns the seigniorage revenue through an inflation tax. To balance the budget, it is assumed that the government returns the revenue as a lump-sum transfer to the household. Therefore, the government's budget constraint (in terms of per capita level) is given by $\dot{M}_t/(L_t P_t) = \dot{m}_t + (\pi_t + g_L)m_t = \zeta_t$.

2.6 Characterization of Equilibrium

The equilibrium in this economy consists of a time path of prices $\{w_t, r_t, i_t, p_{it}, P_t\}_{t=0}^{\infty}$, and a time path of allocations $\{c_t, m_t, l_t, Y_{kt}, Y_t, x_{it}, x_t, L_{ykt}, L_{v,ijt}, L_{hqt}\}_{t=0}^{\infty}$, where $Y_t = \int Y_{kt} dk$ and $x_t = \int_0^{N_t} x_{it} di$. Moreover, at each instant of time,

- individuals maximize utility taking $\{i_t, r_t, w_t\}$ as given;
- the competitive final-goods firms produce $\{y_{kt}\}$ to maximize profits taking $\{P_t\}$ as given;
- the monopolistic intermediate-goods firms produce $\{x_{it}\}$ and choose $\{Y_t, p_{it}\}$ to maximize profits;
- the labor market clears such that $L_{yt} + L_{vt} + L_{ht} = l_t L_t$;
- the final-goods market clears such that $Y_t = C_t + x_t$;
- the asset market clears such that the value of monopolistic firms adds up to the value of the household's assets: $N_t \Pi_{ht} = a_t L_t$; and
- the amount of money borrowed by two types of innovation entrants is $b_t L_t = \xi_v w_t L_{vt} + \xi_h w_t L_{ht}$.

Using (22), we obtain $\int_0^{N_t} A_{it}^{\frac{1}{1-\alpha}} di = A_t^{\frac{1}{1-\alpha}} N_t \int_0^1 z^{\frac{1}{1-\alpha}} F'(z) dz = A_t^{\frac{1}{1-\alpha}} N_t \Gamma^{-1}$. Substituting this equation, $Y_t = \int Y_{kt} dk$, and (10) into (7) yields the equilibrium final-goods production function

$$Y_t = \frac{L_{yt} A_t^{\frac{1}{1-\alpha}} N_t}{\Gamma}. \tag{24}$$

Accordingly, the per-capita consumption and the production-labor shares of outputs are, respectively,

$$c_t = \frac{(1-\alpha^2) l_{yt} A_t^{\frac{1}{1-\alpha}} N_t}{\Gamma}, \tag{25}$$

and

$$w_t = (1-\alpha)\frac{Y_t}{L_{yt}} = \frac{(1-\alpha) A_t^{\frac{1}{1-\alpha}} N_t}{\Gamma}. \tag{26}$$

[9] On the balanced growth path, it can be shown that m_t and c_t grow at the same rate.

In Online Appendix C, we show that the equilibrium dynamics of this model are evaluated by a 2×2 dynamic system with two state variables. Due to its complexity, the dynamic property of our model is explored numerically. It is shown that local determinacy is characterized by a saddle-path stability.

2.7 Balanced-Growth Properties

In this section, we focus on the analysis of the balanced-growth equilibrium properties of the model. In the balanced-growth equilibrium, the fraction of labor supplied to each sector must be constant over time (i.e., $l_{vt} = l_v, l_{ht} = l_h, l_{yt} = l_y$ for all t). Since both g_{At} and g_{Nt} must be constant in a balanced-growth equilibrium, (12) implies that the arrival rate of vertical innovations must be constant as well (i.e., $\phi_t = \phi$ for all t). Furthermore, according to (20) and (21), ι_t must be constant in the balanced-growth equilibrium (i.e., $\iota_t = \iota$ for all t). Thus, the quality and variety growth rates can, respectively, be written as

$$g_A = \sigma \lambda_v l_v^{\delta} \iota, \tag{27}$$

and

$$g_N = \lambda_h l_h^{\gamma} \iota. \tag{28}$$

2.7.1 Economic Growth

Denote by g the growth rate of consumption per capita c_t on the balanced-growth path (and economic growth rate hereafter). Differentiating the consumption per-capita (25) with respect to time yields

$$g = g_N + \frac{1}{1-\alpha} g_A. \tag{29}$$

This equation, called the *iso-growth* condition, demonstrates that on the balanced-growth path (BGP), the growth rate of the measure of industries g_N and the growth rate of the productivity of industries g_A jointly determine the overall rate of economic growth g.

2.7.2 Population-Growth Condition

Differentiating $\iota_t = L_t/(A_t N_t) = \iota$ with respect to time t yields the population-growth condition such that

$$g_L = g_A + g_N. \tag{30}$$

This equation states that to guarantee a BGP, the growth rate of the leading-edge productivity parameter A_t and that of the measure of variety N_t are required to grow in such a way that these growth rates are constrained by the population-growth rate g_L. The intuition behind this constraint is as follows. As the economy grows with higher levels of A_t and N_t, research becomes more complex, and thus the productivity of researchers ι_t falls in response. To maintain a constant innovation rate in g_N and g_A over time as stipulated in (20) and (21), more labor is needed to engage in R&D activities. The population-growth rate g_L determines the rates at which labor resources can be channeled into both horizontal and vertical R&D activities and therefore determines the overall growth rate of the economy.

In addition, examining both equations (29) and (30) yields the following result:

Lemma 1. *In the steady-state equilibrium, the economic growth rate is increasing in the vertical R&D growth rate.*

The intuition underlining this lemma is straightforward. The population-growth condition (30) implies that there is an equal tradeoff between g_A and g_N; an increase in g_A comes at the cost of an identical amount of reduction in g_N to maintain a constant population-growth rate. However, the iso-growth condition (29) reveals that the economic growth rate stems from a larger contribution of g_A than g_N (i.e., $1/(1-\alpha) > 1$). Therefore, an increase in g_A at the sacrifice of g_N comes with a higher economic growth rate. This theoretical attribute is also available in Howitt (1999) and Peretto and Connolly (2007), in which the economic growth rate is eventually supported by the growth from creative destruction (vertical innovation) rather than variety expansion (horizontal innovation) in the steady-state equilibrium.[10]

3 Growth Effects of Monetary Policy

In this section, we analyze the growth effects of monetary policy (in terms of nominal interest rate targeting) for various CIA constraints. To fully comprehend the underlying mechanism, our analysis first proceeds with different scenarios, each of which is subject to one distinct type of CIA constraint. After picking up the intuition behind each scenario, we impose all types of CIA constraints simultaneously and then provide a complete analysis.[11]

3.1 CIA on Consumption

First, we analyze the case in which only a CIA constraint on consumption is present, and the following proposition is obtained:

Proposition 1. *In the presence of a CIA constraint on consumption only (i.e., $\xi_c > 0$, $\xi_v = \xi_h = 0$), a higher nominal interest rate decreases (increases) the economic growth rate if $\gamma < \delta$ ($\gamma > \delta$).*

Fig.1a illustrates the effects of a permanent increase in the nominal interest rate i on the economic growth rate with only a CIA constraint on consumption. Using both the iso-growth condition in (29) and the population-growth condition in (30), we can derive two downward sloping lines with a slope of $-1/(1-\alpha)$ and of -1, respectively, in the (g_A, g_N) space. Thus, the slope of each iso-growth line exceeds the slope of the population-growth condition (in absolute value terms).

To better understand the intuition underlying Proposition 1, first, we analyze the instantaneous effect of raising i starting from the initial balanced-growth equilibrium. When only consumption is subject to the CIA constraint, increasing the nominal interest rate i raises the cost of consumption purchases relative to leisure. As a result, individuals enjoy more leisure by reducing labor supply, and hence the equilibrium labor for both R&D activities l_h and l_v. More importantly, l_h decreases by a smaller (larger) amount than l_v does if horizontal R&D exhibits greater (smaller) diminishing returns than vertical R&D (i.e., $\gamma < (>) \delta$). In Fig.1a, to reflect the case of $\gamma < \delta$, a higher i leads the economy to move from the initial steady state A to B' with a smaller reduction in g_N than in

[10] This implication is consistent with the empirical finding in Garcia-Macia *et al.* (2016), who show that in the US, within the periods 1976-1986 and 2003-2013, the contribution of the aggregate total factor productivity growth from creative destruction is overwhelmingly larger than that from new varieties.

[11] The detailed technical proofs of propositions in this section are available in Online Appendix A.

g_A. By contrast, to reflect the case where $\gamma > \delta$, a higher i shifts the economy from A to C', with a larger reduction in g_N than in g_A.

Next, we follow Segerstrom (2000) to provide an intuitive explanation about how the economy adjusts after its instant shift off the balanced-growth path. The corresponding decreases in g_A and g_N indicate that the research complexity grows at a slower rate than usual. It follows that the research productivity ι_t rises gradually over time, which drives up g_A and g_N again as indicated in (27) and (28), until they are back to a balanced-growth equilibrium, namely, the population-growth condition is satisfied again. Therefore, there are two cases to be considered.

When $\gamma < \delta$, raising i initially drives the economy to move from A to B' (i.e., a larger decrease in g_A than in g_N). Then the research productivity ι_t rises over time, driving up g_A and g_N gradually by a similar magnitude, which induces the economy to move from point B' to the new balanced-growth path B. It is clear that the long-run effect of raising the nominal interest rate raises the horizontal innovation rate g_N at the expense of reducing g_A. Then the economic growth rate will decrease in response as shown in Lemma 1.

When $\gamma > \delta$, raising i initially drives the economy to move from A to C' (i.e., a smaller decrease in g_A than in g_N). This force subsequently induces the economy to move from point C' to the new balanced-growth path C. In this case, the long-run effect of raising the nominal interest rate raises the vertical innovation rate g_A at the expense of reducing g_N. As a result, the economic growth rate will increase in response as implied by Lemma 1.

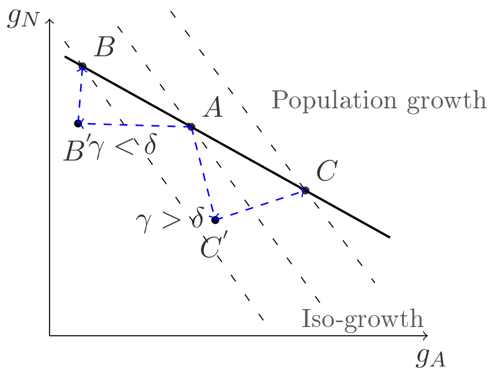
3.2 CIA on Vertical R&D

In this subsection, we analyze the case in which only a CIA constraint on vertical R&D is present, and the following result is obtained:

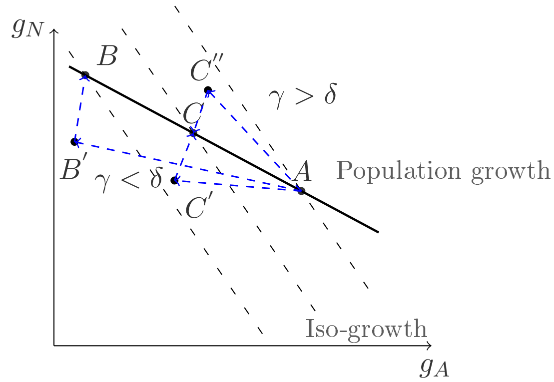
Proposition 2. *In the presence of a CIA constraint on vertical R&D only (i.e., $\xi_v > 0$, $\xi_c = \xi_h = 0$), a higher nominal interest rate decreases the economic growth rate under both $\gamma < \delta$ and $\gamma > \delta$, but by larger amount under $\gamma < \delta$.*

Fig.1b illustrates the effects of a permanent increase in the nominal interest rate i on growth when the model only features a CIA constraint on vertical R&D. Similar to Subsection 3.1, the analysis starts off by exploring the instantaneous effect of raising i from the initial balanced-growth equilibrium.

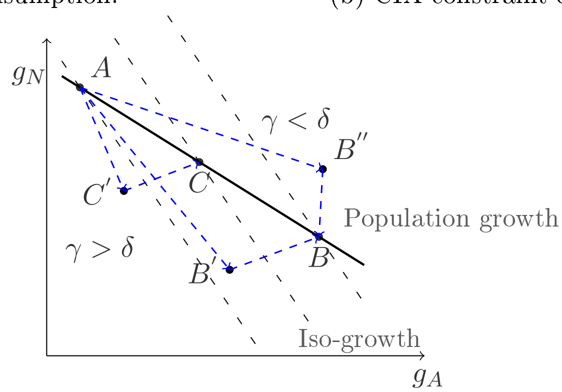
When only vertical R&D is subject to the CIA constraint, an increase in i raises the cost for vertical R&D. The labor force will be reallocated from vertical R&D l_v to production l_y, horizontal R&D l_h, and leisure. Under $\gamma < \delta$, greater diminishing returns to horizontal R&D will reallocate less labor to l_h, allowing only for a smaller increase in g_N; in addition, a high level of δ causes a decrease in l_v to transmit a larger reduction in g_A as shown in (27). In Fig.1b, the economy, therefore, moves from A to B' in this case. By contrast, under $\gamma > \delta$, smaller diminishing returns to horizontal R&D will reallocate more labor to l_h, leading to a higher g_N; in addition, a low level of δ also causes a decrease in l_v to transmit a smaller reduction in g_A. In Fig.1b, the economy will move from A to C' if the gap between γ and δ is small (i.e., γ is slightly larger than δ), and thus the magnitudes of the changes in g_N and g_A are close. Otherwise, the economy will move from A to C'' if the gap between γ and δ is large (i.e., γ is much larger than δ), and thus the size of the increase in g_N is much more significant than that of the decrease in g_A.



(a) CIA constraint on consumption.



(b) CIA constraint on vertical R&D.



(c) CIA constraint on horizontal R&D.

Fig. 1. Adjustment process to new equilibria.

Next, we turn to intuitively explain the adjustment process. There are three scenarios to be considered. First, when $\gamma < \delta$, since the magnitude of the decrease in g_A is much larger than that of the increase in g_N as shown in the movement from point A to point $B^{'}$, the growth of research complexity is driven down to a lower rate than usual. It follows immediately that the research productivity ι_t rises over time. Hence, g_A and g_N grow gradually in a similar manner, inducing the economy to move from point $B^{'}$ to the new balanced-growth path B. Second, when $\gamma > \delta$ and the gap between γ and δ is small, the close magnitudes of the changes in g_A and g_N may still drive down the growth of research complexity to a lower rate than usual. It then follows that ι_t rises over time. Hence, g_A and g_N grow gradually in a similar fashion, inducing the economy to move from point $C^{'}$ to the new balanced-growth path C. Third, when $\gamma > \delta$ and the gap between γ and δ is large, the magnitude of the increase in g_N is greater than that of the decrease in g_A. In this case, the growth of research complexity is driven up to a higher rate than usual. It follows that the research productivity ι_t will fall over time. Hence, g_A and g_N are lowered gradually in a similar manner, inducing the economy to move from point $C^{''}$ to the new balanced-growth path C.

To sum up, the long-run growth effect of raising i is to increase g_N at the expense of reducing g_A regardless of the comparison between γ and δ. Nevertheless, the reduction in g_A turns out to be more significant under $\gamma < \delta$ than under $\gamma > \delta$. Consequently, according to Lemma 1, the economic growth rate is decreasing in i more considerably under $\gamma < \delta$ than under $\gamma > \delta$.

3.3 CIA on Horizontal R&D

In this subsection, we analyze the case in which only a CIA constraint on horizontal R&D is present, and the following result is obtained:

Proposition 3. *In the presence of a CIA constraint on horizontal R&D only (i.e., $\xi_h > 0$, $\xi_v = \xi_c = 0$), a higher nominal interest rate increases the economic growth rate under both $\gamma < \delta$ and $\gamma > \delta$, but by a larger amount under $\gamma < \delta$.*

Fig.1c illustrates the growth effects of a permanent increase in the nominal interest rate i when the model only features a CIA constraint on horizontal R&D. Again, the analysis starts off by studying the instantaneous effect of raising i from the initial balanced-growth equilibrium.

When horizontal R&D is subject to the CIA constraint, the instantaneous effects of raising i are just the opposite of those in Subsection 3.2. An increase in i raises the cost for horizontal R&D, reallocating labors from horizontal R&D l_h to production l_y, vertical R&D l_v, and leisure. On the one hand, when $\gamma < \delta$, namely the diminishing returns to vertical R&D are small, more labor will be reallocated to l_v leading to a larger rise in g_A. In Fig.1c, if the gap between γ and δ is small, then the economy will move from point A to B', since the magnitude of the increase in g_A is not significant compared to the magnitude of the decrease in g_N, as shown in (27) and (28). By contrast, if the gap between γ and δ is large, the economy will move from point A to B'', since the magnitude of the increase in g_A becomes larger than the magnitude of the decrease in g_N. On the other hand, when $\gamma > \delta$, namely the diminishing returns to vertical R&D are large, less labor will be reallocated to l_v leading to a smaller rise in g_A. Therefore, the economy will move from point A to C', given that the size of the decrease in g_N is more significant than that of the increase in g_A.

Now, we turn to intuitively explain the adjustment process. Under $\gamma < \delta$, if the gap between γ and δ is small, the increase in g_A is not significant enough to dominate the decrease in g_N. As a result, the research productivity ι_t grows over time, driving up both g_A and g_N, and therefore the economy moves from point B' to the new balanced-growth path B, as displayed in Fig.1c. However, if the gap between γ and δ is large, the increase in g_A is, instead, more likely to dominate the decrease in g_N, which drives up the growth of research complexity to a higher rate than usual. As a result, the research productivity falls over time and g_A and g_N are reduced, so the economy moves from point B'' to B.

Under $\gamma > \delta$, since the magnitude of the decrease in g_N is much larger than that of the increase in g_A as shown in the movement from point A to point C', the growth of research complexity is driven down to a lower rate than usual. It follows immediately that the research productivity ι_t rises over time. Hence, g_A and g_N grow gradually in a similar manner, inducing the economy to move from point C' to the new balanced-growth path C.

To sum up, the long-run growth effect of raising i is to increase g_A at the expense of reducing g_N regardless of the comparison between γ and δ. Nevertheless, the increase in g_A turns out to be more significant under $\gamma < \delta$ than under $\gamma > \delta$. Consequently, according to Lemma 1, the economic growth rate is increasing in i more considerably under $\gamma < \delta$ than under $\gamma > \delta$.

3.4 CIA on Consumption and R&D

Given the results in the above subsections, we can analyze a more general case by incorporating all types of CIA constraints into the model. To highlight the interesting non-monotonic relationship between inflation and growth, our analysis focuses on the empirically relevant case where $\gamma < \delta$.[12] Accordingly, we obtain the following result.

Proposition 4. *Suppose that $\gamma < \delta$ holds. Then (a) for a sufficiently large gap between ξ_h and ξ_v, the economic growth rate g has a non-monotonic (i.e., an inverted-U) relationship with the nominal interest rate i, and there exists a threshold value i^* below (above) which g is increasing (decreasing) in i; (b) For an insufficiently large gap between ξ_h and ξ_v, g is monotonically decreasing in i.*

To intuitively explain the results of Proposition 4, we need to combine the results obtained in Propositions 1-3. Recall that from Propositions 1 and 2, when $\gamma < \delta$, the CIA constraints on both consumption and vertical R&D yield a negative relationship between the nominal interest rate i and the economic growth rate g, and only Proposition 3 (i.e., the CIA constraint on horizontal R&D) can generate a positive relationship between them. It is obvious that an inverted-U shape requires a positive relationship to emerge at the relatively low levels of i. This implies that the growth effect of i from the CIA constraint on horizontal R&D has to be relatively strong to dominate the other two effects from the CIA constraints on consumption and vertical R&D. At the initial increase in i, the distortions of the CIA constraints are mild, which implies that the above three effects are all weak. Therefore, to ensure a strong positive effect of the constraint on horizontal R&D at the initial increase in i, there must be a sufficiently large extent of the constraint on horizontal R&D relative to vertical R&D (i.e., a sufficiently large gap in $\xi_h > \xi_v$), so that raising i yields a strong reallocation effect from l_h to l_v to generate a high level of g_A to enhance g.

Nevertheless, as i continues to rise, the greater diminishing returns to horizontal R&D relative to vertical R&D (i.e., $\gamma < \delta$) come into play and exert a counter-force that weakens the reallocation effect from l_h to l_v. This counter-force increases non-linearly in i which, henceforth, makes the negative growth effects from the constraints on vertical R&D and consumption stronger than the positive effect from the constraint on horizontal R&D. This implies that there will be a threshold rate of nominal interest i^* across which the two negative growth effects play the dominant role, so that g becomes monotonically decreasing in i.

Finally, if the gap in $\xi_h > \xi_v$ is not sufficiently large, the reallocation effect of i from the constraint on horizontal R&D is weak at the initial increase in i, so it will be dominated by the two negative effects as i rises. Hence, g becomes monotonically decreasing in i for all levels of i.

4 Quantitative Analysis

We first calibrate our model to the US economy in Section 4.1 and then quantitatively analyze the growth and welfare effects of monetary policy in Section 4.2. Section 4.3 provides robustness checks.

4.1 Calibration

To quantitatively analyze the model, the strategy is to assign steady-state values to the following structural parameters $\{\rho, \alpha, \gamma, \delta, \lambda_v, \lambda_h, \xi_c, \xi_v, \xi_h, \theta, \sigma\}$. We follow Acemoglu and Akcigit (2012) to set the discount rate ρ to 0.05. We set the strength of the CIA constraint on consumption ξ_c to

[12] In the case where $\gamma > \delta$, the economic growth rate is monotonically increasing in the inflation rate when all CIA constraints are present. The detailed analysis for this case is available upon request.

0.17 to match the ratio of M1-consumption in the US. According to (29) and (30), we set $\alpha = 0.2$ to ensure $g_A > g_N$ to correspond to the empirical estimate in Garcia-Macia *et al.* (2016), given the conventional population growth rate and economic growth rate that will be chosen below. As for the values of δ and γ, we set $\delta = 0.34$ and $\gamma = 0.12$ to capture the empirical estimates from Audretsch *et al.* (2006) and Acemoglu *et al.* (2014) such that horizontal (radical) innovation suffers greater diminishing returns than vertical (incremental) innovation. Both γ and δ lie in the range of the estimated elasticity of innovation outputs with respect to R&D expenditures documented in Blundell *et al.* (2002), who find that the elasticity of patents with respect to R&D is within the range of $[0.08, 0.5]$.[13]

To calibrate the remaining parameters $\{\lambda_v, \lambda_h, \xi_v, \xi_h, \theta, \sigma\}$, we use the following five empirical moments:[14] (i) the equilibrium economic growth rate; (ii) the Poisson arrival rate of vertical innovations; (iii) the R&D intensity; (iv) the standard time of employment; (v) the population growth rate. For (i), by following Jones and Williams (2000), we consider a value of $g = 1.2\%$. For (ii), we follow Lanjouw (1998) and Laitner and Stolyarov (2013) to consider a creative destruction rate of $\phi = 3.8\%$.[15] For (iii), we use a value of $R\&D/GDP = 2.6\%$, which according to OECD data is the ratio of gross domestic spending on R&D to GDP in the US during 1990-2016. For (iv), the standard time of employment is set to a general value of $l = 1/3$. For (v), the population growth rate is $g_L = 1\%$ according to the Conference Board Total Economy Database during 1990-2016. Furthermore, the market-level nominal interest rate i is calibrated by targeting at $\pi = 2.5\%$, which is the US average annual inflation rate within this period according to the Bureau of Labor Statistics. Hence, the benchmark value of the nominal interest rate is given by $i = r + \pi = g + \rho + g_L + \pi = 0.097$. Table 1 summarizes the values of the parameters and variables in this quantitative exercise.[16]

Table 1: Parameter values in baseline calibration

Targets				g 1.2%	π 2.5%	ϕ 3.8%	l 1/3	R&D/GDP 2.6%
Parameters	ξ_c 0.17	ξ_v 0.1355	ξ_h 0.6165	δ 0.34	γ 0.12	θ 1.7286	σ 0.2105	λ_h 0.0320

4.2 Growth and Welfare Implications of Monetary Policy

Fig.2a displays the quantitative results under the benchmark parameter values, where we find that the rate of economic growth is an inverted-U function of the inflation rate. This result supports the implication of Proposition 4 such that given a sufficiently large difference between ξ_v and ξ_h, at low levels of inflation the positive growth effect of the inflation rate through the CIA constraint

[13]We consider an alternative pair of γ and δ in the sensitivity analysis. We also consider the case where $\gamma > \delta$, but to save space the results are available upon request.

[14]As for the two productivity parameters λ_v and λ_h, since in our model it is the relative productivity of vertical and horizontal innovation functions, instead of their individual values, that matters in the growth and welfare effects of inflation, without loss of generality, we normalize one of them, i.e., λ_v, to one. The remaining λ_h is chosen by ensuring that both ξ_v and ξ_h are within the range of $[0, 1]$ and their values are consistent with the empirical moments.

[15]Lanjouw (1998) estimates the probability of obsolescence to be in the range of 7%-12%, and Laitner and Stolyarov (2013) find a mean rate of creative destruction of 3.5%. Here, we consider an intermediate value of 3.8%.

[16]Detailed calibration procedures are relegated to Online Appendix B.

on horizontal R&D strictly dominates the negative growth effects through the CIA constraints on consumption and vertical R&D. Nevertheless, as the inflation rate rises, this domination becomes increasingly weaker and finally the negative effects overwhelm the positive one. In addition, the threshold value of the inflation rate is roughly 2.4% (i.e., $i = 9.6\%$), which is in line with the empirical estimates of Ghosh and Phillips (1998) (i.e., 2.5%) and Kremer *et al.* (2013) (i.e., 2%).

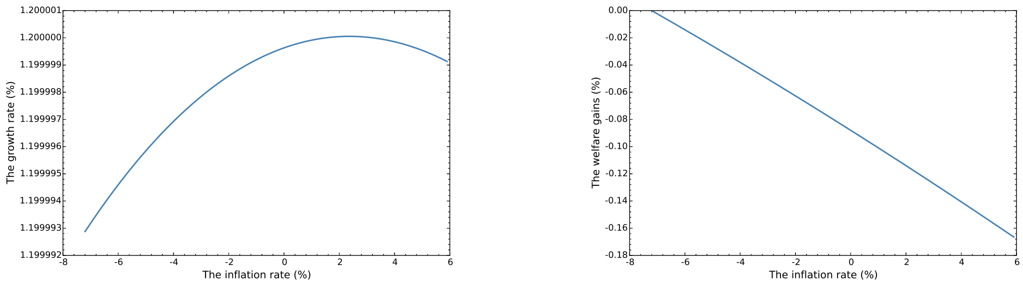


Fig. 2. (a) Inflation and economic growth; (b) Inflation and social welfare.

To explore the welfare effects of monetary policy, we impose a balanced growth condition on (1) to derive the steady-state welfare function such that

$$U = \frac{1}{\rho}\left[\ln c_0 + \frac{g}{\rho} + \theta \ln(1-l)\right] \tag{31}$$

where the exogenous terms A_0 and N_0 have been dropped and $c_0 = (1-\alpha^2)l_y/\Gamma$ is the steady-state level of consumption along the BGP. Fig.2b depicts the welfare effect of inflation. By expressing the welfare gain (loss) at the usual equivalent variations in consumption flow, we find that social welfare is monotonically decreasing in the inflation rate. The intuition can be explained as follows. There are two positive welfare effects of a higher rate of inflation (or raising the nominal interest rate). The first effect stems from the growth effect of an inflation rate that is below the threshold value, as previously discussed. The second effect comes from the increase in leisure, which leads to a higher utility level. However, given our benchmark parameter values, these two positive welfare effects are completely dominated by the negative welfare effect from the decrease in the households' initial consumption level. This occurs due to the CIA constraint on consumption, which reduces labor employment in the final-goods sector and hence the level of c_0. In addition, as the inflation rate increases to a permanently higher rate that is above the threshold, the positive growth effect becomes negative, leading the overall welfare effect of inflation to always be negative. Therefore, this model predicts a monotonically decreasing relationship between welfare and inflation in the benchmark case; in other words, the Friedman rule is optimal.

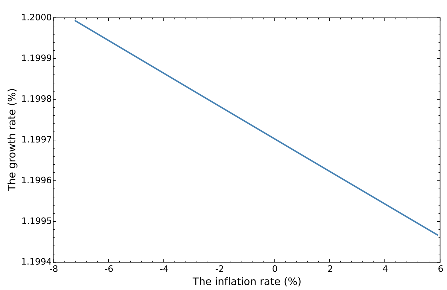
4.3 Sensitivity

In this subsection, we undertake sensitivity checks to test the robustness of our numerical results in terms of quantitative magnitudes. Specifically, we first examine the case of an insufficiently large gap between CIA constraints on the vertical and horizontal R&D activities, ξ_v and ξ_h, to correspond to the theoretical implications of Proposition 4. In doing so, we find that although the growth effect of inflation is similar regardless of the specific difference between ξ_v and ξ_h, the pattern of the welfare effect of inflation relies on its magnitude. Thus, we select two pairs of ξ_v and ξ_h to examine, respectively, how they affect the growth and welfare effects of inflation. We also consider the role of the CIA constraint on consumption.[17] Thereafter, a sensitivity exercise on an alternative couple of γ and δ is conducted. The parameter values that will be altered are summarized in Table 2.

Table 2: Sensitivity analysis

Parameters	ξ_c	ξ_v	ξ_h	δ	γ	θ	σ	λ_h
	0.17	0.4	0.6165	0.34	0.12	1.7286	0.2165	0.0320
	0.17	0.6	0.6165	0.34	0.12	1.7286	0.2165	0.0320
	0.158	0.1355	0.6165	0.34	0.12	1.7286	0.2165	0.0320
	0.17	0.1355	0.6165	0.56	0.12	1.7286	0.2165	0.0320
	0.17	0.0375	0.6165	0.56	0.12	1.7286	0.2165	0.0320

First, some existing empirical studies (e.g.,Vaona (2012) and Barro (2013)) have found a long-run negative effect of inflation on economic growth. In fact, given that the majority of the current calibrated values of the parameters are preserved, our model is also flexible to generate a negative relationship between inflation and economic growth. Fig.3a illustrates this scenario accordingly by narrowing the gap between ξ_v and ξ_h by raising ξ_v to 0.4.[18] It is found that the inflation-growth relationship becomes strictly negative, which still accords with the predictions of Proposition 4. Moreover, Fig.3b shows that the welfare continues to be decreasing in the inflation rate.



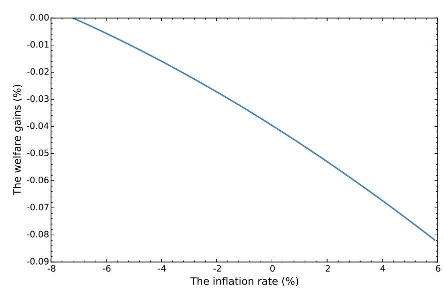


Fig. 3. (a) Inflation and economic growth ($\xi_v = 0.4$); (b) Inflation and social welfare ($\xi_v = 0.4$).

In addition, if the gap between ξ_v and ξ_h is reduced to an even smaller value by raising ξ_v to a larger value (i.e., 0.6), Fig.4a shows that the monotonically decreasing relationship between the inflation rate and the economic growth rate continues to hold, whereas the welfare now turns to be an inverted-U function of inflation as displayed in Fig.4b. In this case, as the inflation rate becomes higher, the positive welfare effect from the resulting increase in leisure will initially dominate the

[17]We have considered the three polar cases in which only one type of CIA constraint is exclusively present, to correspond to our analytical results of Propositions 1, 2 and 3. The numerical results are consistent with the theoretical implications, but to save space, these results are not reported here and are made available upon request.

[18]By holding ξ_v unchanged and reducing ξ_h, we find similar results for the growth and welfare effects of inflation.

negative welfare effects from a lower economic growth rate and a lower level of consumption, but this domination is reversed as the inflation rate increases to its threshold value of -5.2%. This implies a positive welfare-maximizing nominal interest rate of around 2%. Thus, the Friedman rule, which is optimal in the previous cases, becomes suboptimal in this case.

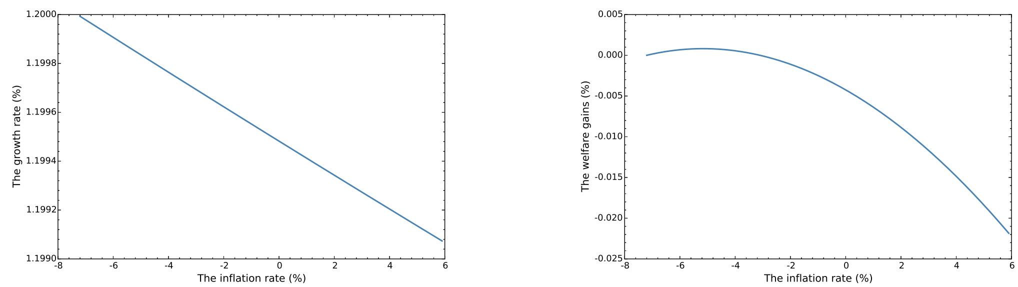


Fig. 4. (a) Inflation and economic growth ($\xi_v = 0.6$); (b) Inflation and social welfare ($\xi_v = 0.6$).

Third, a sensitivity analysis is performed by slightly reducing the value of the parameter ξ_c from 0.17 in the benchmark case to 0.158. A comparison of Fig.2a and Fig.5a shows that a lower value of ξ_c shifts the inflation-growth curve to the right with a higher threshold value of inflation around 7.9%, which is close to some other empirical estimates, for example, from Sarel (1996) and Burdekin *et al.* (2004). Intuitively, a smaller ξ_c weakens the negative inflation-growth effect arising from the CIA constraint on consumption, as shown in Proposition 1. Therefore, for a given level of ξ_v and of ξ_h, which respectively determines the negative inflation-growth effect and the positive one, a larger increase in the inflation rate is required to make the negative inflation-growth effect sufficiently strong to dominate the positive inflation-growth effect from the CIA constraint on horizontal innovation. As for the welfare effect of inflation, Fig.5b shows that the monotonically decreasing effect still holds.

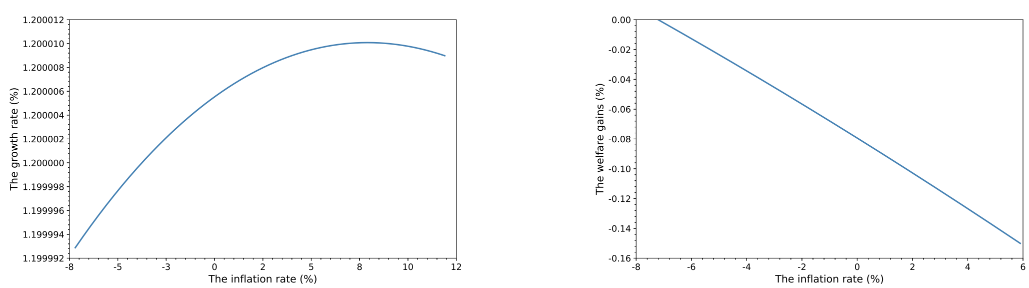


Fig. 5. (a) Inflation and economic growth ($\xi_c = 0.158$); (b) Inflation and social welfare ($\xi_c = 0.158$).

4.3.1 Diminishing returns to R&D

We perform a sensitivity analysis by increasing δ to 0.56 while keeping γ and other parameters unchanged as in the benchmark case. Figs.6a and 6b display the corresponding growth and welfare effect of inflation, both of which are found to be monotonically decreasing. As for the new results on the growth effect of inflation, as compared to the benchmark case, the increase in δ means lower diminishing returns to vertical R&D, and then a lower shifting-out effect of R&D labor due to inflation. Thus, the positive inflation-growth effect from the CIA constraint on horizontal innovation is weakened, leading the growth effect of inflation to be dominated by the other two negative inflation-growth effects even at lower levels of inflation. As a result, a monotonically decreasing relationship between inflation and growth is observed.

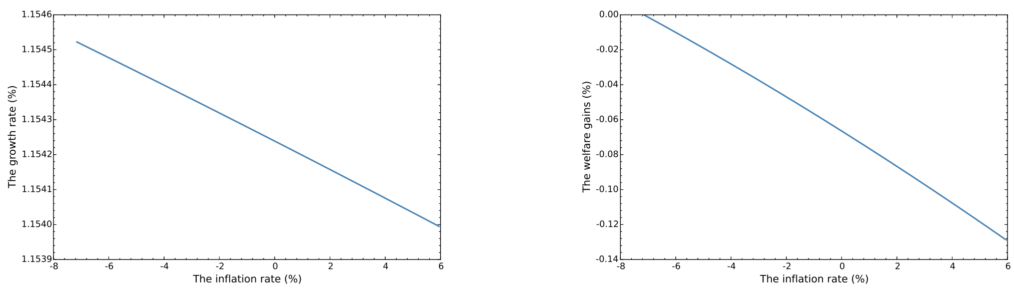


Fig. 6. (a) Inflation and economic growth ($\delta = 0.56$); (b) Inflation and social welfare ($\delta = 0.56$).

By enlarging the gap between ξ_v and ξ_h, through reducing ξ_v to 0.0375, we find that the inverted-U relationship between inflation and the economic growth rate recovers. Fig.7a shows that the inverted-U shaped growth effect of inflation still holds, similar to the case in Fig.2a. Furthermore, a comparison of Fig.6a and Fig.7a indicates that a lower ξ_v weakens the negative inflation-growth effect arising from the CIA constraint on vertical R&D, which in turn makes possible the positive inflation-growth effect to initially dominate the negative ones. However, as explained in the benchmark case, the negative inflation-growth effect from the CIA constraints on consumption and vertical R&D will eventually dominate the positive one as the inflation rate rises. Thus, the inverted-U pattern for inflation and economic growth is generated with a threshold value of approximately 2.6%, which is close to the benchmark case. Finally, the welfare effect of inflation is displayed in Fig.7b, indicating a similar pattern as in the benchmark case.

5 Conclusion

In this study, we explore the growth and welfare effects of monetary policy in an endogenous growth model with both vertical and horizontal innovations by incorporating cash-in-advance constraints on consumption and two R&D sectors. The novel contribution of this work, in contrast to the previous studies, is that our model is flexible enough to generate a mixed (i.e., monotonically decreasing or an inverted-U) relationship between inflation and economic growth, depending on the relative extents of CIA constraints and of diminishing returns to two types of innovation. In particular, in an empirically supportive case where horizontal R&D suffers greater diminishing returns

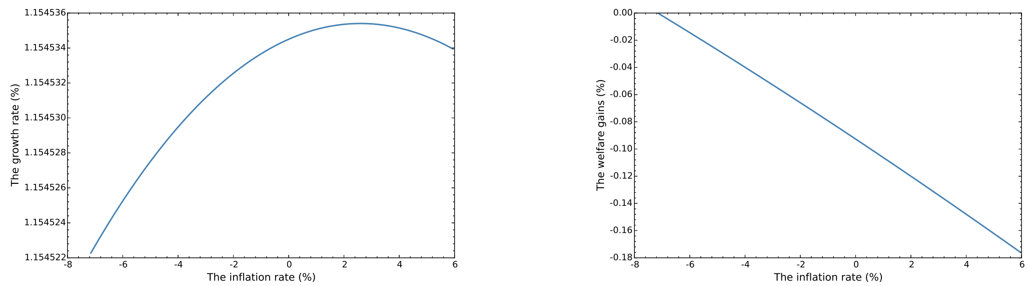


Fig. 7. (a) Inflation and economic growth ($\xi_v = 0.0375$); (b) Inflation and social welfare ($\xi_v = 0.0375$).

than vertical R&D, our model can generate an inverted-U relationship between inflation and growth when a sufficiently strong CIA constraint on horizontal R&D relative to vertical R&D is met. This result holds in our general model setting without scale effects, which well complements Chu *et al.* (2017). Moreover, we calibrate our model by applying the aggregate data of the US economy, and find that the growth-maximizing rate of inflation is around 2.4%, which is closely consistent with recent empirical estimates.

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