Abstract

This paper analyzes the effects of intellectual property rights (IPR) protection on innovation and technology transfer in a North-South quality-ladder model with innovative Northern R&D and adaptive Southern R&D. The degree of IPR protection in two countries differs in terms of patent breadth, which determines the markups of Northern firms and their Southern affiliates, respectively. In this model, stronger IPR protection in the South leads to a permanent decrease in the North-South wage gap, a temporary increase in the Northern innovation rate, and ambiguous effects on technology transfer. By contrast, stronger IPR protection in the North leads to a permanent increase in the North-South wage gap, ambiguous effects on the Northern innovation rate, and a permanent decrease in technology transfer. Finally, we perform a quantitative analysis by calibrating the model to the US-China data, and the numerical results support these policy implications.
1 Introduction

The relationship between intellectual property rights (IPR) protection in developing countries (i.e., the South) and the incentives of developed countries (i.e., the North) to transfer technologies has been a fundamental question in the literature on multinational firms and international trade. This relationship has become even more important since the Trade-Related Intellectual Property Rights (TRIPS) Agreement of the World Trade Organization (WTO), which was signed by the WTO members in 1994 to raise the level of IPR protection around the world, especially in developing countries. So far, the theoretical and empirical conclusions about the impacts of Southern IPR protection on international technology transfer are mixed. For example, the North-South models by Glass and Saggi (2002) and Glass and Wu (2007) show that stronger IPR protection in the South unambiguously reduces the rate of technology transfer, and the empirical analysis of Mayer and Pfister (2001) and Pfister and Defains (2005) finds a negative effect of stronger patent rights on location decisions of French multinationals. However, the implication of North-South models by Helpman (1993), Lai (1998), and Branstetter and Saggi (2011) is consistent with the observation in Lee and Mansfield (1996), Nunnenkamp and Spatz (2004), and Branstetter et al. (2006), such that the increase in foreign direct investment (FDI) by US multinationals results from stronger IPR protection in developing countries. Therefore, this study attempts to reexamine how a strengthening of IPR protection in developing countries affects technology transfer within multinational firms in terms of FDI in order to reconcile the above inconclusive results.

In addition, most existing studies in multinational firms and technology transfer mainly focus on the role of stronger IPR protection in the South. Nevertheless, using the patent rights protection index constructed by Ginarte and Park (1997) and Park (2008), Dinopoulos and Kottaridi (2008) report that during the period 1960-2000, the degree of IPR protection increased significantly not only in developing countries (on average by 70%) but also in developed countries (on average by 50%). In the North-South model setting, stronger IPR protection in the North changes the degree of protection of their intellectual assets, as reflected by the value of patents. This tends to alter the incentives of Northern firms to conduct research and development (R&D) and thus generates a reallocation effect on the resources between production and R&D in the North. This resource reallocation affects the amounts of production shifted from the North to the South and the rate of technology transfer accompanied with it. Consequently, to fully consider the decision of Northern parent firms on innovation and the decision on technology transfer to their Southern affiliates in a more realistic environment, the (long-run) effect of stronger IPR protection in the North should also be taken into account.

To properly address the above issues, this study develops a North-South quality-ladder model with semi-endogenous growth that features innovative R&D in the North and adaptive R&D in

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1See Park (2012) for a detailed survey.
2The types of technology transfer from developed countries to developing countries can be various, such as s FDI, licensing, and illegal imitation. In particular, inward FDI is one of the main modes that becomes increasingly important in developing economies. FDI data from UNCTAD World Investment Report indicate that FDI inflows and inward FDI stock in developing economies grew at an annual rate of about 11.60% and 11.81%, respectively, from 1990 to 2017.
3See, for example, Dinopoulos and Segerstrom (2010), Iwaisako et al. (2011), and Gustafsson and Segerstrom (2011).
4For example, Park (2008) show that from 1960 to 2000, the Ginarte-Park index increase from 3.86 to 4.88 in the US, from 2.85 to 4.67 in Japan, from 3.20 to 4.54 in the UK, and from 2.33 to 4.50 in Germany.
the South to theoretically and quantitatively analyze the cross-country effects of IPR protection on innovation and international technology transfer. Specifically, in this model, Northern firms engage in innovative R&D to develop new higher-quality products, and to increase profit flows, they (in the form of multinational firms) invest in adaptive R&D to transfer their manufacturing of these products from the high-wage North to the low-wage South. Moreover, to model IPR protection, the analysis in this study focuses on the use of the policy instrument: *patent breadth*, in the North and in the South, respectively. The level of patent breadth captures the degree of protection for the state-of-art technology holders against potential imitations, which determines the monopolistic markups charged by multinational firms and the amount of profits generated by the technology in the two regions. Within this open-economy dynamic general equilibrium framework, we derive the following results.

Stronger patent protection in the South leads to a permanent decrease in the North-South wage gap, a temporary increase in the rate of Northern innovation, and an ambiguous effect on the rate of technology transfer from the North to the South depending on the relative size of the two economies.\(^5\) Intuitively, a larger patent breadth in the South raises the cost of imitation, which generates more market power to Southern firms by allowing them to charge a higher markup. Hence, the incentives for relocating manufacturing operations to Southern firms through adaptive R&D increase, yielding a higher demand for R&D labor in the South. As a result, the wage rate in the South rises relative to the North. Furthermore, given that stronger Southern patent protection decreases the North-South relative wage rate, the profit margin of Northern quality leaders also increases, which raises the incentives for innovative R&D. Therefore, there is a labor reallocation from production to R&D in the North, which in turn increases the rate of Northern innovation but only temporarily since the model has the semi-endogenous-growth property. As for the impact on the rate of international technology transfer, there are two opposing effects. On the one hand, a larger Southern patent breadth increases the average product quality through a higher rate of Northern innovation, which raises the difficulty of adaptive R&D under the semi-endogenous-growth setting. This is the negative effect on international technology transfer. On the other hand, a larger Southern patent breadth increases the level of Southern R&D labor through a higher value of Southern firms. This is the positive effect on international technology transfer via FDI, and this effect becomes stronger as the Southern labor force is larger. Consequently, there exists a threshold on the Southern population size above (below) which the overall effect of stronger patent protection in the South on technology transfer would be positive (negative).

Stronger patent protection in the North leads to a permanent increase in the North-South wage gap, a permanent decrease in the rate of international technology transfer, and an ambiguous effect on the rate of innovation in the North depending also on the relative size of the two economies.\(^6\) Intuitively, a larger patent breadth in the North increases the profit margin of Northern firms through a larger markup, which decreases the incentives for adaptive R&D. As

\(^5\)Specifically, we find that an increase in the degree of Southern patent protection would cause a permanent increase (decrease) in the rate of international technology transfer if the Southern population size is greater (smaller) than a threshold value (i.e., \(\alpha\)).

\(^6\)Specifically, we find that strengthening Northern patent protection would cause a temporary increase (decrease) in the rate of Northern innovation if the Southern population size is smaller (greater) than another threshold value (i.e., \(\pi\)).
a result, a lower demand for Southern R&D labor depresses the wage rate in the South relative to the North. Furthermore, given that a larger Northern patent breadth has a negative impact on adaptive R&D, the benefits of remaining as Northern firms increase, which in turn reduces the rate of international technology transfer. Finally, as for the impact on the rate of Northern innovation, there are two contrasting effects: a larger Northern patent breadth raises the demand for Northern R&D labor through a larger markup of Northern firms (i.e., the positive effect) but reduces it through more products being manufactured in the North (i.e., the negative effect). The latter negative effect on the rate of Northern innovation via innovative R&D labor in the North becomes weaker if the Southern labor force is smaller. Therefore, there exists an additional threshold on the Southern population size below (above) which the overall effect of stronger patent protection in the North on the innovation rate would be positive (negative).

We calibrate our model to the China-US data to quantify the cross-country effects of IPR protection in terms of patent breadth. Our numerical analysis shows that increasing the level of patent breadth in China by 5% (percent change) reduces the wage gap between the US and China by 0.250% (percent change), and it raises the average product quality by 13.356% (percent change), implying a temporary higher rate of innovation in the US. The higher patent breadth in China would increase the flow of technology transfer from the US to China by 8.902% (percent change), since the size of China’s population is larger than the threshold \( \bar{\alpha} \). Increasing patent breadth in China also causes an increase in consumption of 5.911% in China and 5.484% in the US. These significant welfare gains are mostly due to the large increase in wage in both countries. Additionally, increasing the level of patent breadth in the US by 5% raises the wage gap between the US and China by 3.590% and decreases the flow of technology transfer from the US to China by 10.471%. The higher patent breadth in the US would raise the average product quality by 7.851%, implying a temporary higher rate of innovation in the US, since the size of China’s population is, by contrast, smaller than another threshold \( \bar{\pi} \) in this case. Increasing patent breadth in the US also causes an increase in consumption of 1.318% in China and 4.780% in the US. Therefore, increasing patent breadth in China leads to a significantly larger increase in the level of welfare for the domestic economy alone and for the two economies in total than increasing patent breadth in the US. These results highlight the importance of strengthening IPR protection in developing countries in raising the benefits of the global economy, which justifies the objective of TRIPS.

### 1.1 Literature review

This paper contributes to the theoretical literature on innovation and technology transfer that models IPR protection in forms other than patent breadth. Yang and Maskus (2001) model stronger IPR protection in terms of technology licensing and explore the impacts of reducing licensing costs and improvements in the licensor’s share of rents. Glass and Saggi (2002) and Glass and Wu (2007) study the effects of stronger IPR protection on innovation and technology transfer, with and without costly FDI, respectively, and the mode of IPR protection in their

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7In other words, our calibration exercise shows that the size of China’s population (i.e., the size of Southern population \( \alpha \) in this study) lies within the two threshold values \((\bar{\alpha}, \bar{\pi})\) implied by Propositions 2 and 3, respectively.

8Tanaka et al. (2007) reexamine the policy analysis of the Yang and Maskus (2001) model by studying the steady state and transitional dynamics, respectively.
models is by means of reducing the rate of imitation intensity in the South. Dinopoulos and Kot- 
taridi (2008) analyze the effects of IPR policy on innovation and technology transfer by modeling 
stronger IPR protection as an increase in patent length and/or a strengthening of patent enforce-
ment. Nevertheless, the analysis of IPR protection in the present paper differs from the above 
papers by focusing on the scope of products that grants to patented firms to produce, which 
is captured by the level of patent breadth. Specifically, in the current quality-ladder model, 
patent breadth represents the degree of quality by which the government in a region permits the 
state-of-art technology holders to produce without potential imitations from competitive fringes, 
which determines the markups and profits of the monopolistic firms in the North and the South, 
respectively. In other words, the different levels of Northern patent breadth and Southern patent 
breadth captures the difference in the market power of the two economies.

This study is closely related to the recent research of Dinopoulos and Segerstrom (2010) and 
Iwaisako et al. (2011), who analyze the effects of IPR protection in North-South quality-ladder 
models, but there are significant differences between these studies and ours. First, in a model 
with innovative R&D in the North and adaptive R&D in the South (i.e., costly FDI), Dinopoulos 
and Segerstrom (2010) investigate the effects of stronger IPR protection in developing countries 
in terms of increasing the exogenous rate of imitation in the South. They find that stronger 
IPR protection in the South leads to an unambiguous impact on (i.e., a permanent increase in) 
the rate of international technology transfer. By contrast, the current study shows that stronger 
IPR protection in the South by broadening patent breadth yields an overall ambiguous effect on 
international technology transfer, depending on the interaction between the benefit of increasing 
adaptive R&D labor and the cost of raising adaptive R&D difficulty, which is determined by 
size of Southern population. However, such an ambiguity in the benefit-cost interaction is ab-
sent in the analysis of Dinopoulos and Segerstrom (2010). Second, Iwaisako et al. (2011) explore 
how strengthening IPR protection in the South by increasing patent breadth affects innovation, 
FDI, and welfare. Nevertheless, similar to the assumption used in Helpman (1993), Lai (1998), 
and Branstetter and Saggi (2011), the setting of Iwaisako et al. (2011) assumes that international 
technology transfer within multinational firms is costless, which is inconsistent with the recent 
evidence that the R&D spending by affiliates of US multinationals increased considerably. Our 
study complements the analysis of Iwaisako et al. (2011) by considering adaptive R&D in South-
ern affiliates as the approach to transfer the intellectual property that facilitates production from 
the North to the South. Third, this study takes into account the effects of tightening IPR protec-
tion in the North, which conforms to the changes in patent rights of developed countries in the 
last few decades. Nonetheless, the impact of Northern IPR protection is neglected in the above 
studies. Therefore, to the best of our knowledge, this is the first study that analyzes the cross-
country effects of patent breadth in a Schumpeterian growth model with North-South technology

9See Chapter 2 in Maskus (2000) for details about the requirement on WTO member countries to strengthen patent 
protection in regard to patent breadth by the TRIPS agreement.

10See Gustafsson and Segerstrom (2010, 2011) and Lorenzck and Newiak (2012), who explore the effects of IPR 
protection in a North-South trade model with increasing product variety.

11See Ohki (2017) for a similar analysis in a framework where both Northern and Southern firms incur technology 
transfer costs.

12According to The National Science Foundation, the series in International Investment and R&D Data Link reports 
that the R&D expenditure of the majority-owned foreign affiliate (MOFA) of US multinationals increased 
transfer and costly FDI.

The present paper is also related to the existing studies of global patent protection. Lai and Qiu (2003) and Grossman and Lai (2004) analyze the welfare incentives of the Northern and Southern governments to protect their intellectual property rights by using patent length as the policy instrument in an open-economy variety-expanding model where both regions invest in R&D, whereas the current paper differs from their interesting studies by focusing on the important role of national patent policies in the form of patent breadth in an open-economy quality-ladder model with international technology transfer. In addition, Chu and Peng (2011) explore the effects of patent breadth in a two-country Schumpeterian model, but their study focuses on the interaction between developed countries by considering an environment with two Northern economies, both of which undertake innovative activities. This study, instead, examines the impacts of the same patent lever in the presence of North-South product cycles and international technology transfer via FDI, so our study complements the analysis of Chu and Peng (2011) by focusing on the interaction between developed and developing economies. Furthermore, the current study adds to the above studies by providing a quantitative analysis on the welfare implications of patent breadth, which shows that tightening IPR protection in a country can lead to a sizable welfare improvement in the other country.

Finally, the present paper relates to a large body of empirical studies that examine the relationship between Southern IPR protection and FDI. However, as argued in the previous subsection, the results in this strand of literature appear to be very mixed. For example, Primo Braga and Fink (1998) find a negative relationship between the degrees of IPR protection in developing countries and overseas sales by US-based multinationals, whereas Javorcik (2004) and Branstetter et al. (2011) find that stronger patent rights in reforming countries have a positive effect on FDI in technology-intensive industries. Furthermore, Fosfuri (2004) does not find any significant relationship between the strengths of IPR protection and multinational investment. Thus, our North-South quality ladder model complements these empirical studies by providing a theoretical rationale for the mixed effect of IPR protection on FDI in developing countries.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 derives the conditions that determine the steady-state equilibrium and the social welfare functions. Section 4 analytically explores the cross-country effects of patent protection. Section 5 performs a quantitative analysis. Section 6 concludes this study.

2 Model

To analyze the respective effects of Northern patent protection and Southern patent protection on innovation and the rate of technology transfer, we extend the Dinopoulos and Segerstrom (2010) North-South quality-ladder model with multinational firms, which is a recent variant of the North-South R&D-based model originating from the seminal work by Grossman and Helpman (1991). In the model of Dinopoulos and Segerstrom (2010), a global economy consists of a high-wage North and a low-wage South, and labor, which grows at the same rate in the two countries, is the only factor of production in products and R&D. Firms hire Northern workers to engage in innovative R&D to produce new higher-quality products, and such firms are called Northern quality leaders since all their production is located in the North. To take the advantage
of lower production costs in the South, a Northern quality leader can transfer its manufacturing operations to the South within multinational firms by hiring Southern workers to engage in adaptive R&D, and such a firm is called a Southern affiliate since all its production is located in the South. Adaptive R&D is considered as a measure of FDI because it represents the cost that multinational firms incur to transfer their technology to foreign affiliates. To introduce IPR protection, we incorporate patent breadth to protect producers from the threat of imitations, which determines the price-marginal-cost markup in each intermediate goods market. The level of patent breadth in the North is assumed to be higher than the one in the South to capture the fact that IPR protection in developed countries is generally stronger than that in developing countries.

2.1 Households

At time $t$, the household in the North (South) has a population size of $L_t^N$ ($L_t^S$). For simplicity, we assume that the population growth rates in both countries are identical and equal to $g_L > 0$. Thus, the total population size in the world is $L_t = L_t^N + L_t^S$. Denote by $\alpha = L_t^S / L_t$ the share of Southern population and $1 - \alpha$ the share of Northern population in the global population, respectively.

The lifetime utility function of the representative household in country $i = \{N, S\}$ is given by

$$U \equiv \int_0^\infty e^{-(\rho - g_L)t} \ln c_i^t dt,$$  

(1)

where $\rho > g_L$ is the discount rate and $c_i^t$ is level of consumption per capita in country $i$. Each household in country $i$ maximizes (1) subject to the following budget constraint:

$$\dot{a}_i^t = (r_t - g_L) a_i^t + w_i^t - c_i^t,$$  

(2)

where in country $i$, $a_i^t$ is the real value of financial assets per capita, $w_i^t$ is the real wage rate, and $r_t$ is the real interest rate that households in both countries face at time $t$. Following Dinopoulos and Segerstrom (2010), we assume that there is a global financial market such that the real interest rates in both countries must be equal. In each country, all prices are expressed in terms of the price of consumption goods.

Solving the standard utility maximization problem gives rise to the familiar Euler equation:

$$\frac{\dot{c}_i^N}{c_i^N} = \frac{\dot{c}_i^S}{c_i^S} = r_t - \rho,$$  

(3)

which implies that the growth rates of consumption in both countries are identical.

2.2 Final goods

Final goods $Y_t$ are all consumed by households and are produced by perfectly competitive firms that aggregate a unit continuum of intermediate goods $x_t(j)$ using the standard CES aggregator such that

$$Y_t = \left\{ \int_0^1 [x_t(j)]^{\sigma-1} dj \right\}^{\sigma \over \sigma-1},$$  

(4)
where $\sigma > 1$ is the elasticity of substitution between intermediate goods. The resource constraint on final goods in the world is

$$Y_t = c_t^N L_t^N + c_t^S L_t^S = [(1 - \alpha)c_t^N + \alpha c_t^S]L_t,$$

(5)

where $c_t^N L_t^N$ and $c_t^S L_t^S$ are the aggregate consumption in the North and South, respectively. Given zero transportation cost, the law of one price holds such that $p_{c, t}^N = \epsilon_t p_{c, t}^S$, where $\epsilon_t$ is the nominal exchange rate and $p_{c, t}^N (p_{c, t}^S)$ is the price of consumption in the North (South). In this study, all variables are expressed in real terms denominated by units of consumption that have the same value in the two countries. Solving this profit-maximizing problems yields the demand function for $x_t(j)$ such that

$$x_t(j) = \frac{Y_t}{p_t(j)^\sigma},$$

(6)

where $p_t(j)$ is the price of $x_t(j)$.

### 2.3 Intermediate goods

The differentiated intermediate goods in each industry $j \in [0, 1]$ is produced by a monopolistic quality leader who holds a patent on the latest innovation. This leader’s products will not be replaced until a new entrant with a more advanced innovation enters the market, which is known as the Arrow replacement effect. Among all intermediate goods, some are produced in the North and the other in the South. If a Northern firm succeeds in inventing a state-of-the-art good, it can register a patent for the good in the Northern and Southern countries. Products are mobile across countries, while labor, as the only production factor of intermediate good, is immobile. The production function of intermediate good by a quality leader in the North is

$$x_t(j) = z_{t,j} L_t^N(j) \equiv x_t^N(j)$$

(7)

where the parameter $z > 1$ measures the step size of a quality improvement, $n_t(j)$ is the number of quality improvements that have occurred in industry $j$ up to time $t$, and $L_t^N(j)$ is the amount of Northern labor employed by the quality leader for manufacturing.

In order to make advantage of cheaper labor force in the South, the quality leader in the North also has an incentive to shift its production to the South. The shift involves adaptive R&D for the Northern quality leader in order to transfer technology to its foreign affiliate. Once the technology transfer is complete, the Southern affiliate of the Northern leader can produce intermediate goods as a monopolist according to

$$x_t(j) = z_{t,j} \delta L_t^F(j) \equiv x_t^F(j)$$

(8)

where $\delta > 0$ is a labor-productivity parameter, capturing the productivity of Southern labor relative to Northern labor. $L_t^F(j)$ is the number of Southern labor employed by the foreign affiliate for production.

**Pricing strategy:** To analyze the pricing strategy of each category of firms, we consider how these firms operate in equilibrium by taking into account the responses of their potential rivals. If the current Northern quality leader, as it shifts the production to the South, it can make use of
cheaper labor in the South but also faces more intense competition due to a lower degree of IPR protection in the South. Thus, the potential rivals for a current Northern quality leader are

(i) **The previous Northern leader** who produces with technology by a step behind the latest generation will exit the market immediately because of losing technology advantage, which causes her no longer to gain patent production from the Northern patent authority and thus be replaced by imitators.\(^\text{13}\)

(ii) **The foreign affiliate of previous Northern leader** will also leave the market because once a new innovation is introduced to the market by the current Northern quality leader, production will be shifted back to the North. The affiliate no longer gains patent protection from the Southern patent authority and will be replaced by Southern imitators.

(iii) **The Northern imitators** who are able to gain access to the newest production technology. However, these imitators, who are not protected by patent policy, will not be active because their Southern counterparts are also possible to use the latest-generation technology and make use of cheaper Southern workers. Due to the cost disadvantage, the Northern imitators will not exist in the market.

(iv) **The Southern imitators** that imitate the top-to-line technology and choose to operate intermediate goods production in the South by hiring the cheaper Southern workers.\(^\text{14}\)

Therefore, the strongest rival against the Northern leader is the Southern imitators. Given the perfect competition among the Southern imitators, the price of intermediate goods is equal to their marginal cost such that

\[
p_t(j) = \frac{w^S_t}{\delta z_n(j)} < \frac{w^N_t}{z_n(j)},
\]

where the last term is the price set by the perfectly competitive Northern imitators. Define \(\omega \equiv w^N_t / w^S_t\) as the relative wage rate. Then (9) can be reduced to the following assumption used to guarantee that the Southern imitators will always win in the competition with the Northern imitators, in which case both types of imitators are not under the protection of patent authorities.

**Assumption 1.** \(\omega \delta > 1\).

As will be shown below, this assumption implies that all other conditions being equal, the cheaper labor force in the South generates incentives for the Northern leader to shift its production to the South.

Following Li (2001), Goh and Olivier (2002), and Iwaisako et al. (2011), we assume that the current Northern quality leader’s markup \(\mu^N_t > 1\), which determines its optimal price, is a policy instrument that can be set by Northern patent authority in the form of patent breadth. Therefore,

\(^{13}\)Similarly, we follow Howitt (1999) and Segerstrom (2000) to assume that once the incumbents stops production and leaves the market, she cannot threaten to reenter.

\(^{14}\)However, due to the limit pricing strategy of Northern leaders and their Southern affiliates, these Southern imitators are not active in the market either.
the standard Bertrand price competition leads to the monopolistic price given by

$$p^N_t(j) = \frac{\mu^N_t \omega^S_t}{\delta z^{n_t}(j)} \left( \frac{\sigma \omega}{\sigma(\delta - 1)} \right),$$

which is the limit price of the Northern quality leader against the Southern and the Northern competitive fringes that undertake potential imitations.\(^{15}\) The unconstrained price is referred to the case in which patent protection in the North is complete and monopolists are able to charge the highest price determined by the intermediate goods market. Given Assumption 1, the first inequality holds and thus the range of Northern patent breadth is given by

$$1 < \mu^N_t \leq \min\{\sigma/(\sigma - 1), \sigma \omega \delta/(\sigma - 1)\} = \sigma/(\sigma - 1).$$

Next, we consider the affiliate of the Northern leader who moves the locus of production to the South to make use of a lower wage rate in the South. The most competitive rival for this affiliate is also the Southern imitators. Nevertheless, the affiliate is protected the policy of Southern patent authority because it enters the Southern market through adaptive innovation. Therefore, the highest price set by the foreign affiliate of the current Northern leader is

$$p^F_t(j) = \frac{\mu^S_t \omega^S_t}{\delta z^{n_t}(j)} \left( \frac{\sigma \omega}{\sigma(\delta - 1)} \right),$$

where $\mu^S_t \in (1, \sigma/(\sigma - 1))$ represents the level of patent breadth in the South, determined by Southern patent authority. Furthermore, we assume that patent protection in the North is stricter than that in the South, so that $\mu^S < \mu^N$.\(^{16}\)

Define the aggregate quality index across industries $j \in [0, 1]$ as

$$Q_t \equiv \int_0^1 q_t(j) dj,$$

where $q_t(j) = [z^{n_t}(j)]^\sigma - 1$. It is also the average quality index given a unit measure of intermediate goods industries in the global economy. The labor demands for an average-quality product produced by a Northern leader and a Southern affiliate can be expressed, respectively, by

$$\bar{L}^N_{x,t} = \int_0^1 L^N_{x,t}(j) dj = Q_t Y_t \left( \frac{\delta}{\mu^N_t \omega^S_t} \right)^\sigma,$$

$$\bar{L}^F_{x,t} = \int_0^1 L^F_{x,t}(j) dj = \frac{1}{\delta} Q_t Y_t \left( \frac{\delta}{\mu^S_t \omega^S_t} \right)^\sigma.$$

\(^{15}\)See Chu and Cozzi (2014) for a detailed discussion on this standard assumption of patent breadth in a quality-ladder growth model.

\(^{16}\)According to Dinopoulos and Kottaridi (2008), the average level of patent protection in developed countries was roughly 33% higher than the counterpart in developing countries during 1960-2000.
Using these equations, we can express the labor demand for product \( j \) as

\[
L_{x,t}^{N}(j) = \frac{q_t(j)}{Q_t} L_{x,t}^{N}; \quad L_{x,t}^{F}(j) = \frac{q_t(j)}{Q_t} L_{x,t}^{F}.
\]  

(15)

The instantaneous profit of the Northern leader is

\[
\pi_t^{N}(j) = \left( \frac{\mu_t^{N} w_t^{S}}{\delta} - w_t^{N} \right) q_t(j) Y_t \left( \frac{\delta}{\mu_t^{N} w_t^{S}} \right)^{\sigma},
\]

(16)

where we have applied (7) and (8). Furthermore, the monopoly profit of the Southern affiliate of the Northern leader is

\[
\pi_t^{F}(j) = \left( \mu_t^{S} - 1 \right) w_t^{S} q_t(j) Y_t \left( \frac{\delta}{\mu_t^{S} w_t^{S}} \right)^{\sigma},
\]

(17)

where again (7) and (8) are used.

To ensure that moving the locus of production to the South is attractive to the Northern leader such that \( \pi_t^{F}(j) > \pi_t^{N}(j) \), the following assumption is imposed:

**Assumption 2.** \( \omega > \frac{1}{\delta} \left[ \mu_t^{N} - \left( \mu_t^{S} - 1 \right) \left( \frac{w_t^{N}}{\mu_t^{N} w_t^{S}} \right)^{\sigma} \right] \).

Intuitively, the benefit of shifting production to the South with a lower wage rate, after taking into account the labor productivity difference \( \delta \), must compensate for the potential loss due to a lower degree of patent protection in the South. Moreover, to ensure that a new innovator has an incentive to perform innovative R&D in the North and that the manufacturing process shifts back to the North when the new innovation arrives, another assumption is required:

**Assumption 3.** \( \omega < \frac{1}{\delta} \left[ \mu_t^{N} - \left( \mu_t^{S} - 1 \right) \left( \frac{w_t^{N}}{\mu_t^{N} w_t^{S}} \right)^{\sigma} \right] \).

Assumption 3 implies that by moving the production technology one step forward, the new quality leader is able to replace Southern affiliates of previous quality leaders.

### 2.4 Innovative and adaptive R&D

Innovative R&D is all performed by entrepreneurs in the North. By employing a number of \( L_{x,t}^{N}(j) \) of Northern labor to engage in innovative R&D in industry \( j \), an R&D entrepreneur will succeed in inventing a newer generation of product in the industry with an instantaneous probability

\[
\lambda_t^{N}(j) = \frac{L_{x,t}^{N}(j)}{\beta q_t(j)},
\]

(18)

where the term \( 1/\beta q_t(j) \) represents the productivity in innovative R&D, with \( \beta > 0 \) being an exogenous parameter and \( q_t(j) \) reflecting the decrease in the productivity of R&D labor as

\[\text{\footnotesize \(17\)}\text{\footnotesize Given the production technology } z^{n}(j) \text{ in the market, the profit flow for a new Northern quality leader in industry } j \text{ by successfully bringing the more advanced technology } z^{n}(j)+1 \text{ is } (\mu_t^{N} / \delta - \omega / z) w_t^{S} q_t(j) Y_t (\beta \mu_t^{N} / w_t^{S})^{\sigma}. \text{ This profit has to exceed the Southern affiliate’s profit } \pi_t^{F}(j) \text{ for the return of production to the North to occur.}\]

11
the product quality increases. The consideration of decreasing R&D labor productivity (i.e., the increasing research complexity), which follows the theoretical studies such as Segerstrom (1998) and Segerstrom (2000) and is consistent with recent empirical findings from Webb et al. (2017), helps to eliminate the counterfactual scale effect.\footnote{See Jones (1999) for a detailed discussion on how semi-endogenous growth models remove the scale effect.}

The expected value of owning the most recent innovation in industry \( j \) is denoted as \( \nu_t^N(j) \). The free entry into R&D implies the following zero-expected profit condition for innovative R&D

\[
v_t^N(j)\lambda_t^N(j) = w_t^N L_{r,t}(j) \Leftrightarrow \nu_t^N(j) = \beta w_t^N q_t(j), \tag{19}
\]

where we have used (18).

Adaptive R&D in the South is performed by local entrepreneurs and the Southern affiliates of Northern industry leaders. By employing \( L_{r,t}^S \) units of Southern labor into adaptive R&D, the Southern affiliate of a Northern leader in industry \( j \) will succeed in shifting the production to the Southern affiliate with an instantaneous probability

\[
\lambda_t^F(j) = \frac{L_{r,t}^F(j)}{\gamma q_t(j)}, \tag{20}
\]

where \( 1/\gamma q_t(j) \) measures the labor productivity in adaptive innovation, and \( \gamma > 0 \) is an exogenous parameter. Similar with the process in innovative R&D, \( q_t(j) \) in the denominator of (20) reflects the increasing research complexity. Denote \( \nu_t^F(j) \) the firm value of the Southern affiliate of a Northern leader. Thus, the expected net profit for the Northern quality leader to invest in adaptive R&D is \( \nu_t^F(j) - \nu_t^N(j) \). The free-entry condition implies the zero-expected profit for adaptive R&D, which can be expressed as

\[
\left[ \nu_t^F(j) - \nu_t^N(j) \right] \lambda_t^F(j) = w_t^S L_{r,t}^F(j) \Leftrightarrow \nu_t^F(j) - \nu_t^N(j) = \gamma w_t^S q_t(j), \tag{21}
\]

where (20) is applied.

Moreover, we follow the standard treatment in this class of models to focus on a symmetric equilibrium in which \( \lambda_t^N(j) = \lambda_t^N \) and \( \lambda_t^F(j) = \lambda_t^F \).\footnote{Cozzi et al. (2007) provide a theoretical justification for the symmetric equilibrium in this strand of Schumpeterian growth model. See Chu et al. (2018) for the same treatment in a monetary Schumpeterian growth model with North-South technology transfer.}

### 2.5 Stock market

The Hamilton-Jacobi-Bellman equation for \( \nu_t^N(j) \) is

\[
r_t \nu_t^N(j) = \pi_t^N(j) - w_t^S L_{r,t}^F(j) - \lambda_t^N(j) \nu_t^N(j) + \lambda_t^F(j) [\nu_t^F(j) - \nu_t^N(j)] + \nu_t^N(j),
\]

which is also the no-arbitrage condition that determines the value of \( \nu_t^N(j) \). In equilibrium, the return on the asset \( \nu_t^N(j) \), \( r_t \nu_t^N(j) \) on the left-hand-side (LHS), equals the sum of the terms on the right-hand-side (RHS), including (i) the flow profits \( \pi_t^N(j) \); (ii) the expenditure for adaptive R&D \( w_t^S L_{r,t}^F(j) \); (iii) the expected capital loss due to creative destruction \( \lambda_t^N(j) \nu_t^N(j) \); (iv) the expected
capital gain once adaptive R&D is successful \( \lambda_t^F(j)[v_t^F(j) - v_t^N(j)] \); (v) the potential capital gain \( v_t^N(j) \). Using (21), the above equation is reduced to

\[
 r_t v_t^N(j) = \pi_t^N(j) - \lambda_t^N(j)v_t^N(j) + \delta_t^N(j). \tag{22}
\]

Similarly, the no-arbitrage condition that determines the value of \( v_t^F(j) \) is given by

\[
 r_t v_t^F(j) = \pi_t^F(j) - \lambda_t^N(j)v_t^F(j) + \delta_t^F(j). \tag{23}
\]

The LHS of this equation is also the return on the asset \( v_t^F(j) \), and this asset return is the sum of the terms on the RHS including (i) monopolistic profits as an affiliate \( \pi_t^F(j) \); (ii) expected capital loss because of creative destruction \( \lambda_t^N(j)v_t^F(j) \); (iii) potential capital gains \( \delta_t^F(j) \).

### 2.6 Decentralized equilibrium

**Definition 1.** The equilibrium is defined as sequences of prices, \( \{r_t, w_t^N, w_t^S, p_t^N(j), p_t^F(j), v_t^N, v_t^F\}_{t=0}^{\infty} \) and allocations, \( \{c_t^N, c_t^S, Y_t, x_t^N(j), x_t^F(j), L_{x,t}^N(j), L_{x,t}^F(j), L_{r,t}^N(j), L_{r,t}^F(j)\}_{t=0}^{\infty} \), for \( j \in [0, 1] \).

Moreover, at each instance of time,

- the representative household in the North maximizes lifetime utility taking \( \{r_t, p_t, w_t^N\} \) as given;
- the representative household in the South maximizes lifetime utility taking \( \{r_t, p_t, w_t^S\} \) as given;
- the competitive final goods firms produce \( Y_t \) to maximize profit taking \( \{p_t^N(j), p_t^F(j)\} \) as given;
- the Northern quality leaders choose \( p_t^N(j) \) and produce \( x_t^N(j) \) to maximize profit taking \( w_t^N \) as given;
- the Southern affiliates choose \( p_t^F(j) \) and produce \( x_t^F(j) \) to maximize profit taking \( w_t^S \) as given;
- entrepreneurs in the North employ \( L_{r,t}^N(j) \) to perform innovative R&D taking \( \{r_t, w_t^N, v_t^N\} \) as given;
- Southern affiliates of Northern quality leaders employ \( L_{r,t}^F(j) \) to perform adaptive R&D taking \( \{r_t, w_t^S, v_t^F\} \) as given;
- the final goods market clears such that \( Y_t = c_t^N L_t^N + c_t^S L_t^S \);
- the labor market-clearing conditions hold in both countries; and
- the nominal exchange rate is determined by the law of one price such that \( \epsilon_t = p_{c,t}^N / p_{c,t}^S \).

### 3 Steady-state equilibrium

In this section, we solve the steady-state equilibrium and analyze how Southern and Northern patent policy affect innovation in the North and international technology transfer, respectively. To do so, we first derive the steady-state number of each type of industries and the expression of quality index. Then, we specify the steady-state labor market condition in the two countries, and by combining these conditions we construct the Southern and Northern steady-state conditions of technology transfer and innovation. Finally, we derive the steady-state welfare in both countries.
3.1 Industry composition and quality dynamics

There are two types of industries in the intermediate goods sector, the Northern quality leaders and the Southern affiliates. Denote \( \theta^N \) and \( \theta^F \) as the steady-state measure of these two types of industries, respectively. Then, these measure of all industries must add up to one such that

\[
\theta^N + \theta^F = 1. \tag{24}
\]

Each industry can switch randomly across these two categories with probabilities that in turn depends on the Poisson arrival rates of innovative and adaptive R&D. In the steady state, the measure of industries in each type must be constant such that the flow in and out of the Southern affiliate must be equal. This relation can be established as the following equation

\[
\theta^N \lambda^F = \theta^F \lambda^N. \tag{25}
\]

It is straightforward that this equation can be also stated as the flow out and into the Northern quality leaders. Solving (24) and (25) yields the measure of these industries such that

\[
\theta^N = \frac{\lambda^N}{\lambda^N + \lambda^F}, \tag{26}
\]

\[
\theta^F = \frac{\lambda^F}{\lambda^N + \lambda^F}. \tag{27}
\]

By definition, the aggregate quality index across industries \( j \in [0, 1] \) is

\[
Q_t \equiv \int_0^1 q_t(j) dj = \int_0^1 \kappa^n_t(j) dj, \tag{28}
\]

where \( \kappa = z^\sigma - 1 > 1 \) is a composite parameter that is increasing in the quality step size \( z \). This quality index can be further decomposed into the following two components:

\[
Q_t = Q^N_t + Q^F_t = \int_{\theta^N} q_t(j) dj + \int_{\theta^F} q_t(j) dj. \tag{29}
\]

The following lemma provides the steady-state expression for the measure of each component of aggregate quality.

**Lemma 1.** In the steady state, the two components of aggregate quality can be expressed as

\[
\frac{Q^N_t}{Q_t} = \frac{\kappa \lambda^N}{\kappa \lambda^N + \lambda^F}, \tag{30}
\]

\[
\frac{Q^F_t}{Q_t} = \frac{\lambda^F}{\kappa \lambda^N + \lambda^F}. \tag{31}
\]

**Proof.** See Appendix A.1.
3.2 Northern labor market

The labor market-clearing condition in the North is given by

\[ L^N_t = L^N_{x,t} + L^N_{r,t} = \int_{\theta^N} L^N_{x,t}(j) dj + \int_{\theta^N} L^N_{r,t}(j) dj. \]  (32)

The amount of labor employed for production by Northern quality leader is

\[ L^N_{x,t} = \int_{\theta^N} \frac{q_t(j)}{Q_t} L^N_{x,t}(j) dj = \frac{Q_t^N}{Q_t} L^N_{x,t}, \]  (33)

where the first equality uses (15). Using (18), the amount of labor employed for innovative R&D is

\[ L^N_{r,t} = \beta \lambda N Q_t \]  (34)

where the symmetry condition \( \lambda N(j) = \lambda N \) is imposed. Substituting (33) and (34) into (32), together with (30), yields the Northern labor market clearing condition in per capita terms such that

\[ 1 = \frac{L^N_{x,t}}{L^N_t} \frac{\kappa \lambda N}{\kappa \lambda N + \lambda F} + \beta \lambda N \Phi, \]  (35)

where \( \Phi_t \equiv Q_t / L^N_t = \Phi \) is defined as the average quality per Northern worker, which is constant over time in any steady-state equilibrium.

3.3 Southern labor market

The labor market-clearing condition in the South is given by

\[ L^S_t = L^F_{x,t} + L^F_{r,t} = \int_{\theta^F} L^F_{x,t}(j) dj + \int_{\theta^N} L^F_{r,t}(j) dj. \]  (36)

The amount of labor employed for production by Southern affiliates is

\[ L^F_{x,t} = \int_{\theta^F} \frac{q_t(j)}{Q_t} L^F_{x,t}(j) dj = \frac{Q_t^F}{Q_t} L^F_{x,t}, \]  (37)

where the first equality uses (15). Using (20) and imposing the symmetry condition \( \lambda F(j) = \lambda F \), the amount of labor employed for adaptive R&D is given by

\[ L^F_{r,t} = \gamma \lambda F Q_t^N \]  (38)

Similarly, substituting (37) and (38) into (36), coupled with (30) and (31), we express the Southern labor market clearing condition in per capita terms such that

\[ 1 = \frac{\lambda F}{\kappa \lambda N + \lambda F} \left[ \frac{L^F_{x,t}}{L^S_t} + \frac{\gamma \kappa \lambda N (1 - \alpha)}{\alpha} \Phi \right], \]  (39)

where \( L^N_t / L^S_t = (1 - \alpha) / \alpha \) is used.
3.4 Innovation and technology transfer

Differentiating (28) with respect to time $t$ yields the growth rate of the quality index

$$
\dot{Q}_t = \int_0^1 \left[ \kappa^{n(j)+1} - \kappa^{n(j)} \right] \lambda^N_t dj = (\kappa - 1)\lambda^N_t Q_t.
$$

Taking the log of $\Phi_t = Q_t/L^N_t$ and differentiating with respect to time yields

$$
\frac{\dot{\Phi}_t}{\Phi_t} = \frac{\dot{Q}_t}{Q_t} - \frac{i^N_t}{L^N_t} = (\kappa - 1)\lambda^N_t - g_L.
$$

Since $\Phi_t$ is stationary in the steady state, (41) implies that the steady-state arrival rate of innovation is completely determined by the exogeneous population growth rate given by

$$
\lambda^N = \frac{g_L}{\kappa - 1}.
$$

This feature originates from the insight that the increasing research complexity acts as a counteractive force to the growing R&D input. As discussed in Segerstrom (2000) and Dinopoulos and Segerstrom (2010), any increase in R&D input leading to a higher product quality makes product more complex and harder for researchers to find further improvement. As a consequence, a growing R&D labor employment is always required to maintain a constant innovation rate over time. In the model, R&D labor is completely determined by the exogenous population growth rate, which leads the steady-state arrival rate of innovation to be exogenously pinned down.

Furthermore, in the steady state, from (22) and (23), the values of assets for the Northern quality leader and the Southern affiliate can be expressed as

$$
v^N_t(j) = \frac{\pi^N_t(j)}{\rho + \lambda^N},
$$

and

$$
v^F_t(j) = \frac{\pi^F_t(j)}{\rho + \lambda^N}.
$$

Substituting (16) and (43) into (19) yields the following steady-state innovative R&D condition:

$$
\left( \frac{\mu^N}{\delta \omega} - 1 \right) \frac{L^N_{x,t}}{Q_t} = \beta (\rho + \lambda^N).
$$

Similarly, substituting (16), (17), (43), and (44) into (21) gives rise to the following steady-state adaptive R&D condition:

$$
\left( \frac{\mu^S - 1}{\delta \omega} \right) \frac{L^F_{x,t}}{Q_t} - \left( \frac{\mu^N}{\delta - \omega} \right) \frac{L^N_{x,t}}{Q_t} = \gamma (\rho + \lambda^N).
$$
Next, substituting (45) into (35) yields the Northern steady-state condition such that

\[ 1 = \beta \lambda N \Phi \left\{ \frac{\kappa (\rho + \lambda N)}{\frac{\mu^N}{\delta \omega (\mu^N, \mu^S)} - 1} (\kappa \lambda N + \lambda^F) + 1 \right\}, \tag{47} \]

which contains two endogenous variables \( \{\Phi, \lambda^F\} \) and features a positive slope and a positive \( \Phi \)-intercept in the \( \{\Phi, \lambda^F\} \) space as shown in Figure 1, where "North" represents the Northern steady-state condition whereas "South" represents the Southern steady-state condition. The intuition behind the positive slope of the Northern steady-state condition can be explained as follows. An increase in \( \lambda^F \) implies that more products are manufactured in the South but less in the North, which in turn leads to a reallocation of labor in the North from production to innovative R&D due to the resource constraint on Northern labor. Thus, the increase in Northern R&D labor raises the average quality per Northern worker (i.e., \( \Phi \)) in the steady state.

\[ \lambda^F \]

\[ \text{North} \]

\[ \mu^N \uparrow \text{ or } \mu^S \uparrow \]

\[ \mu^N \uparrow \]

\[ \text{South} \]

\[ \Phi \]

Figure 1: The steady-state equilibrium.

Then, substituting (45) into (39), together with (46), yields the Southern steady-state condition such that

\[ 1 = \Phi \lambda^F (1 - \alpha) \left\{ \frac{(\rho + \lambda N) [\gamma + \beta \omega (\mu^N, \mu^S)]}{\mu^S - 1} + \gamma \kappa \lambda N \right\}, \tag{48} \]

where the relation \( L^N_t / L^S_t = (1 - \alpha) / \alpha \) is used. The Southern steady-state condition also contains two endogenous variables \( \{\Phi, \lambda^F\} \) but features a negative slope, with no intercepts, in the \( \{\Phi, \lambda^F\} \) space. Intuitively, an increase in \( \lambda^F \) implies that more products are manufactured in the South, which in turn leads to a reallocation of labor in the South from adaptive R&D to production due to the resource constraint on Southern labor. From (38), we have \( L^F_r = \gamma \kappa L^N \Phi \lambda^N \lambda^F / (\kappa \lambda N + \lambda^F) \). Therefore, a higher \( \lambda^F \) is consistent with a smaller amount of adaptive R&D labor when the difficulty level \( \Phi = Q / L^N \) decreases sufficiently (i.e., technologies become sufficiently easy to be transferred to the South). Finally, (47) and (48) are the two conditions that implicitly solve for the steady-state values of \( \{\Phi, \lambda^F\} \). The intersection at point \( O \) in Figure 1 determines the unique steady-state values for \( \Phi \) and \( \lambda^F \).
3.5 Social welfare

In this section, we derive the steady-state social welfare in each country, which will be used to examine the welfare implications of each country’s patent policy in the quantitative analysis. Imposing balanced growth on (i) yields the steady-state welfare of the Northern household given by

\[ U^N = \frac{1}{\rho - g_L} \left( \ln c_0^N + \frac{g_c}{\rho - g_L} \right), \] (49)

where \( g_c = g_L / (\sigma - 1) \) is the growth rate of consumption per capita and determined by exogenous parameters due to semi-endogenous growth. Therefore, the steady-state welfare is determined by the balanced-growth level of consumption. According to (2), using balanced growth condition \( \dot{a}_N / a_N = g_c \) yields

\[ c_t^N = (\rho - g_L) a_t^N + w_t^N. \] (50)

The balanced-growth level of consumption \( c_0^N \) is thus a sum of asset income \( (\rho - g_L) a_0^N \) and wage income \( w_0^N \). Similar conditions also apply to the Southern case. To explicitly derive \( a_0^N \) and \( a_0^S \), we follow Dinopoulos and Segerstrom (2010) to assume that the asset from innovative R&D in the North is owned by the Northern household whereas the asset from adaptive R&D in the South is owned by the Southern household. Under this assumption, we show in Lemma 2 that the balanced-growth levels of consumption \( \{c_0^N, c_0^S\} \) can be expressed as a function of \( \{w_0^N, w_0^S\} \), as similar to Chu et al. (2018).

**Lemma 2.** The balanced-growth level of consumption can be expressed as

\[ c_0^N = w_0^N I^N = \frac{\beta (\rho - g_L) \Phi w_0^N}{\omega} + \frac{w_0^N}{\omega} = \omega w_0^N [\beta (\rho - g_L) \Phi + 1], \] (51)

\[ c_0^S = w_0^S I^S = (\rho - g_L) (\gamma + \beta \omega) \frac{\kappa \lambda^N \Phi (1 - \alpha)}{a (\kappa \lambda^N + \lambda^F)} w_0^S + \frac{w_0^S}{\omega}, \] (52)

where \( \{I^N, I^S\} \) denote income as a ratio of real wages, and

\[ w_0^S = \delta (\Phi L_0^N)^{1-\gamma} \left\{ \frac{\mu^N}{\kappa \lambda^N + \lambda^F} \right\}^{1-\sigma} + \left\{ \frac{\mu^S}{\kappa \lambda^S + \lambda^F} \right\}^{1-\sigma}. \] (53)

**Proof.** See Appendix A.2.

4 Patent policy, innovation, and technology transfer

In this section, we explore the effects of Southern and Northern patent policy \( \{\mu^S, \mu^N\} \) on the innovation rate \( \lambda^N \) and the technology transfer rate \( \lambda^F \), respectively. Before doing so, we examine
the effects of these patent-policy tools on the relative wage $\omega$. From (15), we obtain
\[
\frac{L_{F,t}}{Q_t} = \frac{\mu^N}{\delta} \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{L_{x,t}}{Q_t}.
\]
(54)

Dividing (45) by (46) and making use of (54) yield the following steady-state relative-wage condition:
\[
\frac{\mu^N}{\delta \omega} - \frac{\beta}{\gamma} \omega = 1 - \frac{\beta}{\gamma \delta} \left[ \mu^N - (\mu^S - 1) \left( \frac{\mu^N}{\mu^S} \right)^\sigma \right],
\]
(55)

which is an implicit function that pins down the steady-state equilibrium value of the relative wage $\omega(\mu^N, \mu^S)$. The following proposition shows the effects of patent policy in each country on the relative wage.

**Proposition 1.** Strengthening patent protection in the South lowers the relative wage rate between the North and the South, whereas strengthening patent protection in the North raises the relative wage rate.

**Proof.** See Appendix A.3.

Proposition 1 can be explained as follows. As shown in (17), strengthening Southern patent protection grants a larger market power to Southern affiliates by allowing them to charge a higher markup. Given the wage rates, this raises the benefits of being Southern affiliates, which increases the incentives for adaptive innovation. Thus, the value of Southern firms $v^F_t(j)$ as indicated in (44) (relative to the value of Northern firms as indicated in (43)) tends to rise. Then, the zero-profit condition for adaptive R&D in (21) implies that the increase in the reward for adaptive R&D must correspondingly cause an increase in the R&D cost. This yields a positive effect on the demand for Southern R&D labor. Consequently, strengthening Southern patent protection raises wage rate in the South relative to the North.

By contrast, given the wage rates, strengthening Northern patent protection raises the benefits of remaining as a Northern quality leader through a higher markup. This decreases the incentives for adaptive R&D, and the firm value $v^F_t(j)$ as shown in (44) (relative to the value of Northern firms as shown in (43)) tends to decline. According to the zero-profit condition for adaptive R&D in (21), a decrease in the reward for adaptive R&D corresponds to a decrease in the R&D cost, yielding a negative effect on the demand for Southern R&D labor. Therefore, strengthening Northern patent protection reduces wage rate in the South relative to the North.

Having established the effects of Southern and Northern patent policy $\{\mu^S, \mu^N\}$ on the relative wage rate, we are now in position to explore their effects on the rate of innovation $\lambda^N$ and of international technology transfer $\lambda^F$. First, the following proposition illustrates the results regarding the impacts of an increase in $\mu^S$ on $\lambda^N$ and $\lambda^F$.

**Proposition 2.** Strengthening patent protection in the South yields (i) a temporary higher rate of innovation in the North, and (ii) a positive (negative) effect on the technology transfer from the North to the South if the Southern population size $\alpha$ is greater (smaller) than the value $\tilde{a}$.

**Proof.** See Appendix A.4.

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20In Appendix A.3, we also prove that the relative wage rate $\omega$ is a concave function of Northern patent policy $\mu^N$. 

Proposition 2 can be explained as follows. Graphically, a higher $\mu^S$ shifts both the North and South curves to the right in Figure 1, with $\Phi$ increasing unambiguously whereas $\lambda^F$ inconclusive.\footnote{Precisely, if an increase in $\mu^S$ shifts the North curve to the right in a larger (smaller) magnitude than the South curve to the right, then a decrease (increase) in $\lambda^F$ emerges in response.} As for the impact on $\Phi$, it is known from Proposition 1 that, a rise in $\mu^S$ drives down the relative wage. This tends to increase the profit margin of the Northern quality leaders in (16), because the higher wage rate in the South relative to the North raises the monopolistic price but reduces the marginal cost of intermediate goods production. This makes it more attractive for firms to engage in innovative R&D in the North. Therefore, the lower demand for Northern production labor causes a reallocation of labor in the North from manufacturing to innovative R&D. As a consequence, the rate of Northern innovation increases in the short run, leading to a higher average quality per Northern worker $\Phi$ in the long run, as implied by (41).

As for the impact on $\lambda^F$, the overall effect can be decomposed into two opposing effects. To see this, we use $\lambda^F_i(j) = \lambda^F_i$ and (38) to derive

$$
\lambda^F_i = \frac{L^F_r t}{\gamma Q^N_t} = \frac{1}{\gamma \Phi (1 - \alpha)L_t} \frac{L^F_r t}{Q^N_t}, \quad (56)
$$

where the second equality uses $\Phi = Q_t / L^N_t$ and $L^N_t = (1 - \alpha)L_t$. In the steady state, $Q^N_t / Q_t$ is given by (30), and hence, (56) can be reexpressed as

$$
\frac{\kappa \lambda^N \lambda^F}{\kappa \lambda^N + \lambda^F} = \frac{L^F_r t}{\gamma \Phi (1 - \alpha)L_t}, \quad (57)
$$

where the LHS is increasing in $\lambda^F$. Thus, from (57), we see that Southern patent breadth affects $\lambda^F$ via (i) the difficulty of adaptive R&D viewed from the perspective of the South such that $Q_t / L^S_i = \Phi L^N_i / L^S_i$ (because an increase in the average product quality $\Phi$ raises adaptive R&D difficulty), and (ii) the number of adaptive R&D workers $L^F_r t$. On the one hand, the previous argument shows that a larger Southern patent breadth $\mu^S$ enhances $\Phi$, and thus, this generates a negative effect on the rate of international technology transfer $\lambda^F$ by increasing the difficulty of adaptive R&D (i.e., a higher $Q_t / L^S_i$). On the other hand, an increase in $\mu^S$ also increases the number of adaptive R&D workers $L^F_r t$ since the adaptive R&D firm value relative to the innovative R&D firm value (i.e., $v^F_i(j) / v^N_i(j)$) rises, as explained in Proposition 1.\footnote{Combining (19) and (24), and substituting (55) into the resulting equation yield $v^F_i(j) / v^N_i(j) = (\gamma / \beta)/\omega(\mu^N, \mu^S) + 1$, which shows that $v^F_i(j) / v^N_i(j)$ is increasing in $\mu^S$.}

Accordingly, whether a larger Southern patent breadth $\mu^S$ increases the technology transfer rate $\lambda^F$ depends on the interplay between the effect of adaptive R&D difficulty and the effect of adaptive R&D labor allocation. We find that this interplay is determined by the Southern population size $\alpha$. Specifically, when the size of Southern population $\alpha$ exceeds a certain level (i.e., $\alpha$), then this large size of $\alpha$ tends to reinforce the positive effect of a higher $\mu^S$ on $\lambda^F$ through the channel of $L^F_r t$ to dominate the negative effect through the channel of $\Phi$; namely the incentives for adaptive R&D outweigh its difficulty in the presence of a large population size in the South. In this case, more international technology transfer is realized by strengthening Southern patent protection; otherwise, when $\alpha$ is smaller than $\alpha$, the small size of Southern population weakens...
the positive effect through the channel of \( L_{r,j}^F \), which will be dominated by the negative effect through the channel of \( \Phi \). As a result, less technology transfer occurs.

Next, the following proposition illustrates the results regarding the impacts of an increase in \( \mu^N \) on \( \lambda^N \) and \( \lambda^F \).

**Proposition 3.** Strengthening patent protection in the North yields (i) a temporary higher (lower) rate of innovation in the North if the Southern population size \( \alpha \) is smaller (greater) than the value \( \bar{\alpha} \), and (ii) a lower rate of technology transfer from the North to the South.

**Proof.** See Appendix A.5. □

The intuition for Proposition 3 can be explained as follows. Figure 1 shows that a higher \( \mu^N \) shifts the North curve to the right whereas it shifts the South curve to the left, resulting in an unambiguously decreasing effect on \( \lambda_F \) and an ambiguous effect on \( \Phi \).\(^{23}\) Intuitively, we follow the analysis in Proposition 2 to decompose the overall effect of a larger Northern patent breadth \( \mu^N \) on the rate of international technology transfer \( \lambda^F \) into the following two components: a change in \( \mu^N \) affects \( \lambda^F \) through the level of adaptive R&D labor \( L_{r,j}^F \) and the average product quality \( \Phi \), respectively, as shown in (57). As for the impact on \( L_{r,j}^F \), an increase in \( \mu^N \) decreases \( L_{r,j}^F \) since the adaptive R&D firm value relative to the innovative R&D firm value (i.e., \( v_t^F(j)/v_t^N(j) \)) declines, as explained in Proposition 2.\(^{24}\) As for the ambiguous impact of \( \mu^N \) on \( \Phi \), there are two cases to be considered. If a rise in \( \mu^N \) increases \( \Phi \), then the negative effect through \( L_{r,j}^F \) is reinforced; the rise in its difficulty intensifies the disincentives for adaptive R&D. However, if a rise in \( \mu^N \) reduces \( \Phi \), Proposition 3 implies that the negative effect through \( L_{r,j}^F \) tends to strictly dominate the effect through \( \Phi \), so the overall effect of an increased \( \mu^N \) on \( \lambda^F \) is still negative. In other words, a larger \( \mu^N \) always guarantees sufficient benefits of remaining as a Northern firm, instead of becoming a Southern firm. Therefore, less international technology transfer occurs (i.e., a lower \( \lambda^F \)) under a stronger degree of Northern patent protection (i.e., a higher \( \mu^N \)).

As for the effect on \( \Phi \), one can see from (16) that, a strengthened Northern patent protection \( \mu^N \) causes two contrasting effects as follows. On the one hand, a larger \( \mu^N \) increases the profit margin of Northern quality leaders through allowing for a higher markup, which increases the incentives for innovative R&D. This tends to reallocate labor from production to R&D in the North. On the other hand, a larger \( \mu^N \) decreases the technology transfer rate \( \lambda^F \) (according to Proposition 3 (ii)), which implies that more products will be manufactured in the North. This tends to reallocate labor from R&D to production in the North. Accordingly, whether a larger Northern patent breadth \( \mu^N \) increases the average quality per Northern worker \( \Phi \) in the long run depends on the interplay between the positive effect of \( \mu^N \) on Northern R&D labor through markup and the negative effect through products manufacturing.

We find that this interplay is again determined by the Southern population size \( \alpha \). Specifically, when the size of Southern population \( \alpha \) is smaller than a certain value (i.e., \( \bar{\alpha} \)), the decrease in the number of products manufactured by Southern affiliates is not significant, implying a small increase in the Northern manufacturing operations. Hence, the negative one via products manufacturing becomes relatively weak to be dominated by the positive effect via markup, causing

\(^{23}\) Precisely, if an increase in \( \mu^N \) shifts the North curve to the right in a larger (smaller) magnitude than the South curve to the left, then an increase (decrease) in \( \Phi \) emerges in response.

\(^{24}\) Similarly, combining (19) and (21) yields \( v_t^F(j)/v_t^N(j) = (\gamma/\beta)/\omega(\mu^N, \mu^S) + 1 \), which shows that \( v_t^F(j)/v_t^N(j) \) is decreasing in \( \mu^N \).
a reallocation of labor in the North from manufacturing to innovative R&D. As a result, the rate of Northern innovation increases in the short run, and the average quality per Northern worker $\Phi$ increases in the long run, as implied by (41). Nevertheless, when $\alpha$ exceeds $\pi$, the large size of Southern population implies that the decrease in the number of products manufactured by Southern affiliates is significant. In this case of a large increase in the Northern manufacturing operations, the negative effect through products manufacturing becomes relatively strong to dominate the positive effect through markup. Consequently, the resulting mechanism reverses, and then $\Phi$ declines in the long run, implying that a temporary lower rate of Northern innovation emerges.

5 Quantitative analysis

In this section, we calibrate our model to numerically evaluate the effects of the Northern and Southern patent instruments, respectively. Specifically, we consider China as the South and the US as the North to explore the welfare implications of each country’s patent protection. To do so, we first describe the calibration strategy in Section 5.1. Section 5.2 then provides the benchmark estimation results, and Section 5.3 shows the results of robustness checks by altering the values of parameters and empirical moments.

5.1 Calibration

To perform this numerical analysis, the strategy is to assign steady-state values to the following structural parameters $\{\rho, z, \alpha, \kappa, \sigma, \mu^N, \mu^S, \delta, \beta, \gamma\}$. We choose a conventional value of 0.02 for the discount rate $\rho$. We follow Acemoglu and Akcigit (2012) to set the step size of innovations $z$ to 1.05. The relative population size $\alpha$ is set to 0.829 by using the data from the World Development Indicators on the labor force size of China and the US. As for $\kappa$, we calibrate it by applying the population growth rate $g_L$ and the innovation arrival rate $\lambda^N$. For $g_L$, we follow Jones and Williams (2000) to set it to 1.44% to correspond to the long-run growth rate of the US labor force. For $\lambda^N$, we select an empirically plausible value of 15% and explore other values in Section 5.3. Therefore, equation (42) pins down $\kappa$, and the definition of $\kappa = z^\sigma - 1$ shows that $\sigma = 2.879$, which is in line with the median substitution elasticity estimated by Broda and Weinstein (2006). As for the market-level values of the Northern patent instrument $\mu^N$ and the Southern counterpart $\mu^S$, we choose $\mu^N = 1.3$ according to the estimates of average markup ratio for the US in Christopoulou and Vermeulen (2012) and $\mu^S = 1.2$ according to the estimates for China in Liu and Ma (2017), where the latter ensures that Assumptions 1, 2, and 3 are satisfied.

---

25The data is available at http://wdi.worldbank.org/tables, Table 2.2, Labor Force Structure.
26Studies in the literature have considered different values for the arrival rate of innovations. For instance, Caballero and Jaffe (2002) estimate the mean rate of creative destruction to be roughly 4%. Lanjouw (1998) shows that the probability of obsolescence ranges from 7% for computer patents to 12% for engine patents. A more optimistic estimate by Acemoglu and Akcigit (2012) shows that the average frequency of innovation is about 3 years, implying an innovation-arrival rate of $1/3 = 33%$. Thus, we consider an intermediate value of 15% within this range.
27See also Norrbin (1993) who reports a similar estimate.
28Using the data from the Annual Survey of Industrial Production (ASIP), Liu and Ma (2017) show that the average markup of both importers and non-importers in China are approximately 1.2.
given the value of \( \mu^N \) as chosen above and the calibrated value of \( \delta \) that meets the empirical moments as illustrated below.

The remaining parameters awaiting for calibration are \( \delta, \beta, \) and \( \gamma \). Given that it is the relative R&D productivity \( \gamma/\beta \) (rather than their individual values) that determines the values of variables in equilibrium, we then calibrate \( \gamma/\beta \) by matching the following two empirical moments:\(^{29}\) the relative wage rate between the US and China, and the US R&D intensity. According to the data from the Conference Board on manufacturing hourly compensation costs between the US and China, the North-South relative wage rate \( \omega \) is about 20.101 from 2002 to 2013.\(^{30}\) Moreover, from the OECD database, the gross domestic spending on R&D for the US is about 2.6\% of its GDP.\(^{31}\) Based on this indicator, we thus construct the expression of R&D intensity in the North such that 
\[
\frac{w_0^NL_0^N/(w_0^NL_0^N+c_0^Ni_0^N)}{\omega} = 2.6\%.\]
Together with equations (47), (48) and (55), \{\delta, \gamma/\beta\} and the equilibrium values of \{\Psi, \lambda^F\} are solved. The details of the above calibrated values are reported in Table 1 and Table 2, respectively.

Table 1: Calibrated parameter values

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( z )</th>
<th>( \alpha )</th>
<th>( \kappa )</th>
<th>( \sigma )</th>
<th>( \mu^N )</th>
<th>( \mu^S )</th>
<th>( \delta )</th>
<th>( \gamma/\beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1.05</td>
<td>0.829</td>
<td>1.096</td>
<td>2.879</td>
<td>1.3</td>
<td>1.2</td>
<td>0.063</td>
<td>153.544</td>
</tr>
</tbody>
</table>

5.2 Benchmark estimation results

Given the benchmark calibrated parameter values, we now conduct the following experiments by enhancing patent protection in China and the US, respectively. We start off by exploring the situations in China. First, we raise the level of patent breadth in China from 1.2 to 1.25. As reported in Table 2, we find that the average quality per Northern worker \( \Phi \) increases by 13.356\% (percent change), implying a temporary higher rate of innovation \( \lambda^N \) in the North. Moreover, the international technology transfer rate \( \lambda^F \) increases by 8.902\% (percent change) in response. According to Proposition 2, the size of China’s population (\( \alpha = 0.829 \)) is larger than the threshold value \( \alpha \),\(^{33}\) which makes the positive effect on technology transfer through the increase in adaptive R&D expenditures dominate the negative effect through the increase in technology difficulty. In addition, when expressing the welfare changes as the usual equivalent variation in consumption, we find that a stricter patent policy in China leads to a welfare gain of 5.911\% in China. From Lemma 2, we know that the change in \( c_0^N \) comes from the changes in \( w_0^N \) and \( I^N \). The numerical results show that as \( \mu^S \) rises from 1.2 to 1.25, \( w_0^S \) increases by 5.734\% whereas \( c_0^S \) increases by 5.911\%, implying that \( I^S \) increases marginally by 0.177\%. Therefore, the quantitatively significant welfare gain for China as a result of an increase in \( \mu^S \) is mostly due to the large increase in wage.

\(^{29}\)It is the value of \( \beta \Phi \), which is independent of \( \beta \), that affects equilibrium variables in the model. Therefore, we normalize \( \beta \) to unity for simplicity only when reporting the value of \( \Phi \).

\(^{30}\)The data is included in International Compensations of Hourly Compensation Costs in Manufacturing, 2016 - China and India, Table 4.


\(^{32}\)The subscript of time 0 indicates that the economy is on the initial balanced-growth path before being intervened by changes in patent policy.

\(^{33}\)The threshold value \( \alpha \) given in (A.16) is negative in the benchmark case.
Although the rise in $\mu^S$ narrows the wage gap $\omega$ by 0.250% (percent change), it still raises the wage rate in the US $\omega^N_0$ by 5.470%\textsuperscript{34}. The increase in $\omega^N_0$ in turn causes a welfare gain of 5.484\% in the US, but the size is smaller than that in China.

Then, when continuing to raise $\mu^S$ to a larger value of 1.3,\textsuperscript{35} we find that the pattern of changes in the above economic variables almost remains unchanged. A higher $\mu^S$ still promotes the technology transfer rate $\lambda^F$, but in a less significant size of 8.457%. The wage gap $\omega$ decreases in response to a rise in $\mu^S$; as $\mu^S$ increases from 1.25 to 1.3, the increase in $\omega$ is 0.179\%, which is smaller than its increase (i.e., 0.250\%) when $\mu^S$ rises from 1.2 to 1.25. Nevertheless, the difference in welfare gains between China and the US is still present, indicating that China benefits more from a strengthening of its own patent policy relative to the US.

Furthermore, Table 2 displays that a permanent increase in the level of patent breadth in US from 1.3 to 1.35 raises the average quality per Northern worker $\Phi$ by 7.851\% (percent change), as the result of the Southern population size $a$ being smaller than the threshold value $\pi$,\textsuperscript{36} according to Proposition 3. In this case, the positive effect of a larger $\mu^N$ through markup outweighs the negative effect through products manufacturing, increasing the incentives for Northern innovation. Correspondingly, $\Phi$ rises permanently and the innovation rate $\lambda^N$ in the North rises temporarily. In addition, the technology transfer from the US to China decreases by 10.471\%; it is caused in part by a decrease in adaptive R&D because of a lower level of Southern R&D labor, and an reinforcing effect from the increase in $\Phi$ also makes the technology transfer rate more difficult. Moreover, the US-China wage gap $\omega$ enlarges by 3.590\%, and the wage rate in both countries increases in response to a larger $\mu^N$. Accordingly, a strengthening of patent protection in the US yields a welfare gain of 4.780\% in the US and 1.318\% in China, respectively. In contrast to the change in $\mu^S$, a stronger patent policy in the US leads to more domestic welfare gains than the welfare gains in China, because the wage rate, which is the main contributor to the welfare change, rises in a larger magnitude in the US. Finally, the pattern of changes in these economic variables still holds when $\mu^N$ is raised to a larger value of 1.4.

<table>
<thead>
<tr>
<th>$\mu^S$</th>
<th>$\Phi$</th>
<th>$\lambda^F$</th>
<th>$\omega$</th>
<th>$\Delta \ln \omega^N_0$</th>
<th>$\Delta \ln \omega^S_0$</th>
<th>$\Delta \ln c^N_0$</th>
<th>$\Delta \ln c^S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>0.178</td>
<td>0.059</td>
<td>20.101</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.25</td>
<td>0.202</td>
<td>0.064</td>
<td>20.051</td>
<td>5.470%</td>
<td>5.734%</td>
<td>5.484%</td>
<td>5.911%</td>
</tr>
<tr>
<td>1.3</td>
<td>0.221</td>
<td>0.069</td>
<td>20.015</td>
<td>3.549%</td>
<td>3.735%</td>
<td>3.560%</td>
<td>3.857%</td>
</tr>
<tr>
<td>$\mu^N$</td>
<td>$\Phi$</td>
<td>$\lambda^F$</td>
<td>$\omega$</td>
<td>$\Delta \ln \omega^N_0$</td>
<td>$\Delta \ln \omega^S_0$</td>
<td>$\Delta \ln c^N_0$</td>
<td>$\Delta \ln c^S_0$</td>
</tr>
<tr>
<td>1.3</td>
<td>0.178</td>
<td>0.059</td>
<td>20.101</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.35</td>
<td>0.192</td>
<td>0.053</td>
<td>20.823</td>
<td>4.772%</td>
<td>1.141%</td>
<td>4.780%</td>
<td>1.318%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.207</td>
<td>0.048</td>
<td>21.537</td>
<td>4.568%</td>
<td>1.099%</td>
<td>4.577%</td>
<td>1.287%</td>
</tr>
</tbody>
</table>

\textsuperscript{34}According to International Labour Organization Global Wage Report 2018/19, the increase in the real average wages of emerging G20 economies (by triple) is more significant than the counterpart of advanced G20 economies (by 9\%) during 1999-2017.

\textsuperscript{35}Lu and Yu (2015) estimate the markup levels of China’s two-digit manufacturing industries and find that the markup levels in the majority of industries are within the range of [1,1.3]. Taking into account this fact and our assumption $\mu^S < \mu^N$, it is plausible to consider 1.3 as the upper bound of $\mu^S$ in this exercise.

\textsuperscript{36}According to (A.19), the threshold value $\pi$ is 0.9991.
5.3 Robustness check

We now perform two robustness checks on our numerical exercise to illustrate how the quantitative results will vary under different assumptions. Specifically, we first consider alternative values of the innovation-arrival rate and then of the R&D intensity.\(^{37}\)

5.3.1 Innovation-arrival rate

In this subsection, we consider two alternative values of the innovation-arrival rate \(\lambda^N \in \{0.12, 0.18\}\) and then recalibrate the elasticity of substitution parameter \(\sigma\) while other parameter values remain unchanged as in the benchmark. Similar to the benchmark case, we perform an experiment by raising \(\mu^S\) and \(\mu^N\), respectively, by 0.05 from their benchmark values, i.e., 1.2 and 1.3. Based on the new numerical results as shown in Table 3, it can be seen that the effects of IPR policy in either country are contingent on the value of the elasticity of substitution between intermediate products. A lower substitution elasticity tends to amplify the effects of Southern IPR \(\mu^S\) but weaken the effects of Northern IPR \(\mu^N\) on average product quality \(\Phi\) and the technology transfer rate \(\lambda^F\). Moreover, by enlarging wage changes in China and US as in (53), a lower elasticity of substitution magnifies the welfare effects of IPR policy. For example, in the case of \(\lambda^N = 0.18\), raising the degree of IPR in the US leads to large welfare gains in both the US (5.051%) and China (1.502%) as compared to the benchmark case, which is 4.780% and 1.318% respectively, and raising the degree of IPR in China yields an even large size of welfare gains in the two countries, namely 7.407% in the US and 7.857% in China. However, for a higher or lower value of \(\sigma\), the overall pattern of the cross-country effects of IPR policy remains the same as before. In other words, strengthening IPR in China has larger effects on the global welfare than that in US, and strengthening domestic IPR always causes more welfare gains for itself than the foreign country.

Table 3: Simulation under \(\lambda^N \in \{0.12, 0.18\}\)

<table>
<thead>
<tr>
<th>(\lambda^N = 0.12 (\sigma = 3.323))</th>
<th>(\Delta \Phi)</th>
<th>(\Delta \lambda^F)</th>
<th>(\Delta \omega)</th>
<th>(\Delta \ln \omega_0^N)</th>
<th>(\Delta \ln \omega_0^S)</th>
<th>(\Delta \ln c_0^N)</th>
<th>(\Delta \ln c_0^S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \mu^S = 0.05)</td>
<td>11.845%</td>
<td>10.874%</td>
<td>-0.213%</td>
<td>3.594%</td>
<td>3.815%</td>
<td>3.609%</td>
<td>3.933%</td>
</tr>
<tr>
<td>(\Delta \mu^N = 0.05)</td>
<td>9.292%</td>
<td>-11.508%</td>
<td>3.535%</td>
<td>4.479%</td>
<td>0.912%</td>
<td>4.491%</td>
<td>1.169%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\lambda^N = 0.18 (\sigma = 2.577))</th>
<th>(\Delta \Phi)</th>
<th>(\Delta \lambda^F)</th>
<th>(\Delta \omega)</th>
<th>(\Delta \ln \omega_0^N)</th>
<th>(\Delta \ln \omega_0^S)</th>
<th>(\Delta \ln c_0^N)</th>
<th>(\Delta \ln c_0^S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \mu^S = 0.05)</td>
<td>14.381%</td>
<td>7.583%</td>
<td>-0.274%</td>
<td>7.394%</td>
<td>7.689%</td>
<td>7.407%</td>
<td>7.857%</td>
</tr>
<tr>
<td>(\Delta \mu^N = 0.05)</td>
<td>6.899%</td>
<td>-9.041%</td>
<td>3.625%</td>
<td>5.045%</td>
<td>1.371%</td>
<td>5.051%</td>
<td>1.502%</td>
</tr>
</tbody>
</table>

\(^{37}\)We have also examined the conditions for the case that a rise in \(\mu^S\) may lead to a lower \(\lambda^F\) as shown in Proposition 2 and the case that a rise in \(\mu^N\) may lead to a temporary lower \(\lambda^N\) as shown in Proposition 3, respectively. We find that, as compared to the benchmark parameter values, by increasing the quality step size \(z\) and the relative manufacturing productivity \(\delta\) to a large magnitude, a very small size of Southern population \(\alpha\) causes a negative effect of \(\mu^S\) on \(\lambda^F\), whereas by decreasing the relative R&D productivity \(\gamma/\beta\) and the benchmark innovation arrival rate \(\lambda^N\), a very large size of Southern population \(\alpha\) induces a negative effect of \(\mu^N\) on \(\lambda^N\). Because these cases are less empirically related, we do not report them here and make them available upon request.
5.3.2 R&D intensity

As argued in Comin (2004) and Jones (2016), the aggregate R&D data are likely to underestimate the resources devoted into innovation-related activities. Thus, we follow Wolfe (2014) to adopt the disaggregated-level data of R&D intensity, that is the ratio of domestic R&D performance to domestic sales, for all industries. From the most recent Science and Engineering Indicators 2018,\(^{38}\) we find that the R&D intensity for all industries in the US is 3.9% in 2015. Then, the recalibrated values of \(\delta\) and \(\gamma/\beta\) and the new quantitative results are reported in Table 4. Given that the relative productivity of the Southern manufacturing labor to the Northern one is lower and the relative productivity of adaptive R&D to innovative R&D is higher, increasing Southern patent breadth \(\mu^S\) from 1.2 to 1.25 yields a larger decrease in the relative wage \(\omega\) (0.381% compared to 0.250% in the benchmark), while raising the Northern patent breadth \(\mu^N\) from 1.3 to 1.35 leads to a smaller increase in \(\omega\) (3.465% compared to 3.590% in the benchmark). As compared to the benchmark case, a smaller size in the wage increase and the welfare gain is observed when enhancing IPR in both countries. Again, the overall pattern of the cross-country effects of IPR policy still holds as in the benchmark estimation. Finally, strengthening IPR in China continues to be more conducive than that in US from the perspective of global welfare, whereas strengthening IPR benefits the domestic country more than the foreign country.

Table 4: Simulation under R&D intensity = 0.039

<table>
<thead>
<tr>
<th>(\Delta \Phi)</th>
<th>(\Delta \lambda^F)</th>
<th>(\Delta \omega)</th>
<th>(\Delta \ln \omega_0^N)</th>
<th>(\Delta \ln \omega_0^S)</th>
<th>(\Delta \ln c_0^N)</th>
<th>(\Delta \ln c_0^S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta = 0.062, \gamma/\beta = 94.156)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta \mu^S = 0.05)</td>
<td>13.326%</td>
<td>9.338%</td>
<td>-0.381%</td>
<td>5.310%</td>
<td>5.712%</td>
<td>5.331%</td>
</tr>
<tr>
<td>(\Delta \mu^N = 0.05)</td>
<td>7.538%</td>
<td>-9.932%</td>
<td>3.465%</td>
<td>4.496%</td>
<td>0.996%</td>
<td>4.508%</td>
</tr>
</tbody>
</table>

6 Conclusion

In this study, we analyze the cross-country effects of IPR protection on innovation and technology transfer in an open-economy Schumpeterian model with North-South product cycles. The IPR regime takes patent breadth as the policy instrument in both countries to capture the impacts of market power on the Northern and Southern firms’ R&D incentives. We find that stronger IPR protection in the South leads to a permanent decrease in the North-South wage gap, a temporary increase in the Northern innovation rate, and ambiguous impacts on technology transfer. Nevertheless, stronger IPR protection in the North leads to a permanent increase in the North-South wage gap, ambiguous effects on the Northern innovation rate, and a permanent decrease in technology transfer. In particular, the size of Southern population plays a critically different role in disambiguating the effect of Southern patent protection on technology transfer and the effect of Northern patent protection on innovation, respectively. By calibrating the model to the China-US data, our numerical analysis shows that strengthening IPR protection in China generates larger effects on the global welfare than that in US, and the welfare gain of strengthening IPR protection in the domestic country is more significant than the counterpart in the foreign country.

country. Therefore, this study presents an example in the welfare analysis that sheds some light on the justification for developing countries to make the upgrading of their IPR, following the agreements on TRIPS.

There are two potential dimensions to extend the present paper. First, the current model is based on an open-economy version of the quality-ladder model to explore the effects of patent breadth on innovation and international technology transfer. To characterize the important properties of the innovation structure, these effects could be reexamined in an open-economy version of the variety-expanding model as in Gustafsson and Segerstrom (2011). Moreover, the effects of patent breadth could be investigated in a framework with different modes of international technology transfer that abstract from FDI, such as licensing in Yang and Maskus (2001) and Tanaka et al. (2007) and imitation in Gustafsson and Segerstrom (2010) and Lorenczik and Newiak (2012). These crucial issues can represent interesting directions for future research.
Appendix A

A.1 Proof of Lemma 1

We follow Dinopoulos and Segerstrom (2010) to provide the proof. The dynamics of the quality index for $Q_t^N$ and $Q_t^F$ are given by respectively

$$
\dot{Q}_t^N = \int_{B^N} \left[ \kappa_n^N(j) - \kappa_n^N(j) \right] \lambda_t^N dj + \int_{\beta^N} \kappa_n^N(j+1) \lambda_t^N dj - \int_{B^N} \kappa_n^N(j) \lambda_t^F dj
= (k - 1) \lambda_t^N Q_t^N + \kappa \lambda_t^N Q_t^F - \lambda_t^F Q_t^F,
$$
(A.1)

and

$$
\dot{Q}_t^F = \int_{B^F} \kappa_n^F(j) \lambda_t^F dj - \int_{\beta^F} \kappa_n^F(j+1) \lambda_t^F dj = \lambda_t^F Q_t^N - \lambda_t^N Q_t^F.
$$
(A.2)

Since the industry composition is stationary over time in steady state, the growth rate of average quality in the North and the counterpart in the South must be equal to each other and constant over time. Therefore,

$$
\frac{Q_t^N}{Q_t^N} = \frac{Q_t^F}{Q_t^F}.
$$
(A.3)

Substituting (A.1) and (A.2) into (A.3), together with $Q_t = Q_t^N + Q_t^F$, yields (30) and (31).

A.2 Proof of Lemma 2

Following Dinopoulos and Segerstrom (2010), we assume that the Northern household finances innovative R&D in equilibrium such that $L_t^N a_t^N = \int_0^1 v_t^N(j) dj = \beta w_t^N \int_0^1 q_t(j) dj = \beta w_t^N Q_t$, where the second equality is obtained using (19). Hence,

$$
a_t^N = \beta w_t^N Q_t / L_t^N = \beta w_t^N \Phi.
$$
(A.4)

Using (50) and (A.4), we can show $c_t^N$ in Lemma 2. Moreover, the assumption that adaptive R&D is financed by the Southern household in equilibrium implies that $a_t^S = v_t^F / L_t^S$ and

$$
v_t^F = \int_{B^S} v_t^F(j) dj = \int_{B^S} \left[ \gamma w_t^S q_t(j) + v_t^N(j) \right] dj
= \gamma w_t^S \int_{B^S} q_t(j) dj + \beta w_t^N \int_{B^S} q_t(j) dj = \left( \gamma w_t^S + \beta w_t^N \right) Q_t^N,
$$
(A.5)

where the third equality uses (19) and the last equality uses the definition of $Q_t^N$ in (29). Thus,

$$
a_t^S = \left( \gamma w_t^S + \beta w_t^N \right) \frac{Q_t^N Q_t}{L_t^N} = \left( \gamma w_t^S + \beta w_t^N \right) \frac{\kappa \lambda^0 \Phi(1 - \alpha)}{\alpha (\kappa \lambda^0 + \lambda F)},
$$
(A.6)

where $Q_t^N / Q_t$ is from Lemma 1. Using the relation $c_t^S = w_t^S + (\rho - g_L)a_t^S$, we obtain $c_t^S$ in Lemma 2. Finally, the expression of $w_t^S$ is obtained by substituting (10) and (11) into the aggregate price.
\[
\left\{ \int_0^1 \frac{1}{p_t(j)} \, dj \right\}^{1-\sigma} = 1
\]
\[
\Leftrightarrow \left\{ \int_{\theta^N} \left[ \frac{\mu_N^w j^2}{\partial \omega^N(j)} \right]^{1-\sigma} \, dj + \int_{\theta^F} \left[ \frac{\mu_F^w j^2}{\partial \omega^F(j)} \right]^{1-\sigma} \, dj \right\}^{1-\sigma} = 1
\]
\[
\Leftrightarrow \left\{ \left( \mu^N \right)^{1-\sigma} Q_N^t + \left( \mu^S \right)^{1-\sigma} Q_F^t \right\}^{1-\sigma} = 1
\]
\[
\Leftrightarrow w^S_t = \delta Q_t \left\{ \left( \mu^N \right)^{1-\sigma} \frac{\kappa L_N}{\kappa L_N + \lambda F} + \left( \mu^S \right)^{1-\sigma} \frac{\lambda F}{\kappa L_N + \lambda F} \right\}^{1-\sigma},
\]
where we have used the definitions of \( Q_N^t \) and \( Q_F^t \) in (29), and \( Q_N^t / Q_t \) and \( Q_F^t / Q_t \) from Lemma 1. Using \( Q_0 = \Phi L_0^N \) then yields \( w_0^S \) in Lemma 2.

### A.3 Proof of Proposition 1

First, we examine the effect of \( \mu^S \) on \( \omega \). Differentiating (55) with respect to \( \mu^S \) yields

\[
\frac{\partial \omega}{\partial \mu^S} \left( \frac{\mu^N}{\delta \omega^2} + \frac{\beta}{\gamma} \right) > 0
\] is equivalent to examine whether

\[
\sigma \left( 1 - 1/\mu^S \right) (\mu^N / \mu^S) - (\mu^N / \mu^S) \sigma < 0 \Leftrightarrow \mu^S < \frac{\sigma}{\sigma - 1},
\]
which holds according to the condition implied by (11). Hence, the wage ratio \( \omega \) is decreasing in the degree of Southern patent protection \( \mu^S \).

Next, we examine the effect of \( \mu^N \) on \( \omega \). Differentiating (55) with respect to \( \mu^N \) yields

\[
\frac{\partial \omega}{\partial \mu^N} \left( \frac{\mu^N}{\delta \omega^2} + \frac{\beta}{\gamma} \right) > 0
\] is equivalent to examine whether

\[
\frac{1}{\omega} \geq - \frac{\beta}{\gamma} \left[ 1 - \sigma (\mu^S - 1) (\mu^N)^{\sigma-1} \right].
\]

By defining a function \( f(\mu^N, \mu^S) = 1 - \sigma (\mu^S - 1)(\mu^N)^{\sigma-1}(\mu^S)^{-\sigma} \), we then show that \( f(\mu^N, \mu^S) \) is monotonically decreasing in \( \mu^N \) and \( \mu^S \) such that

\[
\frac{\partial f(\mu^N, \mu^S)}{\partial \mu^N} < 0; \quad \frac{\partial f(\mu^N, \mu^S)}{\partial \mu^S} < 0.
\]
Given that the domains for \( \mu^N \) and \( \mu^S \) are \( 1 < \mu^N \leq \sigma/(\sigma - 1) \) and \( 1 < \mu^S < \mu^N \), respectively, we then have

\[
f(\mu^N, \mu^S) > f_{\min}(\mu^N, \mu^S) = f\left(\frac{\sigma}{\sigma - 1}, \frac{\sigma}{\sigma - 1}\right) = 0. \tag{A.12}
\]

Therefore,

\[
\frac{1}{\omega} > -\frac{\beta}{\gamma} \left[1 - \sigma(\mu^S - 1)(\mu^N)^{\sigma - 1}\right] \Leftrightarrow \partial \omega / \partial \mu^N > 0,
\]

indicating that the wage ratio \( \omega \) is increasing in the degree of Northern patent protection \( \mu^N \).

Furthermore, we can show that \( \omega \) is a concave function of \( \mu^N \). We prove by contradiction. Suppose \( \partial \omega / \partial \mu^N \geq \omega / \mu^N \). Then according to the steady-state relative-wage condition in (55) such that \( \mu^N / (\delta \omega) = (\beta / \gamma)\omega + 1 - [\beta / (\gamma \delta)][\mu^N - (\mu^S - 1)(\mu^N / \mu^S)^\sigma] \), differentiating \( \mu^N / (\delta \omega) \) with respect to \( \mu^N \) yields

\[
\frac{\partial[\mu^N / (\delta \omega)]}{\partial \mu^N} = \frac{\beta}{\gamma} \frac{\partial \omega}{\partial \mu^N} - \frac{\beta}{\gamma \delta} [1 - \sigma(\mu^S - 1)(\mu^N)^{-\sigma} - (\mu^N)^{\sigma - 1}] \\
> \frac{\beta}{\gamma} \frac{\partial \omega}{\partial \mu^N} - \frac{\beta}{\gamma \delta} \frac{1}{\mu^N} \left[ \mu^N - (\mu^S - 1) \left( \frac{\mu^N}{\mu^S} \right)^\sigma \right] \\
> \frac{\partial \omega}{\partial \mu^N} \left( \frac{\partial \omega}{\partial \mu^N} - \frac{\omega}{\mu^N} \right) > 0,
\]

where the inequality in the second line uses \( \sigma > 1 \) and the inequality in the third line uses Assumption 2. However, a direct differentiation of \( \mu^N / (\delta \omega) \) with respect to \( \mu^N \) shows

\[
\frac{\partial[\mu^N / (\delta \omega)]}{\partial \mu^N} = \mu^N \left( \frac{\omega - \partial \omega / \partial \mu^N}{\mu^N - \partial \omega / \partial \mu^N} \right) \leq 0,
\]

which contradicts with (A.13). Therefore, \( \partial \omega / \partial \mu^N < \omega / \mu^N \) must hold, indicating that \( \omega \) is a concave function of \( \mu^N \).

## A.4 Proof of Proposition 2

Given the result in Proposition 1 such that \( \omega(\mu^N, \mu^S) \) is decreasing in \( \mu^S \), it is easy to see both from the Northern steady-state R&D condition (47) and the Southern steady-state R&D condition (48) that as \( \mu^S \) rises, \( \Phi \) increases or \( \lambda^F \) decreases in response. Graphically, a rise in \( \mu^S \) shifts both the North and South curves to the right. As a result, a higher \( \mu^S \) unambiguously raises \( \Phi \), and according to (41), a permanent higher \( \Phi \) must be associated with a temporary increase in the innovation rate \( \lambda^N \) above its steady-state level \( \lambda^N = g_L / (\sigma - 1) \). This completes the proof for (i).

As for (ii), combining (47) and (48) to solve for \( \lambda^F \) yields

\[
\lambda^F = \beta \kappa \lambda^N \frac{\delta(\rho + \lambda^N)}{\mu^N / \omega - \delta} + \lambda^N \left[ (\rho + \lambda^N)(\gamma + \beta \omega) / \mu^S - 1 \right] + \kappa \gamma \lambda^N \left( \frac{1 - \alpha}{\alpha} - \beta \lambda^N \right). \tag{A.15}
\]
Using the condition of
\[
\frac{\gamma + \beta \omega}{\mu^S - 1} = \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{\beta}{\mu^N/\omega - \delta}
\]
from (55) to rewrite (A.15) and then differentiating \( \lambda^F \) with respect to \( \mu^S \), we obtain
\[
\frac{\partial \lambda^F}{\partial \mu^S} \geq 0
\]

\[
\implies \delta(\rho + \lambda^N) \frac{\mu^N}{\omega} \frac{\partial \omega}{(\mu^N/\omega - \delta)^2} \frac{\partial \omega}{\partial \mu^S} \left\{ \left[ \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{\beta(\rho + \lambda^N)}{\mu^N/\omega - \delta} + \frac{\kappa \gamma \lambda^N}{\mu^S} \right] \frac{1 - \alpha}{\alpha} - \beta \lambda^N \right\}
\]

\[
- \frac{1 - \alpha}{\alpha} \left[ \delta(\rho + \lambda^N) \frac{\mu^N}{\omega} \frac{\partial \omega}{\partial \mu^S} \left\{ \left[ \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{\beta(\rho + \lambda^N)}{\mu^N/\omega - \delta} - \frac{\beta \lambda^N \mu^N \delta(\rho + \lambda^N)}{\omega^2(\mu^N/\omega - \delta)^2} \right] \frac{\partial \omega}{\partial \mu^S} \right\}
\right.
\]

\[
+ \frac{1 - \alpha}{\alpha} \frac{\partial \omega}{\partial \mu^S} \left\{ \left[ \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{\beta(\rho + \lambda^N)}{\mu^N/\omega - \delta} - \frac{\beta \lambda^N \mu^N (\rho + \lambda^N)}{\omega^2(\mu^N/\omega - \delta)^2} \right] \left[ \frac{\delta \kappa \gamma - \beta}{\beta} \left( \frac{\mu^N}{\mu^S} \right)^\sigma - \frac{\delta \alpha \beta}{1 - \alpha} \right] \right\} \geq 0
\]

\[
\leq \delta(\rho + \lambda^N) \frac{\mu^N}{\omega} \frac{\partial \omega}{\partial \mu^S} \frac{\partial \omega}{\partial \mu^S} \left\{ \left[ \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{\beta(\rho + \lambda^N)}{\mu^N/\omega - \delta} + \frac{\partial \omega}{\partial \mu^S} \left[ \left( \frac{\mu^N}{\mu^S} \right)^\sigma \frac{\beta(\rho + \lambda^N)}{\mu^N/\omega - \delta} - \frac{\delta \alpha \beta}{1 - \alpha} \right] \right\} \geq 0
\]

\[
\leq \frac{\sigma}{\mu^S} \frac{\mu^N}{\mu^S} \left( \frac{\mu^N}{\mu^S} \right)^\sigma \delta \rho + \lambda^N \frac{\mu^N}{\omega^2} + \frac{\delta \kappa \lambda^N \mu^N}{\omega^2} + \frac{\delta \kappa \lambda^N \mu^N}{\omega^2} \left[ \frac{\gamma - 1 - \frac{\alpha}{\kappa(1 - \alpha)}}{\beta} \right] \geq 0
\]

\[
\leq \frac{\alpha}{\mu^S - 1 - \delta} + \frac{\rho + \lambda^N \mu^N/(\delta \omega)(\gamma \mu^N/\beta + \delta \omega^2)}{\lambda^N \mu^N(1 + \mu^S/\sigma - \mu^S)} \geq 0,
\]

(A.16)

where we have divided both sides by \( \beta(\rho + \lambda^N)/(\mu^N/\omega - \delta)^2 \) to obtain the expression in the fourth inequality, and using (A.8) in the fifth inequality. We can see from the last inequality in (A.16) that there exists a threshold value \( \alpha \). When \( \alpha > \alpha_0, \lambda^F \) is increasing in \( \mu^S \). Otherwise, when \( \alpha < \alpha_0, \partial \lambda^F / \partial \mu^S \) is negative.

### A.5 Proof of Proposition 3

First, how an increase in \( \mu^N \) affects the Northern steady-state condition (47) depends on the effect of \( \mu^N \) on \( \mu^N/(\delta \omega(\mu^N, \mu^S)) \). From (A.14), since \( \omega(\mu^S, \mu^N) \) is concave in \( \mu^N \), we know that \( \mu^N/(\delta \omega(\mu^N, \mu^S)) \) increases as \( \mu^N \) increases. This implies that as \( \mu^N \) increases, \( \Phi \) increases and \( \lambda^F \) decreases in response. Graphically, an increase in \( \mu^N \) shifts the North curve in Figure 1 to the right. Moreover, an increase in \( \mu^N \) drives up \( \omega \) and unambiguously raises \( \Phi \) and \( \lambda^F \) according to the Southern steady-state condition (48). As shown in Figure 1, a higher \( \mu^N \) shifts the South curve

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to the left. Hence, the overall effect of a rise in $\mu^N$ unambiguously lowers $\lambda^F$. This completes the proof for (ii).

As for (i), rewriting $\lambda^F$ from (47) to
\[ \lambda^F = \frac{\beta \lambda^N \Phi_k (\rho + \lambda^N)}{[\mu^N/(\delta \omega) - 1](1 - \beta \lambda^N \Phi)} - \kappa \lambda^N, \] (A.17)
and substituting it into (48) to solve for $\Phi$ yields
\[ \Phi = \frac{1}{\rho + \lambda^N \mu^N/(\delta \omega)} \left[ \frac{\alpha (\rho + \lambda^N)}{(1 - \alpha) \chi_1} + \frac{\mu^N/(\delta \omega) - 1}{\beta \chi_1} \right], \] (A.18)
where
\[ \chi_1 = \frac{(\rho + \lambda^N)(\gamma + \beta \omega)}{\mu^N - 1} + \gamma \kappa \lambda^N. \]

Differentiating $\Phi$ with respect to $\mu^N$ yields
\[
\begin{align*}
\frac{\partial \Phi}{\partial \mu^N} & \geq 0 \\
\Leftrightarrow - \left[ - \frac{\alpha \beta (\rho + \lambda^N)^2}{(1 - \alpha)(\mu^N - 1)\chi_1^2 \partial \mu^N} \frac{\partial \omega}{\delta \omega} + \frac{1}{\beta} \left( \frac{1}{\delta \omega} - \frac{\mu^N \partial \omega}{\delta \omega^2 \partial \mu^N} \right) \right] & \geq \frac{1}{\rho + \lambda^N \mu^N/(\delta \omega)} \\
\Leftrightarrow - \left[ - \frac{\alpha \lambda^N (\rho + \lambda^N)}{(1 - \alpha) \chi_1} + \frac{\lambda^N \mu^N/(\delta \omega) - \lambda^N}{\beta} \right] & \geq \frac{1}{\delta \omega} - \frac{\mu^N \partial \omega}{\delta \omega^2 \partial \mu^N} \\
\Leftrightarrow \frac{1}{\beta} \left( \frac{1}{\delta \omega} - \frac{\mu^N \partial \omega}{\delta \omega^2 \partial \mu^N} \right) & \geq \frac{1}{(\mu^N/(\delta \omega) - 1)\chi_1 - \beta \lambda^N} \geq \frac{\beta (\rho + \lambda^N)(\rho + \lambda^N \mu^N/(\delta \omega))}{(\mu^N/(\delta \omega) - 1)\chi_1} \frac{\partial \omega}{\partial \mu^N} \\
\Leftrightarrow \frac{1 - \alpha}{\alpha} & \geq \frac{\beta^2 \delta \omega (\rho + \lambda^N)[\rho + \lambda^N \mu^N/(\delta \omega)]}{\chi_1^2 (\mu^N - 1)[(\partial \omega/\partial \mu^N) - \mu^N/\omega]} + \frac{\beta \lambda^N}{\chi_1},
\end{align*}
\] (A.19)

where $(1 - \alpha)\chi_1/\alpha > \beta \lambda^N$ in the third inequality must hold because it ensures a positive $\lambda^F$ in (A.15). Denote by $\bar{\pi}$ the expression in the RHS of the last inequality. Thus, when $\alpha < \bar{\pi}$, a rise in $\mu^N$ increases $\Phi$ and then a temporary higher rate of innovation $\lambda^N$ according to (41); otherwise, when $\alpha > \bar{\pi}$, a rise in $\mu^N$ decreases $\Phi$ permanently and $\lambda^N$ temporarily. This completes the proof for (i).
### A.6 Calibration strategy

- **R&D intensity in the US:**

\[
2.6% = \frac{w_0^N L_0^N}{w_{0r}^N L_{0r}^N + c_0^N L_0^N} = \frac{L_0^N}{L_{0r}^N + (c_0^N / w_0^N)L_0^N} \frac{L_0^N}{L_{0r}^N + [\beta\Phi(\rho - g_L) + 1]L_0^N}
\]

where we have used \( c_0^N / w_0^N = \beta\Phi(\rho - g_L) + 1 \) from (51) and (53), and (34) in sequence. Rearranging (A.20) yields

\[
\beta\Phi = \frac{2.6\%}{\lambda N - 2.6\%(\lambda N + \rho - g_L)}.
\]

- **Northern-steady-state condition in (47):**

\[
\delta = \frac{\mu^N(\kappa\lambda^N + \lambda^F)(1 - \beta\lambda^N\Phi)}{\omega\kappa\lambda^N\beta\Phi(\rho + \lambda^N) + \omega(\kappa\lambda^N + \lambda^F)(1 - \lambda^N\beta\Phi)}.
\]

- **Southern-steady-state condition in (48):**

\[
1 = \frac{\beta\Phi\lambda^F(1 - \alpha)}{\alpha(\kappa\lambda^N + \lambda^F)} \left\{ \frac{(\rho + \lambda^N)(\gamma / \beta + \omega)}{\mu^S - 1} + \frac{\gamma\kappa\lambda^N}{\beta} \right\}
\]

\[
\Leftrightarrow \frac{\gamma}{\beta} \left( \frac{\rho + \lambda^N}{\mu^S - 1} + \kappa\lambda^N \right) = \frac{\alpha(\kappa\lambda^N + \lambda^F)}{\beta\Phi\lambda^F(1 - \alpha)} - \frac{\omega(\rho + \lambda^N)}{\mu^S - 1}
\]

\[
\Leftrightarrow \frac{\gamma}{\beta} = \frac{\alpha(\kappa\lambda^N + \lambda^F)(\mu^S - 1) - \omega\beta\Phi\lambda^F(1 - \alpha)(\rho + \lambda^N)}{\beta\Phi\lambda^F(1 - \alpha)[\rho + \lambda^N + \kappa\lambda^N(\mu^S - 1)]}.
\]

- **Steady-state relative wage condition (55):**

\[
\frac{\mu^N}{\delta\omega} - \frac{\beta}{\gamma} \omega = 1 - \frac{\beta}{\gamma}\delta \left[ \mu^N - (\mu^S - 1) \left( \frac{\mu^N}{\mu^S} \right)^\sigma \right]
\]

\[
\Leftrightarrow \frac{\gamma}{\beta} \frac{\mu^N}{\omega} - \delta\omega - \frac{\gamma}{\beta}\delta + \left[ \mu^N - (\mu^S - 1) \left( \frac{\mu^N}{\mu^S} \right)^\sigma \right] = 0.
\]

Therefore, given \( \beta\Phi \) in (A.21) and other exogenously given parameters, substituting both (A.22) and (A.23) into (A.24) solves for \( \lambda^F \). Then, substituting the resulting value of \( \lambda^F \) back into (A.22) and (A.23) yields \( \delta \) and \( \gamma / \beta \), respectively.
References


