Inflation and Growth: A Mixed Relationship in an Innovation-Driven Economy

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Abstract

This paper investigates the effects of monetary policy on long-run economic growth via different cash-in-advance constraints on R&D in a Shumpeterian growth model with vertical and horizontal innovations. The relationship between inflation and growth is contingent on the weights of two innovations on growth, the relative entry cost, and the relative extent of diminishing returns to innovation. The model can generate a mixed (monotonic or non-monotonic) relationship between inflation and growth, provided that the relative strength from the entry costs and from the extent of diminishing returns may change with monetary policy. In particular, when the relative entry cost of horizontal R&D compared to vertical R&D is low, inflation and growth can exhibit an inverted-U shaped relationship depending on the weights of two innovations on economic growth. Finally, the model is calibrated by applying the aggregate data of the US economy, and we find that the threshold value of the inflation rate is around 2.8%, which is closely consistent with recent empirical estimates.

JEL classification: O30; O40; E41.
Keywords: Inflation; Endogenous growth; CIA constraint on R&D

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1 Introduction

The relationship between inflation and growth has long been debated among monetary economists. Is inflation negatively related to long-run economic growth conclusively? And do they maintain a steadily monotonic relationship regardless of the inflation level? Earlier studies indeed find a negative relationship between steady inflation and output/growth across countries (such as Fischer (1983) and Cooley and Hansen (1989)), whereas later works by Bullard and Keating (1995), Bruno and Easterly (1998), and Ahmed and Rogers (2000) seemingly find no robust or even a positive correlation in low-inflation industrialized economies.

More recent empirical works, which challenge most previous studies that document only monotonic relationships between inflation and growth, suggest a non-monotonic relationship that the real growth effect of inflation could be either positive or negative, depending on the status quo inflation rate. This series of studies can be traced back to Fischer (1993), who brings forward the existence of a threshold above and below which the effects of inflation on growth differ. Sarel (1996) identifies a structural break in the function that relates growth rates to inflation, showing that when inflation is low, specifically 8 percent annually, there is no significant negative effect (or even a slightly positive effect) on economic growth. When inflation is high, however, there exists a robust, statistically significant negative effect on growth. Several studies (Ghosh and Phillips (1998); Khan and Senhadji (2001); Burdekin et al. (2004); Eggoh and Khan (2014)) demonstrate successively the nonlinear correlation, but the specific threshold remains inconclusive, varying from 1% to 15-18%. In this study, our model is calibrated to aggregate data of the US economy to provide a quantitative analysis. We find that the growth-maximizing inflation rate is within the range for industrialized economies, i.e., 1-8%. Furthermore, we show that the fraction of CIA constraint on consumption is crucial in the determination of the inflation threshold.

In present article, we reconcile the theories and recent empirical evidences on inflation and growth in the context of an innovation-driven growth model characterized by two-modes of innovation. More precisely, various CIA constraints on R&D are incorporated in this study, which sheds light on how monetary policy can generate a non-monotonic relationship between inflation and growth through these constraints.

In particular, our analysis builds on the growth model developed by Howitt (1999) and Segerstrom (2000) that features both horizontal and vertical innovations. Vertical innovation serves to improve the quality of existing products whereas horizontal innovation aims at expanding product varieties, both of which are conducted by the forward-looking entrepreneurs. Monetary policy, which acts as a nominal interest rate targeting, affects the long-run growth rate by affecting the two types of innovations through the relative extents of different CIA constraints and diminishing returns to two types of R&D.

Imposing (CIA) constraints on R&D is consistent with the following empirical findings. First, monetary evidence (e.g., Hall (1992) and Himmelberg and Petersen (1994)) reports a strong R&D-cash flow sensitivity for firms. Hall and Lerner (2010) reports that more than 50 percent of R&D spending is the wages and salaries of R&D personnel. Hiring scientists and engineers usually

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1 As is documented in López-Villavicencio and Mignon (2011), the reasons for controversies include the frequency of data, the considered framework and the methodologies applied, the countries under study, and the existence of high-inflation observations.
involves a very high adjustment cost. R&D-intensive firms are required to hold cash in order to smooth their R&D spending over time. Brown and Petersen (2011) offers direct evidence that US firms relied heavily on cash reserves to smooth R&D spending during the 1998-2002 boom. The above evidence suggests that relative to the traditional physical investment, R&D activities exhibit a stronger investment-cash flow sensitivity.

In addition, several important empirical findings concerning firm characteristics motivate us to capture these insights through an endogenous growth model with two modes of innovation. First, a large firm size induces relatively a great amount of investment in process and incremental (vertical) R&D, while smaller firms usually involve in more radical (horizontal) product innovation (e.g., Cohen and Klepper (1996); Akcigit (2009); Janiak and Monteiro (2011)). Second, the requirements of cash holdings show distinct patterns to these two modes of innovations. Existing empirical evidence shows that there is a stronger impact from cash holdings on R&D in smaller, younger firms, who are more likely to confront binding liquidity and financing constraints (see Brown and Petersen (2009), Brown and Petersen (2011) and Brown et al. (2012)). Together with the empirical supports that small and young firms undertake more radical and original innovations, it is reasonable to consider that horizontal R&D is subject to a severer CIA constraint than vertical R&D. Accordingly, vertical innovation gains a cost advantage relative to horizontal innovation. Third, Klepper and Simons (2005) document that as firms grow large, its returns to R&D and the R&D investment increase. This thereafter directs to the fact that larger firms engaging in incremental innovative activities are more likely to gain success in their R&D projects.

Taking into consideration various CIA constraints, monetary policy in this study can generate different impacts of inflation on the economic growth subject to the relative extents of the CIA constraints and the different diminishing returns to two innovations. To be specific, with a change in the nominal interest rate, different CIA constraints implies a force that transmits different inflation costs, which distort the incentives and the use of economic resources in different sectors; at the same time, different diminishing returns to R&D implies another force that triggers a reallocation of resources between two types of R&D activities. Both forces jointly determine the long-run relationship between inflation and growth.

We first investigate the cases subject to each single type of CIA constraint and the results are as follows. In the presence of CIA constraint on consumption only, increasing the nominal interest rate increases (decreases) the economic growth rate if horizontal R&D exhibits greater (smaller) diminishing returns than vertical R&D. In this case, the degree of relative diminishing return to R&D plays a crucial role in determining the allocation of R&D resources, whereby along a rise of nominal interest rate, a greater (smaller) diminishing return to horizontal (vertical) R&D allows more R&D resources to be allocated in horizontal (vertical) innovation than in vertical (horizontal) innovation.

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2Because their skills are highly specific and unique, their vacancy may make the whole R&D process fail and dramatically decrease the firm’s profit. See Hall and Lerner (2010).

3Caggese (2015) points out that radical innovation requires a larger replacement of capital and expertise that were used to operate the old technology, resulting in a smaller probability to succeed. Therefore, young firms are more likely to engage in radical innovations, while old firms are endogenously larger and more likely to undertake incremental innovation since they have already attained a relatively high level of productivity and then choose incremental innovation to maintain it. He applies Italian firm-level data, which demonstrate that firms that engage in radical innovation are more financially constrained.

4See Cohen (2010) for a detailed survey about empirical studies on firm sizes and innovation, firm characteristics, and industry characteristics.
increasing the growth of variety (quality) at the expense of the growth of quality (variety) and thus leading to a decrease (an increase) in the long-run economic growth. In contrast, in the presence of CIA constraint on vertical (horizontal) R&D only, increasing the nominal interest rate always decreases (increases) economic growth regardless of the relative diminishing returns to both R&D. The reason is that R&D resources will be always shifted away from the CIA-constrained sector to the non-constrained one no matter which R&D sector exhibits a higher diminishing return. The diminishing returns to R&D in these cases only govern “the amount” but not “the direction” of the R&D resources shift.

More interestingly, incorporating all the CIA constraints into the model yields a diverse relationship between inflation and growth. In particular, focusing on the empirically relevant scenario where horizontal R&D exhibits a greater diminishing return, we find that increasing the nominal interest rate may induce a non-monotonic (inverted-U) relationship between inflation and growth, provided that the CIA constraint on horizontal R&D is sufficiently stronger than that of vertical R&D. Specifically, with a sufficiently large extent of CIA constraint on horizontal relative to vertical R&D, increasing the nominal interest rate from a low level yields a strong positive growth effect from the CIA constraint on horizontal R&D, which dominates the negative growth effects from the CIA constraints on consumption and vertical R&D; thus, a positive relationship between inflation and growth is generated. Nevertheless, when the nominal interest rate increases and then exceeds a threshold, the positive growth effect is dampened and turns to be overwhelmed, leading to a negative relationship between inflation and growth. Overall, a non-monotonic relationship (inverted-U shape) is formed in this circumstance.

By applying the US aggregate data, our quantitative analysis in the benchmark case generates an inverted-U relationship between inflation and growth, showing that the threshold value of the inflation rate is around 2.8%, which is closely in line with the recent empirical estimates in line with Ghosh and Phillips (1998) (i.e., 2.5%), López-Villavicencio and Mignon (2011) (2011) (i.e., 2.7%), and Eggoh and Khan (2014) (2014) (i.e., 3.4%). The welfare, however, is monotonically decreasing in inflation, implying that the Friedman rule is optimal. Interestingly, when the relative extent of the CIA constraint on horizontal to vertical R&D decreases, the inflation-growth relationship turns to be negative, which conforms to our analytical finding. The welfare, instead, turns to take an inverted-U shape in inflation, implying the sub-optimality of the Friedman rule. Finally, to check the robustness of our quantitative results, we also perform a sensitivity analysis with alternative calibrated values for several key parameters.

The literature pertaining to the analysis of monetary policy and growth is too large and diverse for a detailed review. The most closely related works are those that use endogenous growth models with R&D to analyze the effects of monetary policy on long-run growth. The pioneer work is Marquis and Reffett (1994) who explore the effects of monetary policy on growth via a CIA constraint on consumption in the framework of Romer (1990). Subsequent studies (e.g., Chu and Lai (2013) and Chu and Cozzi (2014)) analyze monetary policy in a Schumpeterian quality-ladder model. However, the present study differs from all these works by considering a scale-invariant Schumpetarian growth model that features two dimensions of innovations. Recent studies such as Chu and Ji (2016) and Huang et al. (2015) transpose their analysis into a second-generation fully endogenous growth model that is characterized by both horizontal and vertical innovations. Nonetheless, their models can only generate a monotonic linkage between inflation and long-run growth, whereas our model can yield a non-monotonic relationship between them, depending on the
status quo inflation. Finally, Chu et al. (2017) is another close, but independent theoretical work that is also able to generate an inverted-U relationship between inflation and economic growth. Unlike our analysis on the interaction between two types of innovations, their study examines the effects of monetary policy in a quality-improvement innovation model through endogenous entry of heterogeneous firms.

The remainder of this article proceeds as follows. The basic model is spelled out in section 2. Section 3 analyzes the effects of monetary policy in different cases of CIA constraints. The final section concludes.

2 Model

We consider a monetary variant of Segerstrom (2000) model that features two dimensional innovations. The model is extended to examine the effects of monetary policy by allowing for elastic labor supply and various CIA constraints on consumption and R&D investments. The economy consists of households, firms (including the incumbents for intermediate goods production and entrants for two types of R&D (i.e., vertical and horizontal R&D)), and a government that is solely represented by the monetary authority.

2.1 The Household

Consider a closed economy admits a household that is populated by a mass of individuals $L_t$ with the population size growing at an exponential rate $g_L$. Each individual supplies labor elastically and faces a life-time utility function given by

$$U = \int_0^\infty e^{-\rho t} [\ln c_t + \theta \ln(1 - l_t)] dt,$$

where $\rho$ is the discount rate, $c_t$ is the consumption of final goods per capita at time $t$, $l_t$ is the supply of labor per person at time $t$, and $\theta$ determines the preference for leisure relative to consumption.

An individual maximizes (1) subject to the budget constraint and a CIA constraint, which are respectively given by:

$$\dot{a}_t + \dot{m}_t = (r_t - g_L)a_t + w_t l_t + i_t b_t + \zeta_t - (\pi_t + g_L)m_t - c_t,$$

and

$$\xi_c c_t + b_t \leq m_t,$$

where $a_t$ is the real assets owned by each person. $r_t$ is the real interest rate. Each individual supplies labor $l_t$ to earn a real wage rate $w_t$, and loans out an amount $b_t$ of money to the entrepreneurs, with a return rate $i_t$ (i.e., the nominal interest rate). Each individual receives a lump-sum transfer $\zeta_t$ from the government. Moreover, $m_t$ is the real money balance held by the individual, and $\pi_t$ is the inflation rate.

The CIA constraint in (3) states that the holding of real money balances $m_t$ by each household is used not only to finance the R&D investments but also to partly purchase consumption $c_t$, where $\xi_c \in [0, 1]$ represents the share of consumption required to be purchased by cash/money.
Denote \( \eta_t, \omega_t \) the multipliers associated with the budget constraint in (2) and the CIA constraint in (3), respectively. The utility in (1) is maximized subject to (2) and (3) from which the first-order conditions for \( c_t, l_t, a_t, m_t, \) and \( b_t \) can be derived. After some manipulations, the first-order conditions conditions can be reduced into the following optimality conditions. The standard Euler equation governs the growth of consumption given by

\[
\dot{c}_t = r_t - \rho - gL.
\]  

(4)

The optimal condition determines the consumption-leisure tradeoff such that

\[
w_t(1 - l_t) = \theta c_t(1 + \xi_c \dot{c}_t),
\]  

(5)

and the no-arbitrage condition between all assets and money implies the Fisher equation given by

\[
i_t = r_t + \pi_t.
\]  

(6)

### 2.2 Final Goods

Final goods are produced by a mass of identical perfectly competitive firms that employ labor and a continuum of intermediate inputs according to the same constant returns to scale production technology. The production function of a typical firm \( k \) at time \( t \) is:

\[
Y_{kt} = L_{ykt}^{1-\alpha} \int_0^{N_t} A_i x_{ikt}^\alpha di,
\]  

(7)

where \( L_{ykt} \) is the amount of labor employed by final-good firm \( k \). \( N_t \) is the number of input varieties (or industries). \( A_i \) is the productivity level attached to the latest version of intermediate product \( i \). \( x_{ikt} \) is the \( i \)-th type of intermediate inputs employed by firm \( k \), and \( \alpha \in (0, 1) \) is the elasticity of demand for intermediate products.

Firm \( k \) faces the following profit-maximization problem

\[
\max_{L_{ykt}, x_{ikt}} \pi_{kt} = p_{yt} Y_{kt} - w_t L_{ykt} - p_{it} x_{ikt}
\]

subject to (7) in which the final-good price \( p_{yt} \) is set as the numeraire (i.e., \( p_{yt} \equiv 1 \)). Firm \( k \) chooses the amount of labor \( L_{ykt} \) and intermediate input \( x_{ikt} \) to maximize its profit, taking as given the wage rate \( w_t \) and the price of intermediate input \( p_{it} \). The first-order condition with respect to \( x_{ikt} \) leads to the inverse demand for \( x_{ikt} \):

\[
p_{it} = \alpha A_i (L_{ykt}/x_{ikt})^{1-\alpha}.
\]

Since all firms face the same price \( p_{it} \), the input ratios must be identical across firms (i.e., \( L_{ykt}/x_{ikt} = L_{yit}/x_{it} \)), where \( L_{yit} = \int L_{ykt} dk \) and \( x_{it} = \int x_{ikt} dk \). Therefore, the above expression can be reduced to

\[
p_{it} = \alpha A_i (L_{yt}/x_{it})^{1-\alpha}.
\]  

(8)
Similarly, the inverse demand for $L_{ykt}$ is given by

$$w_t = (1 - \alpha) \int_0^{N_t} A_{it} \left( \frac{x_{it}}{L_{yt}} \right)^{\alpha} di.$$  \hspace{1cm} (9)

### 2.3 Incumbents

There is a continuum of industries $N_t$ producing differentiated intermediate goods. Each industry is occupied by an industry leader who holds a patent on the latest innovation and monopolizes the production of one differentiated intermediate good $i$. The monopolistic leader dominates the market temporarily until its displacement by the next innovation.

The production technology across all incumbent firms is assumed to be identical, in which each incumbent requires $\alpha^2$ units of final goods to produce one unit of intermediate good as in Acemoglu et al. (2012). Accordingly, firm $i$ faces the following profit-maximization problem:

$$\max x_{it} \pi_{it} = p_{it}x_{it} - \alpha^2 x_{it}.$$  \hspace{1cm} (10)

Then the solution yields the optimal price $p_{it} = \alpha$, and thus the quantity of intermediate product $i$ is given by

$$x_{it} = L_{yt} A_{it}^{\frac{1}{1-\alpha}}.$$  \hspace{1cm} (10)

Substituting these results into $\pi_{it}$ yields the equilibrium profit:

$$\pi_{it} = \alpha (1 - \alpha) x_{it} = \alpha (1 - \alpha) L_{yt} A_{it}^{\frac{1}{1-\alpha}}.$$  \hspace{1cm} (11)

The industry leader $i$ possesses this profit flow in each period until the arrival of next innovation.

### 2.4 Entrants

Following Howitt (1999) and Segerstrom (2000), a new firm (an entrant) can enter the market by either engaging in a vertical or a horizontal innovation. An entrant that engages in a vertical innovation targets an existing industrial product line and devotes resources to improve the quality of that product. The product with the improved quality allows the innovator to replace the incumbent of the original product and then become the industry leader until the next innovation in this industry occurs.

An entrant that engages in a horizontal innovation devotes resources to create an entirely new industry. She then becomes a new industry leader with an exclusive patent right to produce a differentiated good until the arrival of next vertical innovation targeted at this industry.

#### 2.4.1 Vertical R&D

First, consider that the entrant $j$ engages in vertical R&D by targeting an existing industry $i$ to improve its product quality at time $t$ with a successful rate of innovation $\phi_{ijt}$ that follows Poisson
process, which is given by

$$\phi_{ijt} = \frac{\lambda(L_{v,ijt})^{\delta}(K_{ijt})^{1-\delta}}{A_t}; \quad 0 < \delta < 1. \quad (12)$$

$\lambda$ is a positive R&D productivity parameter. $L_{v,ijt}$ is the level of firm $j$’s R&D employment. $K_{ijt}$ is the stock of the firm-specific knowledge possessed by firm $j$. $\delta$ measures the degree of diminishing returns to vertical R&D expenditures. $A_t$ is the leading-edge productivity parameter at time $t$ defined as $A_t \equiv \max\{A_{it}; i \in [0, N_t]\}$, and is also interpreted as the force of increasing research complexity. The evolution of $A_t$ will be discussed in detail in the later subsection.

To capture the monetary effect of the CIA constraint on vertical R&D, we assume that a fraction $\xi_v$ of vertical R&D spending is constrained by cash/money. This cash constraint forces the R&D firm to borrow an amount $\xi_v w_t L_{v,ijt}$ of money at the nominal interest rate $i$ from the household for financing the R&D expenditure. Accordingly, the profit-maximization problem for each potential entrant is

$$\max_{L_{v,ijt}} \phi_{ijt} \Pi_v t - w_t L_{v,ijt}(1 - \xi_v) - w_t L_{v,ijt} \xi_v(1 + i_t)$$

$$= \phi_{ijt} \Pi_v t - w_t L_{v,ijt}(1 + \xi_v i_t),$$

where $\Pi_v t = \int_t^{\infty} e^{-\int_t^{\tau}(r+\phi_s)ds} \pi_{t\tau} d\tau$ is the expected present value of the innovative firm’s profit flows before the replacement of the next successful innovation, and $\pi_{t\tau}$ is the monopoly profit flow at time $\tau$ from a firm whose technology is of vintage $t$. As assumed in Howitt (1999) and Segerstrom (2000), each innovation at time $t$ produces a new generation of products in that industry, which embodies the leading-edge productivity parameter $A_t$. This results in a continuous flows of the same monopoly profit $\pi_{t\tau}$ across industries after time $t$ and is given by $\pi_{t\tau} = \alpha(1 - \alpha) L_{y\tau} A_t^{1/(1-\alpha)}$. Moreover, $r$ is the instantaneous interest rate, and $\phi_s$ is the rate of creative destruction, namely the instantaneous flow probability of being displaced by an innovation. Along with the same instantaneous discount rate $r + \phi_s$ applying the same amount of profit flow $\pi_{t\tau}$ earned by each industry leader, it is easy to deduce that the expected reward for vertical innovation $\Pi_v t$ does not vary across industries.

At time $t$, a potential entrant $j$ that targets the vertical R&D at industry $i$ solves the above profit-maximization problem, yielding the first-order condition such that

$$\lambda \delta \Pi_v t \left(\frac{L_{v,ijt}}{K_{ijt}}\right)^{\delta-1} = w_t(1 + \xi_v i_t), \quad (13)$$

which reveals that the marginal expected benefit of an extra unit of vertical R&D equals its marginal cost. It is clear from (13) that the marginal cost is positively correlated with the parameter $\xi_v$, capturing the adverse effect of the nominal interest rate $i$ on the firm’s R&D decision $L_{v,ijt}$ through increasing the marginal cost of vertical innovation.

Following Segerstrom (2000), $K_{ijt}$ is considered to be the same and infinitesimally small for all $j$. Given this assumption, (13) implies that $L_{v,ijt}/K_{ijt} = L_{v,it}/K_{it}$ for all $j$, where $L_{v,it} = \sum_j L_{v,ijt}$ and $K_{it} = \sum_j K_{ijt}$. In addition, we assume that $K_{it} \equiv \sum_j K_{ijt} = L_{it}/N_t$ for all $i$, which is in line

$^5$An infinitesimally small $K_{ijt}$ implies that the optimal amount of firms R&D resources $L_{v,ijt}$ is also infinitesimally small, governed by (13). Hence, the likelihood of any one firm winning a vertical R&D race can be negligible, given that the vertical R&D races are perfectly competitive.
with Romer (1990), Segerstrom (2000), and Ha and Howitt (2007). Thus, (13) can be re-expressed as
\[
\frac{\lambda \delta \Pi_v t}{A_t} (l_v t)^{\delta - 1} = w_t (1 + \xi_v t),
\]
where \(l_v t \equiv L_v t / L_t\) \((L_v t \equiv \sum_j L_{v,i,t})\) is the fraction of total labor employment that is allocated to vertical R&D. We further assume that the returns on conducting vertical R&D are identical across firm \(j\) and across times (see Segerstrom (2000)). This assumption together with the facts that \(l_v t \equiv L_v t / L_t\) and \(K_{it} = L_t / N_t\) indicates that the Poisson arrival rate of vertical innovations in each industry becomes
\[
\phi_t = \sum_j \phi_{ij,t} = \frac{\lambda (L_v t / N_t)^{\delta} (L_t / N_t)^{1-\delta}}{A_t} = \lambda l_v t, \tag{15}
\]
where \(l_t \equiv L_t / (A_t N_t)\). The expression (15) shows that the arrival rate of vertical innovations is increasing in per industry vertical R&D expenditure \(L_v t / N_t\) and the knowledge spillover \(L_t / N_t\) but decreasing in the R&D difficulty term \(A_t\).

### 2.4.2 Horizontal R&D

An entrant \(q\) that engages in horizontal innovations devotes resources to create a new variety (and thus an entirely new industry). She faces the following rate of discovering new innovations, denoted as \(\dot{N}_{qt}\):
\[
\dot{N}_{qt} = \frac{\lambda (L_{hqt}^\gamma (K_{qt})^{1-\gamma})}{A_t}; \quad 0 < \gamma < 1. \tag{16}
\]

\(L_{hqt}\) is the level of firm \(q\)'s R&D employment, \(K_{qt}\) is the firm-specific knowledge possessed by firm \(q\) that is useful for horizontal innovations, and the exponent \(\gamma\) measures the degree of diminishing returns to horizontal R&D expenditures. \(A_t\) reflects the fact of increasing research complexity.

As in Howitt (1999) and Segerstrom (2000), we assume that each horizontal innovation at time \(t\) results in a new intermediate variety whose productivity parameter is drawn randomly from an invariant long-run distribution of the existing productivity parameters \(A_{it}\) across industries \(i\). This assumption makes sure that the process of variety-expanding will not affect the convergence of the distribution of existing parameters \(A_{it}\) to an invariant distribution in the long run. See the detailed discussion in the next subsection.

Next, to capture the monetary effect of the CIA constraint on horizontal R&D, we assume that a fraction \(\xi_h\) of horizontal R&D expenditure is constrained by cash/money. This cash constraint forces the innovative firm to borrow an amount \(\xi_h w_t L_{hqt}\) of money at the nominal interest rate \(i\) from the household for financing the R&D expenditure. In addition, throughout the rest of this study, the assumption that \(\xi_h > \xi_v\) is imposed to capture the empirical evidence that the investment on radical innovations is more constrained by cash/money than that on incremental innovations (e.g., Akcigit (2009) and Caggese (2015)). Accordingly, the profit-maximization problem for horizontal R&D firm \(q\) is
\[
\max_{L_{hqt}} \pi_{hqt} = \dot{N}_{qt} \Pi_{ht} - w_t L_{hqt} (1 + \xi_h i_t),
\]

We mainly follow Ha and Howitt (2007) to capture the insight that the total amount of firm-specific knowledge in each industry equals per industry labor, which grows over time in equilibrium.
\[ \Pi_{ht} = \Gamma^{-1} \Pi_{vt}, \]  
(17)

where \( \Gamma \equiv 1 + [\sigma/(1 - \alpha)] \) and \( \Pi_{ht} \) is the expected value of a successful horizontal innovation. (17) reveals the relationship between \( \Pi_{ht} \) and \( \Pi_{vt} \) from the aforementioned assumption regarding the random draw of the productivity parameters, and the derivation of (17) will be provided in the next subsection.

Then, the first-order condition for horizontal R&D firms profit maximization is given by

\[ \lambda \gamma \Pi_{ht} A_t \left( \frac{L_{ht}}{K_{qt}} \right)^{\gamma - 1} = w_t (1 + \xi_h i_t). \]  
(18)

This equation clearly shows that the marginal cost is positively related to the CIA parameter \( \xi_h \), capturing the negative effect of the nominal interest rate \( i_t \) on the firm’s R&D decision through increasing the marginal cost of horizontal innovation \( w_t (1 + \xi_h i_t) \).

Moreover, (18) states that \( \Pi_{ht} \) only scales \( \Pi_{vt} \) with a constant factor, implying that \( \Pi_{ht} \) is also identical across for all entrants \( q \). Together with the same marginal cost faced by each entrant, the above first-order condition implies that \( L_{ht}/K_{qt} = L_{ht}/K_t \) for all \( q \), where \( L_{ht} \equiv \sum_q L_{hq} \) and \( K_t \equiv \sum_q K_{qt} \). Furthermore, a similar assumption is made such that \( K_{qt} = L_t/N_t \) for all \( q \) as in the previous subsection. Substituting \( K_{qt} = L_t/N_t \) and \( L_{ht}/K_{qt} = L_{ht}/K_t \) into (18) yields:

\[ \frac{\gamma \lambda \Pi_{ht}}{A_t} \left( \frac{l_{ht}}{l_t} \right)^{\gamma - 1} = w_t (1 + \xi_h i_t), \]  
(19)

where \( l_{ht} \equiv L_{ht}/L_t \) is the fraction of labor allocated to horizontal R&D. The growth rate of the measure of industries is the summation of the discovery rates for all the individual firms that engage in horizontal R&D, such that

\[ g_{N_t} \equiv \frac{\dot{N}_t}{N_t} = \sum_q \frac{\dot{N}_{qt}}{N_t} = \frac{\lambda (L_{ht}/N_t)^{\gamma} (L_t/N_t)^{1-\gamma}}{A_t} = \lambda l_{ht}^{\gamma} i_t. \]  
(20)

### 2.4.3 Spillovers

As in Caballero and Jaffe (1993), Howitt (1999), and Segerstrom (2000), the leading-edge productivity parameter \( A_t \) grows over time as a result of knowledge spillovers produced by vertical innovations. The growth rate of \( A_t \) is proposed to take the following standard form

\[ g_{A_t} \equiv \frac{\dot{A}_t}{A_t} = \left( \frac{\sigma}{N_t} \right) (\phi_t N_t) = \sigma \phi_t = \sigma \lambda \phi_{vt} l_{vt}, \]  
(21)

where \( \sigma > 0 \) measures the R&D spillover effect and \( \phi_t = \sum_j \phi_{ij} \) is the Poisson arrival rate of vertical innovations in each industry \( i \in [0, N_t] \) (namely a summation of all potential vertical entrants).

As shown in (21), \( g_{A_t} \) can essentially be decomposed as a product of two factors \( \sigma/N_t \) and \( \phi_t N_t \), where \( \phi_t N_t \) is the aggregate flow of vertical innovations in this economy. (21) states that the growth of knowledge spillover is assumed to be proportional to the aggregate flow of vertical innovations \( \phi_t N_t \). The factor of proportionality \( \sigma/N_t \) measures the marginal effect of each vertical
innovation on the stock of public knowledge. The divisor $N_t$ captures that each vertical innovation has a smaller impact on the aggregate economy as the number of specialized products expands with the development of the economy.

Because the distribution of productivity parameters among new products at any time is identical to the distribution across existing products at that time, one can show that the distribution of relative productivity parameters, which is defined as $z_{it} \equiv A_{it}/A_t$, would converge monotonically to the invariant distribution $Pr\{z_{it} \leq z\} \equiv F(z) = z^{1/\sigma}$, wherein $0 < z \leq 1$. It follows that in the long run:

$$E \left[ \left( \frac{A_{it}}{A_t} \right)^{1/(1-\alpha)} \right] = \Gamma^{-1},$$

where $\Gamma \equiv 1 + \left[ \sigma/(1 - \alpha) \right]$.\(^7\)

Recall that the productivity parameter of each new innovative variety is drawn randomly from the above distribution. This implies that the realized monopoly profit flow for each horizontal R&D firm at date $\tau$ and its realized present value at time $t$ are $\pi_{i\tau} = \alpha(1-\alpha)L_{yt}A_{it}^{1/(1-\alpha)}$ and $\Pi_{ht} = \int_t^\infty e^{-\int_t^\tau (r+\phi_s)ds} \pi_{i\tau} d\tau$, respectively. Along with the fact that a successful vertical innovation gains the profit flow $\hat{\pi}_t \tau = \alpha(1-\alpha)L_{yt}A_{t}^{1/(1-\alpha)}$ with the leading-edge productivity parameter $A_t$, it is easy to deduce that $\Pi_{ht} = \left( A_{it}/A_t \right)^{1/(1-\alpha)} \Pi_{vt}$. Taking expectations on both sides of this equation yields (17).

### 2.5 Monetary Authority

The monetary authority implements its monetary policy by targeting a long-run nominal interest rate level $i_t$. Denote the nominal money supply by $M_t$, thus the growth rate of nominal money supply is $\dot{M}_t/M_t = \mu_t$. Recall that $m_t$ is real money balance per capita and is given by $m_t = M_t/(L_t p_{yt})$, so the growth rate of real money balance per capita is $g_{mt} \equiv \dot{m}_t/m_t = \mu_t - \pi_t - g L$. Substituting this expression and the Euler equation (4) into the Fisher equation (6), along with the fact that $g_{mt} = g_{ct}$ in the steady state,\(^8\) we obtain

$$i_t = r_t + \pi_t = (\rho + g_{ct} + g_L) + (\mu_t - g_{mt} - g_L) = \rho + \mu_t.$$ \hspace{1cm} (23)

This equation illustrates an one-by-one monotonic relationship between nominal interest rate $i$ and the growth rate of nominal money supply $\mu$, which indicates an isomorphic choice of monetary instruments between $i_t$ and $\mu_t$. Specifically, an exogenous increase in $i_t$ corresponds to an endogenous increase in $\mu_t$.

Upon increasing the nominal interest rate $i_t$, the government earns the seignorage revenue through an inflation tax. To balance the budget, it is assumed that the government returns the revenues as a lump-sum transfer to the household. Therefore, the government’s budget constraint (in terms of per capita level) is given by $M_t/(L_t p_{yt}) = \dot{m}_t + (\pi_t + g_L)m_t = \zeta_t$.

---

\(^7\)See Howitt (1999) and Segerstrom (2000) for the detailed proof.

\(^8\)According to the CIA constraint (3), it can be shown that on the balanced growth path, $m_t$ and $c_t$ grow at the same rate.
2.6 Characterization of Equilibrium

The equilibrium in this economy consists of a time path of allocations \( \{c_t, m_t, l_t, Y_{kt}, Y_t, x_{it}, x_t, L_{ykt}, L_{v,ijt}, L_{hqt}\}_{t=0}^{\infty} \) and a time path of prices \( \{w_t, r_t, i_t, p_{it}, p_{yt}\}_{t=0}^{\infty} \), where \( Y_t = \int Y_{kt}dk \) and \( x_t = \int_0^{N_t} x_{it}di \). Moreover, at each instance of time,

- individuals maximize utility taking \( \{i_t, r_t, w_t\} \) as given;
- the competitive final-goods firms produce \( \{y_{kt}\} \) to maximize profits taking \( \{p_{yt}\} \) as given;
- the monopolistic intermediate-goods firms produce \( \{x_{it}\} \) and choose \( \{Y_t, p_{it}\} \) to maximize profits taking \( \{p_{yt}\} \) as given;
- the labor market clears such that \( L_{yt} + L_{vt} + L_{ht} = l_t L_t \);
- the final-goods market clears such that \( Y_t = C_t + x_t \);
- the asset market clears such that the value of monopolistic firms adds up to the value of household’s assets: \( \Pi_{vt} + \Pi_{ht} = a_t L_t \);
- the amount of money borrowed by two types of innovation entrants is given by \( b_t L_t = \xi_v w_t L_{vt} + \xi_h w_t L_{ht} \).

Using (22), we obtain\( \int_0^{N_t} A_1^{1-\alpha} di = A_1^{1-\alpha} N_t \int_0^1 z^{1-\alpha} F'(z)dz = A_1^{1-\alpha} N_t \Gamma^{-1} \). Substituting this equation, (10), and \( Y_t = \int Y_{kt}dk \) into (7) yields the equilibrium final-goods production function:

\[
Y_t = \frac{L_{yt} A_1^{1-\alpha} N_t}{\Gamma}.
\]  

(24)

Accordingly, the per-capita consumption and the production-labor shares of outputs are, respectively,

\[
c_t = \frac{(1 - \alpha^2) L_{yt} A_1^{1-\alpha} N_t}{\Gamma},
\]

(25)

and

\[
w_t = (1 - \alpha) \frac{Y_t}{L_{yt}} = \frac{(1 - \alpha) A_1^{1-\alpha} N_t}{\Gamma}.
\]

(26)

2.7 Balanced-Growth Properties

In this section, we follow Segerstrom (2000) to focus on the analysis of the balanced-growth equilibrium properties of the model. In the balanced-growth equilibrium, the fraction of labor supplied to each sector must be constant over time (i.e., \( l_{vt} = l_v, l_{ht} = l_h, l_{yt} = l_y \) for all \( t \)). Since both \( g_{At} \) and \( g_{Nt} \) must be constant in a balanced-growth equilibrium, (12) implies that the arrival rate of vertical innovations must be constant as well (i.e., \( \phi_t = \phi \) for all \( t \)). Furthermore, according to (21) and (20), \( \iota_t \) must be constant in the balanced-growth equilibrium (i.e., \( \iota_t = \iota \) for all \( t \)). Thus, the quality and variety growth rates can, respectively, be written as

\[
g_A = \sigma \lambda_{i_t}^B, \tag{27}
\]
and
\[ g_N = \lambda \gamma t. \]  

### 2.7.1 Economic Growth

Denote by \( g \) the growth rate of consumption per capita \( c_t \) on the balanced-growth path (and economic growth rate thereafter). Differentiating the per-capita consumption share of outputs (25) with respect to time yields
\[ g = g_N + \frac{1}{1-\alpha} g_A. \]  
This equation, called the *iso-growth* condition, demonstrates that on the balanced-growth path (BGP), the growth rate of the measure of industries \( g_N \) and the growth rate of productivity of industries \( g_A \) jointly determine the overall rate of economic growth \( g \).

### 2.7.2 Population-Growth Condition

Moreover, differentiating \( \iota_t = \frac{L_t}{A_t N_t} = \iota \) with respect to time \( t \) yields the population-growth condition
\[ g_L = g_A + g_N. \]  
This equation states that to guarantee a BGP, the growth rate of the leading-edge productivity parameter \( A_t \) and that of the measure of variety \( N_t \) are required to grow in a manner such that these growth rates are constrained by the population-growth rate \( g_L \). The intuition behind this constraint is as follows. As the economy grows with higher levels of \( A_t \) and \( N_t \), research becomes more complex, and thus the productivity of researchers \( \iota_t \) falls in response. To maintain a constant innovation rate in \( g_N \) and \( g_A \) over time as stipulated in (20) and (21), more labors are needed to devote into R&D activities. The population-growth rate \( g_L \) determines the rates at which labor resources can be devoted into both horizontal and vertical R&D activities and therefore determines the overall growth rate of the economy.

Additionally, examining both equations (29) and (30) yields the following result.

**Lemma 1.** In the steady-state equilibrium, the economic growth rate is increasing in the vertical R&D growth rate.

The intuition of this lemma is straightforward. The population-growth condition (30) implies that there is an equal tradeoff between \( g_A \) and \( g_N \) (i.e., an increase in \( g_A \) comes with the cost of an identical amount of reduction in \( g_N \) to maintain a constant population-growth rate). However, the iso-growth condition (29) reveals that the economic growth rate features a larger contribution of \( g_A \) than \( g_N \) (i.e., \( 1/(1-\alpha) > 1 \)). Therefore, an increase in \( g_A \) at the sacrifice of \( g_N \) comes with a higher economic growth rate. This theoretical attribute is also available in Howitt (1999) that the economic growth rate is eventually supported by the growth from creative destruction (vertical innovation) rather than variety expansion (horizontal innovation) in the steady-state equilibrium. In addition, this implication is consistent with empirical finding by Garcia-Macia et al. (2016), who decompose the aggregate TFP growth for the US within periods 1976-1986 and 2003-2013, and find that the contribution of growth from creative destruction is overwhelmingly larger than that from new varieties.
3 Growth Effects of Monetary Policy

In this section, we analyze the growth effects of monetary policy (in terms of nominal interest rate targeting) on growth with various CIA constraints. To fully comprehend the underlying mechanism, we first proceed our analysis in different scenarios, each of which is subject to one distinct type of CIA constraint. After picking up the intuition behind each scenario, we impose all types of CIA constraints simultaneously and then provide a complete analysis.

3.1 CIA on Consumption

First, we analyze the case in which only a CIA constraint on consumption is present, and the following proposition is obtained.

Proposition 1. In the presence of a CIA constraint on consumption only (i.e., $\xi_c > 0$, $\xi_v = \xi_h = 0$), a higher nominal interest rate increases (decreases) the economic growth rate if $\gamma > \delta$ ($\gamma < \delta$).

Fig. 1 illustrates the effects of a permanent increase in the nominal interest rate $i$ on the economic growth rate when the model only features a CIA constraint on consumption. Using both the iso-growth condition in (29) and the population-growth condition in (30), we can derive two downward sloping lines with a slope of $-1/(1-\alpha)$ and of $-1$, respectively, in the $(g_A, g_N)$ space. Thus, the slope of each iso-growth line exceeds the slope of the population-growth condition (in absolute value).

To better understand the intuition underlying Proposition 1, first, we analyze the instant effect of raising $i$ starting from the initial balanced-growth equilibrium. When only consumption is subject to the CIA constraint, increasing the nominal interest rate $i$ raises the cost for consumption purchase relative to leisure. As a result, individuals enjoy more leisure by reducing labor supply, which drives up the real wage rate provided that the labor demanded by firms is unaffected in the absence of CIA constraints on R&D. A higher wage rate immediately pushes down the equilibrium labor for both R&D activities $l_h$ and $l_v$. More importantly, $l_h$ decreases by a smaller (larger) amount than $l_v$ does if horizontal R&D exhibits greater (smaller) diminishing returns than vertical R&D (i.e., $\gamma < (>\delta)\delta$). In Fig. 1, to reflect the case of $\gamma < \delta$, a higher $i$ leads the economy to jump from the initial steady state $A$ to $B'$ with a smaller reduction in $g_A$ than in $g_N$. In contrast, to reflect the case of $\gamma > \delta$, a higher $i$ shifts the economy from $A$ to $C'$, with a larger reduction in $g_N$ than in $g_A$.

Next, we follow Segerstrom (2000) to provide an intuitive explanation about how the economy adjusts after its instant shift off the balanced-growth path. The corresponding decreases in $g_A$ and $g_N$ indicate that the research complexity grows at a slower rate than usual. It follows that the research productivity $\iota_t$ rises gradually over time, which drives up $g_A$ and $g_N$ again as indicated in (27) and (28), till they are back to balanced-growth equilibrium. That is, the population-growth condition is satisfied again. Therefore, there are two cases to be considered.

When $\gamma < \delta$, raising $i$ initially drives the economy to jump from $A$ to $B'$ (i.e., a larger decrease in $g_A$ than in $g_N$). Then the research productivity $\iota_t$ rises over time, driving up $g_A$ and $g_N$ gradually in a similar magnitude, which induces the economy to move from point $B'$ to the new balanced-growth path $B$. It is clear that the long-run effect of raising the nominal interest rate boots the horizontal innovation rate $g_N$ at the expense of reducing $g_A$. Then the economic growth rate will decrease in response as shown in Lemma 1.
When \( \gamma > \delta \), raising \( i \) initially drives the economy to jump from \( A \) to \( C' \) (i.e., a smaller decrease in \( g_A \) than in \( g_N \)). This force subsequently induces the economy to move from point \( C' \) to the new balanced-growth path \( C \). In this case, the long-run effect of raising the nominal interest rate boots the vertical innovation rate \( g_A \) at the expense of reducing \( g_N \). As a result, the economic growth rate will increase in response as implied by Lemma 1.

![Fig. 1. Adjustment process: CIA constraint on consumption.](image)

### 3.2 CIA on Vertical R&D

In this subsection, we analyze the case in which only a CIA constraint on vertical R&D is present, and the following result is obtained.

**Proposition 2.** In the presence of a CIA constraint on vertical R&D only (i.e., \( \xi_v > 0, \xi_c = \xi_h = 0 \)), a higher nominal interest rate decreases the economic growth rate under both \( \gamma < \delta \) and \( \gamma > \delta \), but with a larger size under \( \gamma < \delta \).

Fig.2 illustrates the effects of a permanent increase in the nominal interest rate \( i \) on growth when the model only features a CIA constraint on vertical R&D. Similar to Subsection 3.1, the analysis starts off by exploring the instant effect of raising \( i \) from the initial balanced-growth equilibrium.

When only vertical R&D is subject to the CIA constraint, an increase in \( i \) raises the cost for vertical R&D, reducing the firms’ demand for R&D labor on vertical innovation. Given an unchanged labor supply due to the absence of CIA constraint on consumption, the wage rate declines, which reallocates labors from vertical R&D \( l_v \) to production \( l_y \), horizontal R&D \( l_h \), and leisure. Under \( \gamma < \delta \), greater diminishing returns to horizontal R&D will reallocate less labor force to \( l_h \), allowing only for a smaller increase in \( g_N \); in addition, a high level of \( \delta \) causes a decrease in \( l_v \) to transmit a larger reduction in \( g_A \) as shown in (27). In Fig.2, the economy, therefore, moves from \( A \) to \( B' \) in this case. By contrast, under \( \gamma > \delta \), smaller diminishing returns to horizontal R&D will reallocate more labor force to \( l_h \), leading to a higher \( g_N \); in addition, a low level of \( \delta \) also causes a decrease in \( l_v \) to transmit a smaller reduction in \( g_A \). In Fig.2, the economy would move from \( A \) to \( C' \) if the gap between \( \gamma \) and \( \delta \) is small (i.e., \( \gamma \) is slightly larger than \( \delta \)), and thus the magnitudes of the changes in \( g_N \) and \( g_A \) are close. Otherwise, the economy would move from \( A \) to \( C'' \) if the gap
between $\gamma$ and $\delta$ is large (i.e., $\gamma$ is much larger than $\delta$), and thus the size of the increase in $g_N$ is much more significant than that of the decrease in $g_A$.

![Diagram of economic growth]

**Fig. 2.** Adjustment process: CIA constraint on vertical R&D.

Next, we turn to intuitively explain the adjustment process. There are three scenarios to be considered. First, when $\gamma < \delta$, since the magnitude of the decrease in $g_A$ is much larger than that of the increase in $g_N$ as shown in the movement from point $A$ to point $B'$, the growth of research complexity is driven down to a lower rate than usual. It follows immediately that the research productivity $\iota_t$ rises over time. Hence, $g_A$ and $g_N$ grow gradually in a similar manner, inducing the economy to move from point $B'$ to the new balanced-growth path $B$. Second, when $\gamma > \delta$ and the gap between $\gamma$ and $\delta$ is small, the close magnitudes of the changes in $g_A$ and $g_N$ may still drive down the growth of research complexity to a lower rate than usual. It then follows that $\iota_t$ rises over time. Hence, $g_A$ and $g_N$ grow gradually in a similar fashion, inducing the economy to move from point $C'$ to the new balanced-growth path $C$. Third, when $\gamma > \delta$ and the gap between $\gamma$ and $\delta$ is large, the magnitudes of the increase in $g_N$ is greater than that of the decrease in $g_A$. In this case, the growth of research complexity is driven up to a higher rate than usual. It follows that the research productivity $\iota_t$ will fall over time. Hence, $g_A$ and $g_N$ are lowered gradually in a similar manner, inducing the economy to move from point $C''$ to the new balanced-growth path $C''$.

In summary, the long-run growth effect of raising $i$ increases $g_N$ at the expense of reducing $g_A$ regardless of the comparison between $\gamma$ and $\delta$. Nevertheless, the reduction in $g_A$ turns out to be more significant under $\gamma < \delta$ than under $\gamma > \delta$. Consequently, according to lemma 1, the economic growth rate is decreasing in $i$ more considerably under $\gamma < \delta$ than under $\gamma > \delta$.

### 3.3 CIA on Horizontal R&D

In this subsection, we analyze the case in which only a CIA constraint on horizontal R&D is present, and the following result is obtained.

**Proposition 3.** In the presence of a CIA constraint on horizontal R&D only (i.e., $\xi_h > 0$, $\xi_v = \xi_c = 0$), a higher nominal interest rate increases the economic growth rate under both $\gamma < \delta$ and $\gamma > \delta$, but with a larger size under $\gamma > \delta$. 
Fig. 3 illustrates the growth effects of a permanent increase in the nominal interest rate $i$ when the model only features a CIA constraint on horizontal R&D. Again, the analysis starts off by studying the instant effect of raising $i$ from the initial balanced-growth equilibrium.

When horizontal R&D is subject to CIA constraint, the instant effects of raising $i$ are just oppose to those in Subsection 3.2. An increase in $i$ raises the cost for vertical R&D, reducing the firm’s demand for R&D labor on vertical innovation. Given an unchanged labor supply due to the absence of a CIA constraint on consumption, the wage rate decreases, reallocating labors from horizontal R&D $l_h$ to production $l_y$, vertical R&D $l_v$, and leisure.

On the one hand, when $\gamma < \delta$, namely the diminishing returns to vertical R&D are small, more labor force will be reallocated to $l_v$ leading to a larger rise in $g_A$. In Fig. 3, if the gap between $\gamma$ and $\delta$ is small, then the economy would move from point $A$ to $B'$, since the magnitude of the increase in $g_A$ is not significant compared to the magnitude of the decrease in $g_N$, as shown in (27) and (28). By contrast, if the gap between $\gamma$ and $\delta$ is large, the economy would move from point $A$ to $B''$, since the magnitude of the increase in $g_A$ becomes larger than the magnitude of the decrease in $g_N$. On the other hand, when $\gamma > \delta$, namely the diminishing returns to vertical R&D are large, less labor force will be reallocated to $l_v$ leading to a smaller rise in $g_A$. Therefore, the economy the economy would move from point $A$ to $C'$, given that the size of the decrease in $g_N$ is significantly larger than that of the increase in $g_A$.

Now, we turn to intuitively explain the adjustment process. Under $\gamma < \delta$, if the gap between $\gamma$ and $\delta$ is small, the increase in $g_A$ is not significant enough to dominate the decrease in $g_N$. As a result, the research productivity $\iota$ grows over time, driving up both $g_A$ and $g_N$, and therefore the economy moves from point $B'$ to the new balanced-growth path $B$, as displayed in Fig. 3. However, if the gap between $\gamma$ and $\delta$ is large, the increase in $g_A$ is, instead, more likely to dominate the decrease in $g_N$, which drives up the growth of research complexity to a higher rate than usual. As a result, the research productivity falls over time and $g_A$ and $g_N$ are reduced, so the economy moves from point $B''$ to $B$.

Under $\gamma > \delta$, since the magnitude of the decrease in $g_N$ is much larger than that of the increase in $g_A$ as shown in the movement from point $A$ to point $C'$, the growth of research complexity is driven down to a lower rate than usual. It follows immediately that the research productivity $\iota$ rises over time. Hence, $g_A$ and $g_N$ grow gradually in a similar manor, inducing the economy to move from point $C'$ to the new balanced-growth path $C$.

In summary, the long-run growth effect of raising $i$ increases $g_A$ at the expense of reducing $g_N$ regardless of the comparison between $\gamma$ and $\delta$. Nevertheless, the increase in $g_A$ turns out to be more significant under $\gamma < \delta$ than under $\gamma > \delta$. Consequently, according to Lemma 1, the economic growth rate is increasing in $i$ more considerably under $\gamma < \delta$ than under $\gamma > \delta$.

### 3.4 CIA on Consumption and R&D

After building up the intuition underlying each scenario in which only one type of the CIA constraints is present, we are now in a position to analyze a more general case by incorporating all types of CIA constraints into the model. To avoid distraction and highlight the interesting non-monotonic relationship between inflation and growth, our analysis is simplified to focus on the
empirically relevant case where $\gamma < \delta$.\footnote{The analysis of the relationship between inflation and growth with all CIA constraints in the case of $\gamma > \delta$ is available upon request.} Accordingly, we obtain the following result.

**Proposition 4.** Suppose that $\gamma < \delta$ holds. Then (a) for a sufficiently large gap between $\xi_h$ and $\xi_v$, the economic growth rate $g$ has an non-monotonic (i.e., an inverted-U) relationship with the nominal interest rate $i$, and there exists a threshold value $i^*$ below (above) which $g$ is increasing (decreasing) in $i$; (b) For an insufficiently large gap between $\xi_h$ and $\xi_v$, $g$ is monotonically decreasing in $i$.

To intuitively explain the results of Proposition 4, we need to combine the results obtained in Propositions 1-3. Recall that from Propositions 1 and 2, in the case of $\gamma < \delta$, the CIA constraints on both consumption and vertical R&D yields a negative relationship between the nominal interest rate $i$ and the economic growth rate $g$, and only Proposition 3 (i.e., the CIA constraint on horizontal R&D) can generate a positive relationship between them. It is obvious that an inverted-U shape requires a positive relationship between the nominal interest rate and economic growth at the relatively low levels of $i$. This implies that the growth effect of $i$ from the CIA constraint on horizontal R&D has to be relatively strong to dominate the other two effects from the CIA constraints on consumption and vertical R&D. At the initial increase in $i$, the distortions of the CIA constraints are mild, which implies the above three effects are all weak. To ensure a stronger positive effect of the constraint on horizontal R&D at the initial increase in $i$, there must be a sufficiently large extent of the constraint on horizontal R&D relative to vertical R&D (i.e., a sufficiently large gap in $\xi_h > \xi_v$), so that raising $i$ yields a strong reallocation effect from $l_h$ to $l_v$ to generate a high level of $g_A$ to enhance $g$.

Nevertheless, as $i$ increases over time, the diminishing returns to horizontal R&D relative to vertical R&D become large (i.e., $\gamma < \delta$), which tend to weaken the reallocation effect from $l_h$ to $l_v$. Therefore, the negative growth effects from the constraints on vertical R&D and consumption turn to be stronger than the positive effects from the constraint on horizontal R&D. This implies that there will be a threshold rate of nominal interest $i^*$ across which the two negative growth effects play the dominant role, so that $g$ becomes monotonically decreasing in $i$.\footnote{The analysis of the relationship between inflation and growth with all CIA constraints in the case of $\gamma > \delta$ is available upon request.}
Finally, if the gap in $\xi_h > \xi_v$ is not sufficiently large, the reallocation effect of $i$ from the constraint on horizontal R&D is weak at the initial increase in $i$, so it will be dominated by the two negative effects as $i$ rises. Accordingly, it is straightforward that $g$ becomes monotonically decreasing in $i$ for all levels of $i$.

4 Quantitative Analysis

In this section, our model is calibrated to quantify the growth effects of monetary policy. We show that using an empirically plausible range of parameter values, there exists an inverted-U relationship between the nominal interest rate and economic growth in the calibrated economy, which is consistent with the empirical findings. In addition, we evaluate the effects of monetary policy on (steady-state) social welfare.

4.1 Calibration

To make the quantitative analysis more realistic, our model is calibrated to match the aggregate data of the US economy. Our model features the following set of parameters \( \{\rho, \alpha, g_L, \xi_c, \xi_v, \xi_h, \gamma, \delta\} \) and the policy variable $i$. The discount rate is set to a standard value $\rho = 0.02$ as in Grossmann et al. (2013). We follow Jones and Williams (2000) to set the capital share to a standard value such that $\alpha = 0.36$, which is an estimate of the US economy during 1951-2000. According to the Conference Board Total Economy Database, $g_L$ is thereafter set to 1.2% to correspond the population growth rate in the US within this period.

As for the degree of various types of CIA constraints \( \{\xi_c, \xi_v, \xi_h\} \), the degree of the CIA constraint on consumption $\xi_c$ is set to 0.29, which lies in a reasonable range of M1-consumption ratios (see, for example, Chu et al. (2010) and Dotsey and Sarte (2000)). We focus on the case that $\xi_h > \xi_v$ to capture the empirical findings that young and small firms that tend to engage in radical innovation are more constrained by cash/money than their old and large counterparts that conduct incremental innovation (e.g., Akcigit (2009) and Caggese (2015)). Additionally, following Chu et al. (2015), the strengths of CIA constraints on horizontal and vertical innovative activities are set to $\xi_h = 0.6$ and $\xi_v = 0.4$, respectively, as the benchmark values, which features a sufficiently large gap between $\xi_h$ and $\xi_v$. We will also choose $\xi_h = 0.5$ and $\xi_v = 0.4$, and $\xi_h = 0.45$ and $\xi_v = 0.4$ for analysis, corresponding to the case of an insufficiently large gap in the CIA constraints between the two R&D activities.

Next, as for the values of two R&D diminishing returns $\gamma$ and $\delta$, we choose the empirically plausible case $\gamma < \delta$ in the benchmark, as shown in Cohen (2010).\(^{10}\) Specifically, we set $\delta = 0.8$ and $\gamma = 0.6$,\(^{11}\) respectively, both of which lie in the range of the estimated elasticity of innovative outputs with respect to R&D expenditures documented in Acs et al. (1994) and Anselin et al. (1997).\(^{12}\)

\(^{10}\) We examine $\gamma > \delta$ in the sensitivity check to complete the quantitative illustrations.

\(^{11}\) We also conduct an sensitivity analysis by choosing alternative values to show that our basic results are robust to this change.

\(^{12}\) Acs et al. (1994) documents a range of the estimated elasticity of innovation citations with respect to R&D expenditure, which is around [0.55, 0.95]. Anselin et al. (1997) and Acs et al. (2002) estimate this elasticity by using the number of patents as a proxy of new knowledge, resulting in a range of [0.54, 0.85].
Moreover, we follow Jones and Williams (2000) to set the equilibrium rate of economic growth in the benchmark as $g = 1.25\%$. Then the market-level nominal interest rate $i$ is calibrated by targeting at $\pi = 2.5\%$, which is in line with the average inflation rate in the US economy. Given the above calibrated parameter values, we calibrate $\sigma$ and $\theta$ simultaneously to match the equilibrium growth rate and the standard time of employment $l = 0.33$ by using the labor-leisure choice (5), the first-order conditions for the vertical and horizontal R&D (13) and (18), the iso-growth condition (29), the population growth condition (30), and the labor-market-clearing condition. The detailed calibration procedure is relegated to the Appendix. The parameter values are summarized in Table 1.

Table 1: Calibration

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<tr>
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<td>0.0125</td>
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Parameters | $\xi_h$ | $\xi_v$ | $\xi_c$ | $\gamma$ | $\delta$ | $\theta$ | $\sigma$ | $i$ |
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4.2 Growth and Welfare Implications of Monetary Policy

Fig. 4 displays the quantitative result under the benchmark parameter values, wherein we find that the rate of economic growth is an inverted-U function of the inflation rate given $\xi_h - \xi_v = 0.2$. This result supports the implication of Proposition 4; when the difference between $\xi_v$ and $\xi_h$ is relatively large, at the low levels of $i$, the positive growth effect of $i$ through the CIA constraint on horizontal R&D strictly dominates the negative growth effects through the CIA constraints on vertical R&D and consumption. Nevertheless, as $i$ rises, this domination becomes increasingly weaker and finally the negative effects overwhelm the positive one. In addition, the threshold value of $i$ is roughly 2.8\%, which is in line with the empirical estimates of Ghosh and Phillips (1998) (i.e., 2.5\%), López-Villavicencio and Mignon (2011) (i.e., 2.7\%), and Eggoh and Khan (2014) (i.e., 3.4\%).

To explore the welfare effects of monetary policy, we impose balanced growth on (1) to derive...
the steady-state welfare function

\[ U = \frac{1}{\rho} \left[ \ln c_0 + \frac{g}{\rho} + \theta \ln(1 - l) \right] \]  \hspace{1cm} (31)

where the exogenous terms have been dropped and \( c_0 = (1 - \alpha^2)l_y/\Gamma \) is the steady-state level of consumption along the BGP. Fig.5 accordingly depicts the welfare effect of the inflation rate. It is shown that the level welfare is monotonically decreasing in the inflation rate. Specifically, a 10 percentage points increase in the inflation rate, from -4.45% (corresponds to the zero nominal interest rate) to 5.55%, leads to approximately a welfare loss of 0.587%. The intuition can be explained as follows. There are two positive welfare effects of of a higher rate of inflation (or raising the nominal interest rate). The first effect stems from the growth effect for an inflation rate that is below the threshold value, as aforementioned. The second effect comes from the increase in leisure, which leads to a higher utility level. However, given our benchmark parameter values, these two positive welfare effects are completely dominated by the negative welfare effect from the decrease in households’ initial income level. This mainly arises from the CIA constraint on consumption, which reduces labor employment in the final-goods sector and hence the level of \( c_0 \). Furthermore, as the inflation rate increases to a permanently higher rate that is above the threshold, it seems that the positive growth effect turns to be negative, making the overall welfare effect of inflation to be always negative. Therefore, this model predicts a monotonically decreasing relationship between welfare and inflation in the benchmark case.

Nevertheless, some existing empirical studies (e.g., Vaona (2012) and Barro (2013)) also find a long-run negative effect of inflation on economic growth. In fact, given that the majority of the current calibrated values of parameters are preserved, our model is also flexible to generate a negative relationship between inflation and economic growth. Fig.6 illustrates this scenario accordingly. We recalibrate the values of the parameters when the gap between \( \xi_v \) and \( \xi_h \) from 0.2 (i.e., \( \xi_v = 0.4, \xi_h = 0.6 \)) is shrunk to 0.1 (i.e., \( \xi_v = 0.4, \xi_h = 0.5 \)). It is found that the inflation-growth relationship turns to be strictly negative, which is still consistent with the predictions of the analytical part. The welfare is also decreasing in the increase of inflation rate, as shown in Fig.7. There is approximately 0.180% of welfare loss when the inflation rate is increased by 10 percentage points (from -4.45% to 5.55%), and a larger welfare loss (i.e., 0.592%) is thereafter attained when the inflation rate continues to increase from 5.55% to 15.55%.

Moreover, if the the gap between \( \xi_v \) and \( \xi_h \) is shrunk to an even smaller value of 0.05 (i.e., \( \xi_v = 0.4, \xi_h = 0.45 \)), Fig.8 shows that the monotonically decreasing relationship between inflation...
rate and economic growth rate still holds. Interestingly, a higher inflation rate generates an inverted-U shaped effect on welfare in this case (see Fig.9). As the inflation rate becomes higher, the positive welfare effect for the increase in leisure will initially dominate the negative welfare effects through a lower economic growth and a lower level of consumption, but the domination is reversed as the inflation rate continues to increase. It in turn implies that the Friedman rule, which is optimal in the aforementioned cases, becomes suboptimal. Accordingly, our model predicts a welfare-maximizing inflation rate of 1.7% in this case.

4.3 Sensitivity

In this subsection, we undertake sensitivity checks to test the robustness of our numerical results in terms of quantitative magnitudes. Specifically, this sensitivity exercise is conducted by varying several key parameters. The parameter values that will be altered are summarized in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\xi_h$</th>
<th>$\xi_v$</th>
<th>$\xi_c$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
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First, we perform a sensitivity analysis by examining the analytical results in three special scenarios, namely $\xi_c > 0(\xi_v = \xi_h = 0)$, $\xi_v > 0(\xi_c = \xi_h = 0)$, and $\xi_h > 0(\xi_c = \xi_v = 0)$, respectively. Fig.10, Fig.12, and Fig.14 illustrate that the inflation rate and economic growth rate are negatively correlated in the cases of the CIA constraint on only consumption and on only vertical R&D, but positively in the case of the CIA constraint on only horizontal R&D. These quantitative results are consistent with our analytical implications in Propositions 1-3. In addition, the corresponding effect of a higher inflation rate on social welfare is depicted in Fig.11, Fig.13, and Fig.15, respectively.
That is, the inflation-welfare relationship is inverted-U-shaped under $\xi_c > 0 (\xi_v = \xi_h = 0)$, positive under $\xi_v > 0 (\xi_c = \xi_h = 0)$, and negative under $\xi_h > 0 (\xi_c = \xi_v = 0)$, respectively.

Second, a sensitivity analysis is performed by changing the value of parameter $\xi_c$ from 0.29 in the benchmark to 0.26. Comparing Fig. 4 and Fig. 16, a lower degree of $\xi_c$ shifts the inflation-growth curve to the right, implying a higher threshold value of inflation $i^*$. A smaller $\xi_c$ weakens the negative inflation-growth effect arising from the CIA constraint on consumption, as shown in Proposition 1. Therefore, for a given level of $\xi_v$ and of $\xi_h$, which, respectively, determines the negative inflation-growth effect and the positive one, a larger increase in the inflation rate (and also the nominal interest rate) is required to make the negative inflation-growth effect sufficiently strong to dominate the positive inflation-growth effect from the CIA constraint on horizontal innovation. As a result, the threshold value $i^*$ increases to around 8%, which is close to the empirical estimates.
in Sarel (1996), Burdekin et al. (2004), and Yilmazkuday (2013) (i.e., 8%). As for social welfare, a 10 percentage points increase in the inflation rate slightly enlarges the welfare loss under a smaller $\xi_c$ (i.e., 0.773%) as compared to the benchmark case (i.e., 0.587%).

Third, our model predicts that under $\delta < \gamma$, the economic growth rate is monotonically increasing in the inflation rate. To capture the impacts of diminishing returns in the two types of innovations, we consider an alternative case in which $\delta > \gamma$, although it is less empirically realistic. As displayed in Fig.18, the quantitative result is consistent with the prediction of the model; an increase in the nominal interest rate (and then an increase in the inflation rate) monotonically raises the economic growth rate. As illustrated in Proposition 1, $\delta > \gamma$ leads to a positive inflation-growth effect in the presence of the CIA constraint on consumption, and thereby, together with another positive effect from the CIA constraint on horizontal R&D, an increase in the inflation rate always raises the economic growth rate. In addition, the inflation rate is monotonically increasing in social welfare (see Fig.19). In particular, raising the inflation rate by 10 percentage points from -4.45% to 5.55% yields a welfare gain of 3.889%. In this case, the positive welfare effect, stemming from the positive growth effect under a higher inflation rate, reinforces another positive welfare effect from a higher leisure to strictly dominate the negative welfare effect due to a lower initial income level. Therefore, a higher inflation rate results in welfare gains.

Forth, we perform a sensitivity analysis by altering the values for $\gamma$ and $\delta$ to $\gamma = 0.5$ and $\delta = 0.75$, respectively, while other parameters remain unchanged. In this case, the growth rate is monotonically decreasing in the inflation rate (see Fig.20). Moreover, the welfare level is an inverted-U function of the inflation rate (see Fig.21), in which the welfare-maximizing rate of inflation is roughly 2%, implying that the Friedman rule is suboptimal.
Finally, we consider a case in which the gap between $\xi_v$ and $\xi_h$ is enlarged from 0.2 to 0.3 by singly raising $\xi_h = 0.7$, while other parameters remain unchanged. The purpose of this experiment is to show that the inverted-U relationship between inflation and economic growth rate is robust to the changes in the two parameters that govern the extent of diminishing returns. Fig. 22 displays that the inverted-U relationship between inflation and economic growth still holds, which is similar to Fig. 4. A comparison of Fig. 20 and Fig. 22 shows that a larger $\xi_h$ strengthens the positive inflation-growth effect arising from the CIA constraint on horizontal R&D, which in turn makes possible the positive inflation-growth effect to initially dominate the negative inflation-growth effects. As explained in the benchmark case, the negative inflation-growth effects from the CIA constraints on consumption and vertical R&D eventually dominate the positive effect as the inflation rate becomes higher. Thus, the inverted-U pattern for inflation and economic growth is generated, and the threshold value in this case is approximately 1.8%, which is close to the empirical estimates in Khan and Senhadji (2001) (i.e., 1-3%) and Omay and Kan (2010) (i.e., 2%). In addition, social welfare is also monotonically decreasing in the inflation rate as shown in Fig. 23. For example, a 10 percentage points increase in the inflation rate, from -4.45% to 5.55%, leads to a welfare loss of roughly 0.215%, which is smaller as compared to the benchmark case (i.e., 0.587%).

5 Conclusion

In this study, we explore the effects of monetary policy on economic growth and social welfare in an endogenous growth model that include two dimensions of innovations through vertical R&D and horizontal R&D, and cash-in-advance constraints on consumption and two R&D sectors are
incorporated in the setting. The novel contribution of this study is that our model is flexible enough to generate a mixed (i.e., monotonically decreasing or an inverted-U) effect of monetary policy on economic growth, depending on the weights of two innovations on growth, the relative entry cost, and the comparison in the diminishing returns to two innovations. Therefore, our analysis differs from the existing literature that adopts a similar approach but mainly predicts a monotonic nexus between inflation and long-run economic growth: either negative or positive.

However, Chu et al. (2017) is one exception in this strand of literature, who also find an inverted-U relationship between inflation and growth in a canonical Schumpeterian growth model featuring random quality improvement. Our results complement theirs in several aspects. First, their framework only considers vertical innovations, whereas our model considers vertical innovations in addition to horizontal innovations, and these two types of innovations are shown to play very different roles in explaining the impact of monetary policy on economic growth. Second, the model in Chu et al. (2017) removes scale effects by normalizing the number of population, whereas our model is made to be scale invariant by taking into account the increasing complexity of research. Furthermore, when taking elastic labor supply into consideration, the result of an inverted-U relationship between inflation and growth does not hold in Chu et al. (2017), whereas our model is still able to produce this result with more realistic setting. Finally, our quantitative analysis reveals that the welfare-maximizing rate of inflation is around 2.8%, which is more complied with recent empirical estimates as compared to Chu et al. (2017).
Appendix

A.1. Proofs of Proposition 1, 2, and 3.

To analytically prove these propositions, first, we follow Segerstrom (2000) to establish the mutual R&D condition. This condition is derived from the no-arbitrage conditions (14) and (19) for vertical R&D and horizontal R&D. As for the expected profit of each successful vertical innovator, substituting (11) into (14) yields

\[ \Pi_{vt} = \int_t^{\infty} e^{-t} \int_t^{\infty} (r + \phi_s)ds \pi_{te}dt \times \frac{\alpha(1 - \alpha)L_{yt}A_t^{1-\alpha}}{\rho + g_L + \left(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}\right)gA}. \]  

(A.1.1)

Thus, the two R&D conditions are, respectively,

\[ \frac{\delta \Gamma \alpha \lambda_y t}{\rho + g_L + \left(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}\right)gA} l_v^{\delta-1} = 1 + \xi_{vi}, \]  

(A.1.2)

and

\[ \frac{\gamma \alpha \lambda_y t}{\rho + g_L + \left(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}\right)gA} l_h^{\gamma-1} = 1 + \xi_{hi}. \]  

(A.1.3)

Combining (A.1.3) and (A.1.2) yields the mutual R&D condition such that

\[ \frac{\delta \Gamma l_v^{\delta-1}}{1 + \xi_{vi}} = \frac{\gamma l_h^{\gamma-1}}{1 + \xi_{hi}}. \]  

(A.1.4)

Also, using (27) and (28), (A.1.4) can be re-expressed as a relationship with two growth rates such that

\[ g_N = \sigma \Omega \frac{1}{\gamma \xi_{vi}} l_v^{\frac{1}{\gamma} - \delta} gA, \]  

(A.1.5)

where \( \Omega = \frac{1 + \xi_{vi}}{\gamma \xi_{vi}} \Psi \), and \( \Psi = \frac{\alpha}{\gamma} \). Plugging (24), (26), and \( c_t = C_t/L_t \) into the individuals’ consumption-leisure condition (5) yields the relationship between leisure and the production labor such that

\[ l = 1 - \theta (1 + \alpha)(1 + \xi_{ci})l_y. \]  

(A.1.6)

Then, using (A.1.4), (A.1.6), and the labor-market-clearing condition \( l_y + l_v + l_h = l \) yields

\[ l_y = \frac{1 - l_v - \Omega \frac{1}{\gamma - \delta} l_v^{\frac{1}{\gamma} - \delta}}{\Upsilon}, \]  

(A.1.7)

where \( \Upsilon = 1 + \theta (1 + \alpha)(1 + \xi_{ci}) \). Substituting (A.1.7) into (A.1.3) yields the general R&D condition:

\[ gA \left\{ \frac{1 - l_v}{(1 + \xi_{vi})l_v} - \Omega \frac{1}{\gamma - \delta} l_v^{\frac{1}{\gamma} - \delta} \frac{1 + \sigma \left(\frac{1}{1-\alpha} - 1\right)}{\Gamma \delta \alpha} \right\} = \frac{\sigma \Upsilon (\rho + g_L)}{\Gamma \delta \alpha}. \]  

(A.1.8)
In addition, substituting (A.1.6) into the population-growth condition (30) yields

\[ g_L = \left( 1 + \sigma \Omega^{\frac{\gamma}{\gamma - 1} l_v^{\frac{\gamma - \delta}{\gamma - 1}}} \right) g_A. \]  

(A.1.9)

Consequently, (A.1.8) and (A.1.9) represent a system of two equations with two unknowns \((l_v, g_A)\), which can be solved for a balanced-growth equilibrium.

**Lemma 2.** The model has a unique balanced-growth equilibrium. In the equilibrium with a CIA constraint on consumption only, a permanent increase in the nominal interest rate \(i\) (a) decreases the fraction of labor allocated to vertical R&D \(l_v\) and increases the long-run product-quality growth rate \(g_A\) if \(\gamma > \delta\), and (b) decreases \(l_v\) and \(g_A\) if \(\gamma < \delta\).

**Proof.** Imposing \(\xi_v = \xi_h = 0\) reduces (A.1.5), (A.1.8) and (A.1.9) to

\[ g_N = \sigma \Psi \frac{\gamma}{\gamma - 1} l_v^{\frac{\gamma - \delta}{\gamma - 1}} g_A, \]  

(A.1.10)

\[ g_A \left\{ \frac{1 - l_v}{l_v} - \Psi \frac{1}{\gamma - 1} l_v^{\frac{\gamma - \delta}{\gamma - 1}} - \frac{\Upsilon \left[ 1 + \sigma \left( \frac{1}{1 - \alpha} - 1 \right) \right]}{\Gamma \delta \alpha} \right\} = \frac{\sigma \Upsilon (\rho + g_L)}{\Gamma \delta \alpha}, \]  

(A.1.11)

and

\[ g_L = \left( 1 + \sigma \Psi \frac{\gamma}{\gamma - 1} l_v^{\frac{\gamma - \delta}{\gamma - 1}} \right) g_A, \]  

(A.1.12)

respectively. The last two equations are graphed in Fig.24 assuming that \(\gamma > \delta\). The curve for the R&D condition (A.1.11) is unambiguously upward sloping and goes through the origin, whereas the curve for the population-growth condition (A.1.12) is unambiguously downward sloping and has a strictly positive vertical intercept. As illustrated in Fig.24, there is a unique intersection of these two curves at point \(A\), which pins down the balanced-growth equilibrium values of \(l_v\) and \(g_A\). With these values determined, (A.1.10) pins down \(g_N\), (27) pins down \(\iota\), and thereby (28) pins down \(l_h\). Thus, the model has a unique balanced-growth equilibrium for \(\gamma > \delta\).

The effect of permanently increasing the nominal interest rate \(i\) is illustrated in Fig.24 by the movement from point \(A\) to point \(B\). An increase in \(i\) unambiguously causes the curve for the R&D condition (A.1.11) to shift up, whereas it has no effect on the curve for the population-growth condition (A.1.12). Thus, a higher nominal interest rate decreases \(l_v\) and increases \(g_A\) if \(\gamma > \delta\).

Equations (A.1.11) and (A.1.12) are graphed in Fig.25 assuming that \(\gamma < \delta\). For \(\gamma < \delta\), the slope of the curve for the population-growth condition turns to be positive because a higher \(l_v\) is correlated with a higher \(g_A\), whereas the positiveness of the slope of the curve for the R&D condition remains unchanged. Again, there is a unique intersection of these two curves at point \(A\), which pins down the balanced-growth equilibrium values of \(l_v\) and \(g_A\) in addition to other variables. Thus, the model also has a unique balanced-growth equilibrium if \(\gamma < \delta\).

The effect of permanently increasing the nominal interest rate \(i\) is illustrated in Fig.25 by the movement from point \(A\) to point \(B\). An increase in \(i\) unambiguously shifts the curve for the R&D condition (A.1.11) upward, whereas it has no effect on the curve for the population-growth condition (A.1.12). Thus, a higher nominal interest rate decreases \(l_v\) and decreases \(g_A\) if \(\gamma < \delta\).

**Proof of Proposition 1.** Based on the above results, we now proceed to the analysis of the overall
effects of monetary policies on $g_A$ and $g_N$. In the $(g_A, g_N)$ space, the slope of each iso-growth line (i.e., $1/(1-\alpha)$) exceeds the slope of the population-growth condition (i.e., 1) (in absolute value). The effects of a higher nominal interest rate are illustrated in Fig. 26 accordingly. The mutual R&D condition (given by (A.1.13)) is an upward-sloping line that goes through the origin in the $(g_A, g_N)$ space, when $l_v$ is fixed at the initial equilibrium value. An increase in $i$ shifts down the mutual R&D condition to a new intersection $C$ if $\gamma > \delta$, leading to an increase in $g_A$ as shown in Lemma 2. In contrast, an identical increase in $i$ shifts up the mutual R&D condition to another new intersection $B$ if $\gamma < \delta$, leading to a decrease in $g_A$. Combining (29) with (30), the aggregate economic growth rate is exclusively expressed as the vertical innovation growth rate such that $g = g_L + \left[1/(1-\alpha) - 1\right]g_A$. It then shows that an increase in $i$, which leads to a decrease in $g_A$ for $\gamma < \delta$, decreases the long-run growth rate $g$ (i.e., the movement from $A$ to $B$); whereas an identical increase in $i$, which results in an increase in $g_A$ for $\gamma > \delta$, increases the long-run growth rate $g$ (i.e., the movement from $A$ to $C$).

**Lemma 3.** The model has a unique balanced-growth equilibrium. In the equilibrium with a CIA constraint on vertical R&D only, a permanent increase in $i$ (a) decreases $l_v$ and $g_A$ if $\gamma > \delta$, and
(b) decreases $l_v$ and $g_A$ with a larger magnitude if $\gamma < \delta$.

**Proof.** We make use of $\xi_c = \xi_h = 0$ to reduce (A.1.5), (A.1.8) and (A.1.9) to

\[
g_N = \sigma \Psi^\gamma (1 + \xi_v i)^\gamma l_v^{\gamma - \delta} g_A,
\]

(A.1.13)

\[
g_A \left[ \frac{1 - l_v}{l_v(1 + \xi_v i)} - \Psi^\gamma (1 + \xi_v i)^\gamma l_v^{\gamma - \delta} - \frac{1 + \theta + \theta \alpha (\Gamma - \sigma)}{\Gamma \delta \alpha} \right] = \frac{\sigma(1 + \theta + \theta \alpha)(\rho + g_L)}{\Gamma \delta \alpha},
\]

and

\[
g_L = \left[ 1 + \sigma \Psi^\gamma (1 + \xi_v i)^\gamma l_v^{\gamma - \delta} \right] g_A,
\]

(A.1.14)

respective. Equations (A.1.14) and (A.1.15) are graphed in Fig.27 given $\gamma > \delta$. There is a unique intersection of these two curves at point $A$, which pins down the balanced-growth equilibrium values of all endogenous variables as in the previous case (in which only the CIA constraint on consumption is present). Thus, the model has a unique balanced-growth equilibrium for $\gamma > \delta$. The effect of permanently increasing the nominal interest rate $i$ is illustrated in Fig.27 by the movement from point $A$ to point $B$. An increase in $i$ unambiguously causes the curve for the R&D condition (A.1.14) (the negative sign means that the value of those terms overall decreases as $i$ increases) to shift upward, and unambiguously causes the curve for the population-growth condition (A.1.15) to shift downward. Hence, a higher rate of nominal interest surely decreases $l_v$.

As to determine the effect on $g_A$, first, suppose that for some $\gamma > \delta$, an increase in $i$ increases (or has no effect on) $g_A$. According to (A.1.15), this implies that $(1 + \xi_v i)^\gamma l_v^{\gamma - \delta} / (1 - l_v)$ must decrease (or remain unchanged) when $i$ increases. That is, $[(1 + \xi_v i) l_v]^{-1} l_v^{\delta / \gamma}$ must increase (or remain unchanged). Given that $l_v$ decreases as $i$ increases, $[(1 + \xi_v i) l_v]^{-1}$ must increase in response. Therefore, (A.1.14) implies that $(1 - l_v) / [(1 + \xi_v i) l_v] - \Psi^\gamma (1 + \xi_v i)^\gamma l_v^{\gamma - \delta} / (1 - l_v)$ must increase, and thereby $g_A$ will decrease. This yields a contradiction. As a result, $g_A$ must always decrease in response to an increase in $i$ for $\gamma > \delta$.

![Fig. 27. The effect of a higher nominal interest rate](image1)

![Fig. 28. The effect of a higher nominal interest rate](image2)
Equation (A.1.14) and (A.1.15) for $\gamma < \delta$ are graphed in Fig. 28. There is still a unique intersection of these two curves at point $A$, so the model has a unique balanced-growth equilibrium for $\gamma < \delta$. The effect of permanently increasing $i$ is illustrated in Fig. 28 by the movement from point $A$ to point $B$. An increase in $i$ unambiguously causes the curve for the R&D condition (A.1.14) to shift upward, while it unambiguously shifts the curve for the population-growth condition (A.1.15) downward. Hence, a higher $i$ unambiguously decreases $l_v$. A similar proof applies for the change in $g_A$. The only difference in $\gamma < \delta$ from $\gamma > \delta$ is that a higher $i$ amplifies the negative effect on $g_A$ in the former case, leading to a larger magnitude of the reduction in $g_A$.

![Graph](image)

Fig. 29. The growth effect of a higher $i$ with CIA constraint on vertical R&D.

**Proof of Proposition 2.** The effects of a higher rate of nominal interest on the aggregate rate of economic growth $g$ are displayed in Fig. 29. An increase in $i$ shifts up the line for the mutual R&D condition (given by (A.1.13)), which eventually decreases the vertical R&D growth rate if $\gamma > \delta$ (namely the movement from $A$ to $C$). Also, a higher $i$ continues to shift the line for the mutual R&D condition if $\gamma < \delta$, but the scale becomes larger, implying that the reduction in $g_A$ (namely the movement from $A$ to $B$) is larger. In other words, the overall effect of a higher nominal interest rate is to increase the product-variety growth rate at the expense of the product-quality growth rate, with a larger sacrifice in vertical innovation growth rate when $\gamma < \delta$. Combining (29) with (30), the aggregate economic growth rate is exclusively expressed as the vertical innovation growth rate such that $g = g_L + [1/(1 - \alpha) - 1]g_A$. It states that a movement on the population-growth condition in the northwest direction ($g_N$ increases and $g_A$ decreases) is growth-retarding given $1 < 1/(1 - \alpha)$. Therefore, a larger sacrifice in the product-quality growth rate $g_A$ in the case of $\gamma < \delta$ implies a larger decrease in the aggregate economic growth rate as compared to the case of $\gamma > \delta$.

**Lemma 4.** The model has a unique balanced-growth equilibrium. In the equilibrium with a CIA constraint on horizontal R&D only, a permanent increase in $i$ (a) increases $l_v$ and $g_A$ if $\gamma > \delta$, and (b) increases $l_v$ and $g_A$ with a larger magnitude if $\gamma < \delta$.

**Proof.** In an analogous fashion of the proof of Lemma 3, imposing $\xi_c = \xi_v = 0$ reduces (A.1.5),
(A.1.8) and (A.1.9) to
\[ g_A = \frac{1}{l_v} \left[ 1 - \Psi \frac{1}{\gamma}(1 + \xi_h)^{-\gamma}(1 - v)^{-\gamma} l_v^{\frac{\gamma-\delta}{\gamma}} g_A. \right] \quad (A.1.16) \\
\text{and}
\[ g_L = \left[ 1 + \sigma \Psi \frac{1}{\gamma}(1 + \xi_h)^{-\gamma}(1 - v)^{-\gamma} l_v^{\frac{\gamma-\delta}{\gamma}} \right] g_A, \quad (A.1.18) \]
respectively. Equations (A.1.17) and (A.1.18) are graphed in Fig.30 given \( \gamma > \delta \). There is a unique intersection of these two curves at point A, which pins down the balanced-growth equilibrium values of all endogenous variables. Thus, the model also has a unique balanced-growth equilibrium for \( \gamma > \delta \). The effect of permanently increasing the nominal interest rate \( i \) is illustrated in Fig.30 by the movement from point A to point B. An increase in \( i \) unambiguously causes the curve for the R&D condition (A.1.17) to shift downward, and it unambiguously causes the curve for the population-growth condition (A.1.18) to shift upward. Hence, a higher \( i \) surely increases \( l_v \).

As to determine the effect on \( g_A \), first, suppose that for some \( \gamma > \delta \), an increase in \( i \) decreases (or does not change) \( g_A \). Then, (A.1.18) implies that \( (1 + \xi_h)^{-\gamma/(1-\gamma)} l_v^{\gamma-\delta/(1-\gamma)} \) increases (or remains unchanged) when \( i \) increases, from which it follows that \( [(1 + \xi_h)^{-\gamma/(1-\gamma)} l_v^{\gamma-\delta/(1-\gamma)} \) increases (or remains unchanged). Since \( l_v \) increases in response to an increase in \( i \), \( [(1 + \xi_h)^{-\gamma/(1-\gamma)} l_v^{\gamma-\delta/(1-\gamma)} \) increases and \( [(1 + \xi_h)^{-\gamma/(1-\gamma)} l_v^{\gamma-\delta/(1-\gamma)} \) decreases. According to (A.1.17), \( \frac{1}{l_v} - \Psi \frac{1}{\gamma}(1 + \xi_h)^{-\gamma}(1 - v)^{-\gamma} l_v^{\frac{\gamma-\delta}{\gamma}} \]
\( \left[ (1 + \xi_h)^{-\gamma}(1 - v)^{-\gamma} l_v^{\frac{\gamma-\delta}{\gamma}} \right] \) must decrease and \( g_A \) must increase. This yields a contradiction. As a result, \( g_A \) must always increase in response to an increase in \( i \) for \( \gamma > \delta \).

\[ \text{Fig. 30. The effect of a higher nominal interest rate Fig. 31. The effect of a higher nominal interest rate for } \gamma > \delta. \]

Equation (A.1.17) and (A.1.18) for \( \gamma < \delta \) are graphed in Fig.31. There is also a unique intersection of these two curves at point A, and the model has a unique balanced-growth equilibrium for \( \gamma < \delta \). The effect of permanently increasing the nominal interest rate \( i \) is illustrated in Fig.31 by the movement from point A to point B. An increase in \( i \) unambiguously causes the curve for the R&D condition (A.1.17) to shift downward, whereas it unambiguously shifts the curve for the
population-growth condition (A.1.18) upward. Thus, a higher $i$ unambiguously increases $l_v$. A similar proof applies for the change in $g_A$. The difference in $\gamma < \delta$ from $\gamma > \delta$ is that a higher $i$ amplifies the positive effect on $g_A$ in the former case, leading to a larger magnitude of the increase in $g_A$.

**Proof of Proposition 3.** The effects of a higher rate of nominal interest on the aggregate rate of economic growth $g$ are displayed in Fig. 32. An increase in the nominal interest rate $i$ shifts down the line for mutual R&D condition (given by (A.1.16)), which eventually increases the vertical R&D growth rate if $\gamma > \delta$ (namely the movement from $A$ to $C$). Also, a higher $i$ shifts down the line for the mutual R&D condition if $\gamma < \delta$, but the scale becomes larger, which implies that the increase in $g_A$ is larger (namely the movement from $A$ to $B$). In other words, the overall effect of a higher nominal interest rate is to increase the product-quality growth rate at the expense of the product-variety growth rate, with a larger sacrifice in horizontal innovation growth rate when $\gamma < \delta$. A combination of (29) and (30) yields $g = g_L + [1/(1 - \alpha) - 1] g_A$, which implies that a movement on the population-growth condition in the southeast direction ($g_A$ increases and $g_N$ decreases) is growth-promoting given $1 < 1/(1 - \alpha)$. Therefore, a larger sacrifice in the product-variety growth rate $g_N$ in the case of $\gamma < \delta$ implies a larger increase in the aggregate economic growth rate as compared to the case of $\gamma > \delta$.

![Fig. 32](image-url). The growth effect of a higher $i$ with CIA constraint on horizontal R&D.

**A.2. Proof of Proposition 4**

To prove Proposition 4, the model is solved in a slightly different way. Given the equation (A.1.7), equation (A.1.2) is used to set up another correlation between $l_{yt}$ and $l_{vt}$. To do this, $\iota$ needs to be eliminated. Rewriting the economic growth rate solely as the vertical innovation growth rate by combining (29) and (30) yields

$$g = g_L + \left( \frac{1}{1 - \alpha} - 1 \right) g_A.$$
Together with \( g_A = \sigma \lambda l^\delta_t \) and \( g_N = \lambda l^\gamma_{ht} \), we obtain
\[
g_L = i\lambda \left( \sigma l^\delta_t + \Omega^{\gamma / \gamma - 1} l_L^{\gamma - 1} \right). \tag{A.2.1}
\]

Then, we reduce \( \iota \) in (A.1.2) by making use of (A.2.1) to derive \( l_y \) as
\[
l_y = \frac{(1 + \xi v i) [\rho + g_L + (\frac{1}{1 - \alpha} - 1 + \frac{1}{\sigma}) g_A]}{\delta \alpha \Gamma \lambda t} l_v^{1 - \delta}
= \frac{(1 + \xi v i) (\rho + g_L) [\sigma l^\delta_t + \lambda \Omega^{\gamma / \gamma - 1} l_L^{\gamma - 1}]}{\delta \alpha \Gamma \lambda l} l_v^{1 - \delta} + \frac{(1 + \xi v i) (\frac{1}{1 - \alpha} - 1 + \frac{1}{\sigma}) \sigma l^\delta_t l_v^{1 - \delta}}{\delta \alpha \Gamma \lambda t} \tag{A.2.2}
= (1 + \xi v i) \left( \Theta l_v + \Lambda \Omega^{\gamma / \gamma - 1} l_L^{\gamma - 1} \right)
\]
where \( \Theta = \frac{\rho \sigma + g_L \Gamma}{\delta \alpha \Gamma g_L} \), and \( \Lambda = \frac{\rho + g_L}{\delta \alpha \Gamma g_L} \). By plugging (A.2.2) into (A.1.4), the labor-market-clearing condition can be rewritten as
\[
l_v [\gamma \Theta (1 + \xi v i) + 1] + \Omega^{\gamma / \gamma - 1} l_v^{1 - \delta} [\gamma \Lambda (1 + \xi v i) + \Omega^{-1}] = 1. \tag{A.2.3}
\]

Finally, to find the relationship between \( i \) and \( g \), we need to derive a function of \( g \) on \( l_v \). By combining (29) with (30) and using the expression of \( i \), we obtain
\[
g = g_L \left[ 1 + \left( \frac{1}{1 - \alpha} - 1 \right) \frac{\sigma}{\sigma + \Omega^{\gamma / \gamma - 1} l_v^{\gamma - 1}} \right]. \tag{A.2.4}
\]

Differentiating \( g \) with respect to \( i \) yields
\[
\frac{\partial g}{\partial i} = \frac{-g_L (\frac{1}{1 - \alpha} - 1) \sigma}{\left( \sigma + \Omega^{\gamma / \gamma - 1} l_v^{\gamma - 1} \right)^2} \left\{ \frac{\gamma}{\gamma - 1} \Omega^{\gamma / \gamma - 1} \frac{\partial \Omega}{\partial l_v} l_v^{\gamma - 1} - \frac{\delta}{\gamma - 1} \frac{\partial l_v}{\partial i} l_v^{\gamma - 1} \right\}
= \frac{\sigma g_L (\frac{1}{1 - \alpha} - 1) \Omega^{\gamma / \gamma - 1} l_v^{\gamma - 1} l_v^{\gamma - 1}}{(1 - \gamma) \left( \sigma + \Omega^{\gamma / \gamma - 1} l_v^{\gamma - 1} \right)^2} \left\{ \gamma \Psi \frac{\xi h - \xi v}{(1 + \xi v i)} l_v^{1 - \delta} + (\delta - \gamma) \Psi \frac{1 + \xi v i}{(1 + \xi v i)} \right\} \tag{A.2.5}
= \frac{\sigma \delta g_L \Omega^{\gamma / \gamma - 1} l_v^{\gamma - 1} \left( \frac{1}{1 - \alpha} - 1 \right)}{(1 - \gamma)(1 + \xi v i)^2 \left( \sigma + \Omega^{\gamma / \gamma - 1} l_v^{\gamma - 1} \right)^2} \left\{ (\xi h - \xi v) + (\delta - \gamma)(1 + \xi v i)(1 + \xi v i) \frac{\partial l_v}{\partial \gamma l_v} \right\}.
\]

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Therefore, the sign of $\partial g/\partial i$ depends on the sign of $[(\xi_h - \xi_v) + (\delta - \gamma)(1 + \xi_v i)(1 + \xi_h i)\partial l_v/\partial i]$. Differentiating (A.2.3) with respect to $i$ to derive $\partial l_v/\partial i$ (note that $\Psi, \Theta, \text{and } \Lambda$ are unrelated to $i$) yields

$$
\left\{\begin{array}{l}
[\gamma \Theta (1 + \xi_v i) + 1] + \frac{1 - \delta}{1 - \gamma} \Omega^{-\gamma} l_v^{\frac{\gamma - \delta}{\gamma - 1}} [\gamma \Lambda (1 + \xi_v i) + \Omega^{-1}] \end{array}\right\} \frac{\partial l_v}{\partial i} \\
\chi_1 > 0
$$

$$
= \left\{\begin{array}{l}
(\xi_h - \xi_v) \left[\frac{\gamma \Lambda + \gamma \Psi^{-1} (1 + \xi_h i) + \frac{1}{\Psi(1 + \xi_h i)^2}}{(1 - \gamma)(1 + \xi_h i)} - \frac{\Lambda [\theta \xi_e (1 + \alpha)(1 + \xi_v i) + \gamma \Psi]}{\chi_3 > 0} \right] \Omega^\gamma l_v^{\frac{\gamma - \delta}{\gamma - 1}} \\
- \frac{\Theta [\theta \xi_e (1 + \alpha)(1 + \xi_v i) + \gamma \Psi]}{\chi_4 > 0} l_v
\end{array}\right\}
$$

(A.2.6)

It is apparent that $\chi_2$ is monotonically decreasing in $i$, whereas $\chi_3$ and $\chi_4$ are monotonically increasing in $i$. Thus, for a positive $\chi_1$, $\partial l_v/\partial i$ monotonically decreases as $i$ increases. Therefore, $\partial g/\partial i$ eventually goes to negative (positive) on the condition of $\gamma < (>)\delta$. To see whether $\partial g/\partial i > 0$, one can substitute (A.2.6) into $[(\xi_h - \xi_v) + (\delta - \gamma)(1 + \xi_v i)(1 + \xi_h i)\partial l_v/\partial i]$ to show that

$$
\left(\frac{\partial g}{\partial i}\right)_{i=0} > 0
$$

$$
\Leftrightarrow (\xi_h - \xi_v) + (\delta - \gamma) \left\{\frac{\Psi^{\gamma} l_v^{\frac{\gamma - \delta}{\gamma - 1}}}{\gamma \chi_1} \left[(\xi_h - \xi_v) \chi_2 - \chi_3\right] - \frac{\chi_4}{\gamma \chi_1}\right\}_{i=0} > 0
$$

(A.2.7)

$$
\Leftrightarrow (\xi_h - \xi_v) > \left\{\frac{(\delta - \gamma) \left(\frac{\chi_4 + \chi_3 \Psi^{\frac{\gamma - \delta}{\gamma - 1}}}{\chi_2 \Psi^{\gamma} l_v^{\frac{\gamma - \delta}{\gamma - 1}}}\right)}{\gamma \chi_1 + (\delta - \gamma) \chi_2 \Psi^{\frac{\gamma - \delta}{\gamma - 1}}}\right\}_{i=0} > 0,
$$

where $l_v$ is determined in the implicit function (A.2.3) evaluated at $i = 0$. Accordingly, a large $(\xi_h - \xi_v)$ is a sufficient and necessary condition for a local maximum of function $g(i)$. It in turn implies that as $i$ increases, $g$ increases when $i < i^*$ and decreases when $i > i^*$, where $i^*$ can be solved by

$$
(\xi_h - \xi_v) = \frac{(\delta - \gamma) \left(\frac{\chi_4 + \chi_3 \Omega^{\gamma} l_v^{\frac{\gamma - \delta}{\gamma - 1}}}{\gamma \chi_1 + (\delta - \gamma) \chi_2 \Omega^{\gamma} l_v^{\frac{\gamma - \delta}{\gamma - 1}}}\right)}{\gamma \chi_1 + (\delta - \gamma) \chi_2 \Omega^{\gamma} l_v^{\frac{\gamma - \delta}{\gamma - 1}}}.
$$

(A.2.8)

In addition, if $\gamma > \delta$, the condition (A.2.7) surely holds, because $(\xi_h - \xi_v) > 0$ while the RHS of (A.2.7) turns to be negative. Therefore, $(\partial g/\partial i)_{i=0} > 0$. However, when $\gamma > \delta$, $\partial g/\partial i$ remains positive as $i$ increases. Therefore, under the condition of $\gamma > \delta$, $g$ is monotonically increasing in $i$. 

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A.3. Calibration Strategy

In this section, given the values of all other parameters, we illustrate the strategy that is used for calibrating $\sigma$ and $\theta$ simultaneously to match the growth rate of 1.25% and the standard time of employment $l = 0.33$. Using the individual’s optimal decision on the labor-leisure choice (5), the equilibrium final-goods production function (24), the per-capita consumption share of outputs (25), the production-labor share of outputs (26), and $c_t = C_t/L_t$, we obtain

$$l = 1 - \theta(1 + \alpha)(1 + \xi c i l y).$$

Together with (A.2.2), we derive the first equation for calibration such that

$$l = 1 - \theta(1 + \alpha)(1 + \xi c i)(1 + \xi v i)(1 + \xi v i)\left\{\Theta l_v + \Lambda \Omega \frac{1}{\nu - \gamma}\right\}$$

Given all other parameters, there are three unknowns $\{\sigma, \theta, l_v\}$, and another two equations are needed for solution, which are (A.2.3) and (A.2.4), namely

$$l_v[\nu \Theta (1 + \xi v i) + 1] + \Omega \frac{1}{\nu - \gamma} l_v^{\gamma - 1} [\nu \Lambda (1 + \xi v i) + \Omega^{-1}] = 1$$

and

$$g = g_L \left[1 + \left(\frac{1 + \alpha}{\nu - \gamma} - 1\right)\frac{\sigma}{\sigma + \Omega \frac{1}{\nu - \gamma} l_v^{\gamma - 1}}\right].$$

Finally, we have three equations to pin down the above three unknowns.

References


