Monetary policy in a Schumpeterian economy with endogenous fertility and human capital accumulation*

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Abstract

This study investigates the growth and welfare effects of monetary policy in a Schumpeterian economy featuring cash-in-advance (CIA) constraints and two engines of growth: innovation from R&D and human capital accumulation from endogenous fertility. Our theoretical analysis considers the cases of various CIA constraints. When the CIA constraint is only on consumption, higher inflation retards economic growth by weakening human capital accumulation. When the CIA constraint is only on R&D, higher inflation would generate a negative or U-shaped effect on economic growth, depending on the interplay between inflationary effects on innovation and human capital accumulation. When the CIA constraint is only on manufacturing, the growth effect of inflation could be positive (negative) if the positive growth effect from technological progress dominates (is dominated by) the negative effect from human capital accumulation. Our quantitative analysis finds a generally negative inflation-growth relationship in the calibrated economy. Moreover, the welfare effect of inflation is also negative, implying that the Friedman rule is optimal.

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1 Introduction

In recent years, researchers have investigated the link between demographic changes and inflation (see, for example, Bullard *et al.* 2012, Broniatowska 2019, Juselius and Takáts 2021), considering that the interaction between demographic changes and inflation may affect the effectiveness of monetary policy. Monetary policy, as a common technique of policy regulation, plays a crucial role in economic growth and development. However, demographic changes have received relatively less attention in analyzing how they impact monetary-policy outcomes. This paper aims to revisit the effects of monetary policy when demography is taken into account and reveals a subtle and novel channel – fertility choices – for monetary policy to impact economic growth and social welfare.

Linking fertility choices to monetary policy may seem unrelated at first glance. However, humans, if rational, optimize their fertility choices along with other decisions such as consumption and investments. More importantly, existing empirical studies provide evidence for the correlation between fertility and monetary policy (see He 2018a and Affuso *et al.* 2022). Given these facts and evidence and considering that fertility choices and human capital accumulation goes hand in hand,[1] this study develops a dynamic general framework to reexamine the impacts of monetary policy on economic growth by incorporating endogenous fertility and human capital into the Schumpeterian quality-ladder model in Grossman and Helpman (1991), where long-run economic growth depends on two independent growth engines: technological progress and human capital accumulation. In this study, the endogenous fertility choice operates through the growth engines in a different fashion. On the one hand, a higher fertility rate crowds out households' time allocation to education and dilutes human capital per household member, stifling human capital accumulation. On the other hand, a higher fertility rate crowds out the households' time allocation to working activities, slowing technical progress. Consequently, endogenous fertility makes the two growth engines independent upon each other, clarifying the mechanism for the impacts of monetary policy through different channels; this is the main contribution of this study.

Given the growth-theoretic framework in this study, money is introduced by distinct cash-in-advance (CIA) constraints. In addition to the well-established CIA constraints on consumption as in Lucas (1980) and Dotsey and Sarte (2000), we consider CIA constraints on R&D and manufacturing, which is supported by empirical findings on firms' liquidity constraints. First, an earlier empirical study by Bates *et al.* (2009) shows that the average cash-to-assets ratio for US industrial firms has increased dramatically and more than doubled from 1980 to 2006, indicating that firms' behavior is severely constrained by liquidity. Liu *et al.* (2008) provide evidence that firms' manufacturing activities are constrained by cash in advance. Moreover, recent studies reveal that R&D investments suffer from liquidity constraints.[2] For example, Hall and Lerner (2010) show that over 50% of R&D spending is wage payment to highly educated scientists and engineers, and high adjustment costs lead R&D firms to hold cash in order to smooth their R&D spending over time.[3] Furthermore, Brown *et al.* (2012) and Brown and Petersen (2015) suggest that firms

[1] Li *et al.* (2008) provide evidences for the quantity-quality tradeoff in having children.

[2] See Chu *et al.* (2015) for the literature review on the presence of a CIA constraint on R&D.

[3] The innovation efforts of high technology workers are firms' knowledge base that generates profits. Firing R&D workers could result in large hiring and training costs as well as a dramatic decrease in firms' profits. See Hall and Lerner (2010) for more details.

tend to use cash reserves to build and manage a buffer stock of liquidity to smooth their R&D expenditures.

In this monetary Schumpeterian growth model with endogenous fertility and human capital, we analytically examine the polar cases that are subject to only one type of CIA constraint. We show that the growth effects of monetary policy can be diverse, which stands in stark contrast to existing studies by generalizing their results. First, in the presence of a CIA constraint on consumption, a higher nominal interest rate retards long-run economic growth by increasing the fertility rate and thereby decreasing human capital accumulation, given that the growth rate of technology is unaffected by the nominal interest rate.

Second, in the presence of a CIA constraint on R&D, a higher nominal interest rate shifts the human-capital-embodied labor the CIA-constrained sector to the non-constrained one, hence the growth rate of technology is decreasing in the nominal interest rate. At the same time, human capital accumulation exhibits a U-shaped relationship with the nominal interest rate. The reason stems from the inverted U-shaped relationship between the nominal interest rate and the fertility rate:[4] raising the nominal interest rate generates a positive effect and three negative effects on the opportunity cost of fertility. The interplay among all these effects leads the cost of having children to initially decrease in the nominal interest rate until it reaches a threshold value i^*. Beyond this value i^*, the fertility rate will first increase and then decrease in i. Due to the quantity-quality tradeoff of having children, human capital accumulation becomes a U-shaped function of the nominal interest rate. Therefore, with a CIA constraint on R&D, the overall effect of the nominal interest rate on economic growth can be either U-shaped or negative, depending on whether a threshold value exists to make the positive effect of i on g_h dominate its negative effect on g_Z.

Finally, in the presence of a CIA constraint on manufacturing, technological progress is increasing in the nominal interest rate, whereas human capital accumulation is decreasing in it. In this case, the overall effect on the long-run economic growth depends on these two counteracting engines. If the step size is sufficiently large, the overall impact of the nominal interest rate on economic growth would be positive; otherwise, there exists a threshold value of i^+ above (below) which human capital accumulation dominates (is dominated by) R&D-based innovation, leading to a negative (positive) growth effect. Through the above cases, our theoretical analysis outlines the roles of various CIA constraints in the interaction between human capital accumulation and R&D-based innovation in a monetary Schumpeterian growth framework.

By applying the US aggregate data, our quantitative analysis finds a negative relationship between inflation and growth in the benchmark case, which is in line with the results in existing empirical findings (see Vaona 2012; Barro 2013; Chu *et al.* 2014). In addition, we quantify the impacts of inflation on human capital accumulation, innovation and long-run economic growth under different CIA constraints. As for social welfare, it also monotonically decreases in inflation, indicating that the Friedman rule (i.e., the zero nominal interest rate) is optimal in the benchmark

[4] On the one hand, some empirical studies provide evidence for the positive correlation between fertility and monetary policy. For example, He (2018a) finds a significant, positive effect of inflation on fertility rate in instrumental variables (IV) estimation with a panel data for 12 advanced countries during 2000-2014. Moreover, by using a structural VAR model and a Toda–Yamamoto causality test on the annual US data between 1975-2020, Affuso *et al.* (2022) shows that an interest rate shock significantly increases the fertility rate. On the other hand, some studies, such as Juselius and Takáts (2015) and Juselius and Takáts (2016), find that population aging is inflationary, which indicates a negative correlation between fertility and inflation.

case. These quantitative results continue to hold in our robustness checks.

2 Literature review

This study is closely related to existing studies that analyze the relationship between inflation and economic growth in endogenous R&D-based growth models. The pioneering study by Marquis and Reffett (1994) examines the growth effects of inflation by introducing a transaction service sector and a CIA constraint on consumption to the Romer model. A large number of subsequent studies (see, for example, Chu and Lai 2013; Chu and Cozzi 2014; Huang *et al.* 2017; Zheng *et al.* 2021; Hu *et al.* 2021) analyze monetary policy in a Schumpeterian model. Nevertheless, the majority of these studies focus on the framework in which the engine of growth is only endogenous technological progress.[5] Our study is mostly related to Chu *et al.* (2019b), who introduce endogenous human capital into a monetary Schumpeterian growth model and find that the additional long-run growth effect under endogenous human capital accumulation amplifies the welfare effect of monetary policy. One implication of Chu *et al.* (2019b) suggests that long-run growth is solely determined by human capital accumulation, since the long-run growth rate of technology depends on the growth rate of human capital in their model, which further indicates that endogenous human capital accumulation would strengthen the effects of the nominal interest rate on economic growth. In contrast to their study, in our model, the growth rate of technology and the growth rate of human capital are independently determined, and endogenous human capital accumulation may either strengthen the growth effect of the nominal interest rate or weaken its growth effect, leading to diverse inflation-growth relationships. Therefore, our study complements their interesting study and contributes to this literature by allowing for endogenous fertility and human capital accumulation and developing a monetary model with two independent growth engines. Moreover, in addition to CIA constraints on consumption and R&D, our study also considers the CIA constraint on manufacturing.

Furthermore, existing literature attempts to explore various channels through which monetary policy influences growth and welfare.[6] In particular, our study is highly related to He (2018a), who reveals a novel channel of fertility choice through which monetary policy impacts economic growth in a quality-ladder model. He finds a positive effect of the nominal interest rate on fertility, which in turn reduces labor supply to production and R&D and thereby decreases the long-run economic growth. Different from his study, in our model, endogenous fertility is not only a crucial channel for monetary policy to impact innovation but also a determinant of human capital accumulation. Moreover, we consider the roles of CIA constraints on other activities (i.e., R&D and manufacturing) in addition to the counterpart of a CIA constraint on consumption. To the best of our knowledge, this is the first study that analyzes the growth and welfare effects of monetary policy in a dynamic general equilibrium (DGE) model featuring endogenous fertility and human capital accumulation.

[5]See exceptions such as Chu *et al.* (2019a), Chu *et al.* (2019b), and Gil and Iglésias (2020).

[6]Mao *et al.* (2019) model a banking sector to examine the effects of additional monetary policy instruments, such as the required reserve ratio and the leverage ratio. Lin *et al.* (2020) introduce credit constraint to analyze the relationship between inflation, economic growth and financial development. He *et al.* (2023) show that inflation can promote long-run economic growth when the spirit of capitalism is strong.

This study also relates to the literature on the interaction between innovation and human capital. Previous studies have investigated many issues related to this topic. For example, some earlier studies (such as Blackburn *et al.* 2000; Zeng 2003) attempt to remove the scale effect by endogenizing human capital. Strulik (2005) focus on the relationship between population growth and economic growth. Galor (2005) and Strulik and Weisdorf (2008) endogenize people's fertility in a unified growth framework to explain demographic transition and economic development from stagnation to growth through the interplay among population, human capital and technological progress. Our study differs from their interesting studies by focusing on how monetary policy impacts the interaction between endogenous technical progress and human capital accumulation.

The rest of this study is organized as follows. Section 3 describes the model setup. Section 4 defines the equilibrium and solves the model. In Sections 5 and 6, we examine the implications of monetary policy on economic growth and social welfare, analytically and numerically. Section 7 concludes this study.

3 A monetary Schumpeterian growth model with endogenous fertility and human capital accumulation

In this section, we follow Chu *et al.* (2013) to extend the Schumpeterian quality-ladder model of Grossman and Helpman (1991) by allowing for endogenous fertility and human capital, generating two independent growth engines: endogenous technological progress and human capital accumulation. Moreover, we introduce CIA constraints on consumption, R&D, and manufacturing, respectively, as in Chu and Cozzi (2014) and Zheng *et al.* (2021). Throughout the analysis, the nominal interest rate serves as the monetary policy instrument and we explore the implications of monetary policy on growth and welfare by altering the nominal interest rate.

3.1 Households

Suppose that a closed economy admits a unit continuum of identical households whose lifetime utility is given by[7]

$$U = \int_0^\infty e^{-\rho t} \left(\ln c_t + \alpha \ln n_t \right) dt, \tag{1}$$

where $\rho > 0$ is the discount rate and $\alpha > 0$ determines households' preference for fertility relative to consumption. c_t is the per capita consumption of the final good (numeraire) and n_t is the number of births per person at time t. Following Chu *et al.* (2013), we assume zero mortality, so n_t also denotes the population growth rate. Assuming that N_t is the size of the population, the total number of births is $\dot{N}_t = n_t N_t$.

Each household maximizes (1) subject to the asset-accumulation equation and a CIA con-

[7] Following Chang *et al.* (2013), in addition to consumption, the fertility rate also enters each individual's utility function, since individuals enjoy the happiness in the process of raising children influenced by an emotive aspect. This setting is in line with the standard treatment of endogenous fertility as in Razin and Ben-Zion (1975) and Yip and Zhang (1997).

straint, which are, respectively, given by

$$\dot{a}_t + \dot{m}_t = (r_t - n_t)\, a_t + w_t l_t + i_t b_t - c_t - (\pi_t + n_t)\, m_t + \tau_t, \tag{2}$$

and

$$\eta_c c_t + b_t \leq m_t, \tag{3}$$

where a_t is the value of real assets that each individual owns (in the form of equity shares in monopolistic intermediate goods firms) and r_t is the real interest rate. Each person supplies l_t units of human-capital-embodied labor to earn a real wage rate w_t and lends an amount b_t of money to firms to finance R&D and/or manufacturing, with the rate of return i_t (i.e., the nominal interest rate). m_t and π_t are the real money balance held by each person and the inflation rate that measures the cost of money holding, respectively. An increase in n_t reduces the number of assets per capita and the real money balance per capita, referred to as the asset-diluting effect of fertility and the money-balance-diluting effect. Equation (3) indicates that the holding of real money balance m_t by each person is used to finance R&D and manufacturing and to support partial consumption, where $\eta_c \in [0,1]$ denotes the strength of the CIA constraint on consumption.

Each person owns one unit of time endowment to allocate between non-working and working activities. Non-working activities include the production of children and education that produces human capital, whereas working activities include the production of intermediate goods and R&D. In line with Chu *et al.* (2013), we assume that the time spent on fertility is given by $n_t/\theta < 1$, where $\theta > 0$ is a parameter that measures the time cost of fertility. Combining with the stock of human capital per capita h_t, each individual spends her remaining time endowment $1 - n_t/\theta$ on education and work such that

$$h_t\,(1 - n_t/\theta) = l_t + e_t, \tag{4}$$

where l_t is the level of human capital devoted to work and e_t is the level of human capital devoted to education. Notably, as n_t increases, less time is available for work and education, reflecting the forgone-wage effect of fertility. The law of motion for human capital per capita is given by

$$\dot{h}_t = \xi e_t - (n_t + \delta)\, h_t, \tag{5}$$

where $\xi > \rho$ denotes the productivity parameter of human capital accumulation and $\delta \geq 0$ is the depreciation rate of human capital. Note that $n_t h_t$ captures the human-capital-diluting effect of children as in Strulik (2005). The standard dynamic optimization (see Appendix A.1) implies the following optimality conditions. Specifically, the optimality condition for consumption is

$$c_t = \frac{1}{\mu_t\,(1 + \eta_c i_t)}, \tag{6}$$

where μ_t is the Hamiltonian co-state variable on (2). The familiar Euler equation is

$$\frac{\dot{c}_t}{c_t} = -\frac{\dot{\mu}_t}{\mu_t} = r_t - n_t - \rho. \tag{7}$$

The optimal condition that determines the consumption-fertility tradeoff is

$$\frac{\alpha}{n_t} = \frac{1}{c_t(1+\eta_c i_t)} \left[a_t + m_t + \left(\frac{1}{\theta} + \frac{1}{\xi} \right) w_t h_t \right], \tag{8}$$

which indicates that the marginal utility of fertility (given by the LHS of (8)) equals the marginal cost of fertility (given by the RHS of (8)). As in Chu *et al.* (2013) and He (2018a), the first term $a_t/[c_t(1+\eta_c i_t)]$ captures the asset-diluting effect of fertility, which is positively related to the value of a_t (i.e., a higher a_t means a higher asset-dilution effect of newborn). The second term $m_t/[c_t(1+\eta_c i_t)]$ captures the money-balance-diluting effect of fertility; similarly, this effect is positively correlated with the value of m_t (i.e., an increase in m_t raises the money-balance-diluting effect of children). The third term $\theta^{-1}w_t h_t/[c_t(1+\eta_c i_t)]$ captures the foregone-wage effect of fertility, and the last term $\xi^{-1}w_t h_t/[c_t(1+\eta_c i_t)]$ captures the human-capital-diluting effect of fertility. These two effects are both positively related to the wage rate w_t. The presence of the CIA constraint on consumption $\eta_c i_t$ mitigates the above effects of fertility.

Additionally, we derive an equilibrium condition equating the returns on assets and human capital such that

$$r_t = \frac{\dot{w}_t}{w_t} + \xi \left(1 - \frac{n_t}{\theta} \right) - \delta. \tag{9}$$

This condition determines the equilibrium growth rate of human capital. Finally, the no-arbitrage condition between all assets and money implies the Fisher equation given by $i_t = r_t + \pi_t$.

3.2 Final goods

By aggregating differentiated intermediate goods, competitive firms produce final goods according to the following Cobb-Douglas production function:

$$Y_t = \exp \left(\int_0^1 \ln X_t(j) dj \right) \tag{10}$$

where $X_t(j)$ denotes the quantity of intermediate goods $j \in [0, 1]$. Taking the price of each variety of intermediate goods $p_t(j)$ as given, final-good firms' profit maximization yields the conditional demand for $X_t(j)$ such that

$$X_t(j) = \frac{Y_t}{p_t(j)}. \tag{11}$$

3.3 Intermediate goods

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is dominated by an industry leader who owns a patent of the latest invention. That is, an industry leader occupies the industry until the new invention arrives, and the owner of the new invention will replace the previous leader and become the new industry leader. The production function for the leader of industry j is given by

$$X_t(j) = \gamma^{q_t(j)} L_{x,t}(j), \tag{12}$$

where the parameter $\gamma > 1$ represents the step size of a quality improvement, and $q_t(j)$ represents the number of quality improvements that have taken place in industry j as of time t. $L_{x,t}(j)$ is the human-capital-embodied labor for production in industry j. To impose a CIA constraint on manufacturing, we follow Chu and Cozzi (2014) and Arawatari *et al.* (2018) to assume that the leader in industry j borrows an amount of $\eta_m w_t L_{x,t}(j)$ cash from households at the nominal interest rate i to finance the wage payment for production workers, where $\eta_m \in [0,1]$ denotes the strength of the CIA constraint on manufacturing. We here adopt a cost-reducing view of vertical innovation as in Peretto (1998): given $\gamma^{q_t(j)}$, the marginal cost of production for the leader in industry j is $mc_t(j) = (1+\eta_m i_t)w_t/\gamma^{q_t(j)}$. Bertrand competition implies that the profit-maximizing price $p_t(j)$ is given by $p_t(j) = \gamma mc_t(j)$.[8] Therefore, the monopolistic profits in industry j is

$$\Pi_t(j) = \left(\frac{\gamma-1}{\gamma}\right) p_t(j)X_t(j) = \left(\frac{\gamma-1}{\gamma}\right) Y_t, \tag{13}$$

where the second equality applies (11). In addition, we can derive the wage payment in industry j such that

$$(1+\eta_m i_t)w_t L_{x,t}(j) = \frac{1}{\gamma} p_t(j)X_t(j) = \frac{1}{\gamma} Y_t, \tag{14}$$

where the second equality applies (11) again.

3.4 R&D

Equation (13) implies that $\Pi_t(j) = \Pi_t$ for all intermediate goods $j \in [0,1]$. Denote by $v_t(j)$ the market value of the monopolistic firm in industry j. Following the standard approach in the literature (see, for example, Cozzi *et al.* 2007), we focus on the symmetric equilibrium such that $v_t(j) = v_t$. Then the no-arbitrage condition for v_t is given by

$$r_t v_t = \Pi_t + \dot{v}_t - \lambda_t v_t, \tag{15}$$

where λ_t is the arrival rate of the next successful innovation. This equation implies that the asset return $r_t v_t$ equals the sum of monopolistic profits Π_t, the capital gain $\dot{v}_t$, and the potential loss $\lambda_t v_t$ due to creative destruction.

Innovations on quality improvement in each industry are proceeded by a unit continuum of R&D firms indexed by $k \in [0,1]$. Each firm hires human-capital-embodied labor $L_{r,t}(k)$ for innovation. Similar to the setup in the intermediate-good sector, financing wage payments to R&D workers also requires money borrowed from households. Therefore, free entry into the R&D sector implies the following zero-expected-profit condition of R&D firm k:

$$\lambda_t(k) v_t = (1+\eta_r i_t) w_t L_{r,t}(k), \tag{16}$$

where $\eta_r \in [0,1]$ captures the strength of the CIA constraint on R&D. Furthermore, we follow Laincz and Peretto (2006) and Chu and Cozzi (2014) to formulate the firm-level arrival rate of

[8] As in Grossman and Helpman (1991), the patent holder is assumed to have complete protection from imitation such that the markup over the marginal cost is equal to the step size of innovation.

innovation $\lambda_t(k)$ such that

$$\lambda_t(k) = \varphi \frac{L_{r,t}(k)}{h_t N_t}, \tag{17}$$

where the presence of $h_t N_t$ captures the dilution effect to remove the scale effect. In equilibrium, the aggregate arrival rate of innovation is

$$\lambda_t = \int_0^1 \lambda_t(k) dk = \varphi \frac{L_{r,t}}{h_t N_t} = \varphi s_{r,t}, \tag{18}$$

where $L_{r,t} = \int_0^1 L_{r,t}(k), k \in [0,1]$. We define a transformed variable $s_{r,t} \equiv L_{r,t}/(h_t N_t)$ as the share of human capital devoted to R&D. Similarly, the share of human capital devoted to production is $s_{x,t} \equiv L_{x,t}/(h_t N_t)$.

3.5 Monetary authority

In this study, the nominal interest rate i_t serves as the monetary policy instrument, which is exogenously set by monetary authority. Given i_t, other monetary variables such as the inflation rate π_t and the growth rate of nominal money supply denoted by $\psi_t = \dot{M}_t/M_t$ will be determined endogenously. First, the inflation rate π_t is endogenously determined according to the Fisher equation such that $\pi_t = i_t - r_t$, where the real interest rate r_t is derived from the Euler equation in (7). Moreover, the growth rate of nominal money supply is given by $\psi_t = \pi_t + \dot{m}_t/m_t + n_t$.[9] Under the condition that m_t and c_t grow at the same rate along the balanced growth path, the growth rate of nominal money supply will be endogenously determined by $\psi_t = i_t - \rho$.[10]

When the nominal interest rate i_t increases, monetary authority receives seigniorage revenues through inflation tax. To balance the budget, seigniorage revenues are returned to households by means of a lump-sum transfer in the sense that $\tau_t = \dot{M}_t / (N_t P_t) = \dot{m}_t + (\pi_t + n_t) m_t$.

4 Solving the model

This section proceeds to solve the model. Section 4.1 defines the decentralized equilibrium. Section 4.2 characterizes the balanced growth path (BGP) of the model and demonstrates the balanced-growth equilibrium properties.

4.1 Decentralized equilibrium

The equilibrium consists of a time path of prices $\{w_t, i_t, r_t, p_t(j), v_t\}_{t=0}^{\infty}$ and a time path of allocations $\{c_t, a_t, m_t, n_t, h_t, l_t, N_t, Y_t, X_t(j), L_{x,t}(j), L_{r,t}(k)\}_{t=0}^{\infty}$, which satisfy the following conditions at each instance of time:

- the representative household maximizes the utility taking prices $\{r_t, w_t, i_t\}$ as given;

[9] Recall that the real money balance per capita is given by $m_t = M_t / (N_t P_t)$, where P_t denotes the nominal price of final goods. Therefore, the growth rate of nominal money supply is $\psi_t = \pi_t + \dot{m}_t/m_t + n_t$, where $\pi_t = \dot{P}_t/P_t$ denotes the inflation rate of the price of final goods.

[10] Along the BGP, the fact that $\dot{m}_t/m_t = g_c = r_t - n_t - \rho$ implies $\psi_t = \pi_t + r_t - \rho$. Combining it with the Fisher equation yields $\psi_t = i_t - \rho$.

- competitive final-good firms produce $\{Y_t\}$ to maximize their profits taking $\{p_t(j)\}$ as given;
- monopolistic intermediate-good firm j produces $X_t(j)$ and chooses $\{L_{x,t}(j), p_t(j)\}$ to maximize its profits taking $\{w_t\}$ as given;
- competitive R&D entrepreneurs choose $L_{r,t}(k)$ to maximize their expected profits taking $\{w_t, v_t, i_t\}$ as given;
- the labor market clears such that $l_t N_t = L_{x,t} + L_{r,t}$;
- the final-good market clears such that $Y_t = c_t N_t$;
- the value of monopolistic firms equals the value of households' assets such that $v_t = a_t N_t$;
- the amount of money borrowed by entrepreneurs is $\eta_r w_t L_{r,t} + \eta_m w_t L_{x,t} = b_t N_t$.

Then substituting (12) into (10) yields the aggregate final-good production function such that

$$Y_t = Z_t L_{x,t}, \tag{19}$$

where $L_{x,t} = \int_0^1 L_{x,t}(j)dj$ is the aggregate manufacturing labor. The aggregate technology Z_t is defined as

$$Z_t = \exp\left(\int_0^1 q_t(j)dj \ln\gamma\right) = \exp\left(\int_0^t \lambda_s ds \ln\gamma\right), \tag{20}$$

where the second equality uses the law of large numbers. Consequently, the growth rate of aggregate technology Z_t is

$$g_Z \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln\gamma, \tag{21}$$

where λ_t is given by (18).

4.2 Balanced growth path

In this subsection, we first prove that the economy jumps to a unique and stable balanced growth path (BGP). Then we derive the steady-state equilibrium growth rates of technology and human capital, which together determine the growth rate of consumption per capita.

Proposition 1. *With a constant nominal interest rate i, the economy immediately jumps to a unique and stable BGP along which each variable grows at a constant (possibly zero) growth rate.*

Proof. See Appendix A.2. □

From Proposition 1, the equilibrium labor allocations are stationary along the BGP (i.e., $s_{r,t}$ and $s_{x,t}$ are constant on the BGP). Therefore, the arrival rate of innovation is also stationary on the BGP and the steady-state growth rate of technology is given by

$$g_Z = \varphi s_r \ln\gamma. \tag{22}$$

From (19), we obtain the steady-state growth rate of consumption per capita (i.e., the economic growth rate) such that

$$g_c = g_Y - n = g_Z + g_h, \tag{23}$$

which implies that the long-run economic growth g_c is determined by two engines of growth, namely technical progress g_Z and human capital accumulation g_h. Combining (14) and (19) yields $w_t = Z_t/\gamma$, which implies

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} + n_t + \rho + \delta - \xi\left(1 - \frac{n_t}{\theta}\right), \tag{24}$$

where the second equality uses (7) and (9). Then substituting (24) into (23), we obtain the steady-state growth rate of human capital per capita such that

$$g_h = \xi\left(1 - \frac{n}{\theta}\right) - n - \rho - \delta, \tag{25}$$

which implies the negative effects of fertility on the growth rate of human capital per capita. The first term $-\xi n/\theta$ represents the crowding-out effect of fertility on time endowment, and the second term $-n$ represents the diluting effect of fertility on human capital per capita. Next, we derive the balanced growth rate of consumption per capita such that

$$g_c = g_Z + g_h = (\varphi \ln \gamma)s_r - \left(1 + \frac{\xi}{\theta}\right)n + \xi - \rho - \delta, \tag{26}$$

which indicates that economic growth is increasing in s_r and decreasing in n. As Strulik (2005) shows, population growth and economic growth could exhibit a negative relationship once human capital accumulation is incorporated into the R&D-based growth model. Furthermore, introducing endogenous fertility to our model generates an additional negative effect on human capital accumulation through its crowding-out effect, thereby leading to an additional negative growth effect.

From (5), the steady-state growth rate of human capital g_h is

$$g_h \equiv \frac{\dot{h}_t}{h_t} = \xi\frac{e_t}{h_t} - n - \delta. \tag{27}$$

Equating (25) and (27) yields

$$\frac{e_t}{h_t} = 1 - \frac{n}{\theta} - \frac{\rho}{\xi}. \tag{28}$$

Using the labor-market clearing condition, it is straightforward to obtain the first equation for solving the model:

$$s_r + s_x = \frac{l_t}{h_t} = \frac{\rho}{\xi}, \tag{29}$$

where the second equality applies (28) and the time-endowment constraint in (4). Moreover, Proposition 1 implies $\dot{v}_t/v_t = \dot{\Pi}_t/\Pi_t = \dot{Y}_t/Y_t = \dot{c}_t/c_t + n$ along the BGP. Therefore, equation

(15) implies that the firm value on the BGP is

$$v_t = \frac{\Pi_t}{\rho + \lambda}. \tag{30}$$

Using (13), (14), (16) and (30), we obtain the second equation for solving the model:

$$\frac{\lambda_t v_t}{(1+\eta_r i)L_{r,t}} = w_t = \frac{Y_t}{\gamma(1+\eta_m i)L_{x,t}} \Leftrightarrow \frac{s_r}{s_x} = \frac{(1+\eta_m i)(\gamma-1)\lambda}{(1+\eta_r i)(\rho+\lambda)}, \tag{31}$$

with the arrival rate of innovation $\lambda = \varphi s_r$.

Now, we are ready to solve the steady-state equilibrium $\{s_x, s_r\}$. Combining (29) and (31) yields the equilibrium labor allocation such that

$$s_x = \rho\left(\frac{1}{\xi} + \frac{1}{\varphi}\right)\frac{1+\eta_r i}{(1+\eta_m i)(\gamma-1) + (1+\eta_r i)}, \tag{32}$$

$$s_r = \frac{\rho}{\xi} - \rho\left(\frac{1}{\xi} + \frac{1}{\varphi}\right)\frac{1+\eta_r i}{(1+\eta_m i)(\gamma-1) + (1+\eta_r i)}. \tag{33}$$

Finally, we derive the equilibrium fertility rate n by applying $a_t = v_t / N_t$, $w_t = Y_t/\gamma(1+\eta_m i_t)L_{x,t}$, $\eta_r w_t L_{r,t} + \eta_m w_t L_{x,t} = b_t N_t$ and $\lambda_t = \varphi s_{r,t}$ to reexpress (8) as

$$\begin{aligned}\frac{\alpha}{n} = \frac{1}{(1+\eta_c i)}\Bigg[&\frac{\gamma-1}{\gamma}\frac{1}{\varphi s_r + \rho} + \eta_c + \frac{\gamma-1}{\gamma}\frac{\varphi s_r \eta_r}{(1+\eta_r i)(\varphi s_r + \rho)} \\ &+ \frac{\eta_m}{\gamma(1+\eta_m i)} + \left(\frac{1}{\theta} + \frac{1}{\xi}\right)\frac{1}{\gamma(1+\eta_m i)s_x}\Bigg].\end{aligned} \tag{34}$$

This equilibrium fertility rate n is used to determine the long-run growth rates in (25) and (26).

5 Growth implications of monetary policy

In this section, we analyze the growth effects of monetary policy (in terms of the nominal interest rate) in different scenarios. To better understand the mechanism, we impose the CIA constraint in each distinct sector. Specifically, Section 5.1 examines the case where only consumption is subject to a CIA constraint, Section 5.2 examines the case where only R&D is subject to a CIA constraint, and finally Section 5.3 examines the case where only manufacturing is subject to a CIA constraint.

5.1 CIA constraint on consumption

In this subsection, we first analyze the case in which the CIA constraint is only imposed on consumption, and the result is summarized by the following proposition.

Proposition 2. *In the presence of a CIA constraint on consumption only (i.e., $\eta_c > 0, \eta_r = \eta_m = 0$), a higher nominal interest rate i does not affect the growth rate of technology g_Z, whereas it decreases the growth rate of human capital g_h. Therefore, the overall effect of i on the economic growth rate g_c is negative.*

Proof. Proven in the text. □

When the CIA constraint is imposed on consumption, the price of consumption increases from 1 to $(1+i)$, resulting in a decrease in consumption. The consumption-fertility tradeoff implies that a reduction in consumption would induce an increase in fertility, which in turn reduces the growth rate of human capital.[11] In addition, equations (32) and (33) show that the effect of the nominal interest rate i on human capital allocation does not operate through the channel of the cash constraint on consumption, implying that the growth rate of technology is unaffected when the CIA constraint is applied to consumption only. Therefore, the overall growth effect of the nominal interest rate is negative, which is consistent with He (2018a), though his model considers a different mechanism.[12]

To highlight the importance of endogenous fertility regarding the growth effects of monetary policy in the study, we also consider the case with exogenous fertility. In this case, the steady-state growth rate of human capital is determined by exogenous fertility such that $g_h = \xi(1-\bar{n}/\theta) - \bar{n} - \rho - \delta$, where $\bar{n}$ is an exogenous parameter. Then the balanced growth rate of consumption per capita in (26) becomes $g_c = (\varphi \ln \gamma)s_r - (1+\xi/\theta)\bar{n} + \xi - \rho - \delta$. That is, human capital accumulation is invariant of monetary policy and R&D-based innovation becomes the unique channel through which monetary policy affects economic growth. Therefore, the nominal interest rate has no effect on economic growth since its negative effect on the growth rate of human capital g_h is absent. This result of the exogenous-fertility case is in line with Chu and Cozzi (2014), who show that in the presence of a CIA constraint only on consumption, the nominal interest rate does not affect economic growth when labor supply is inelastic.

5.2 CIA constraint on R&D

Next, we consider the case in which the CIA constraint is only imposed on R&D activities and summarize the result as follows.

Proposition 3. *In the presence of a CIA constraint on R&D only (i.e., $\eta_r > 0, \eta_c = \eta_m = 0$), a higher nominal interest rate i reduces the growth rate of technology g_Z. Furthermore, the nominal interest rate i has a U-shaped effect on the growth rate of human capital g_h. Therefore, the overall effect of i on economic growth g_c can be U-shaped (negative) if there exists (does not exist) a threshold value i^* such that that the positive effect of i on g_h dominates the negative effect on g_Z when $i > i^*$.*

Proof. Proven in the text. □

[11] Li *et al.* (2008) provide evidence for the negative effects of fertility on human capital accumulation.

[12] In He (2018a)'s model, the CIA constraint on consumption also raises the fertility rate, but the increased fertility rate tends to reduce labor supply and thereby R&D labor, which would in turn decrease the growth rates of technology and economic growth. By contrast, in our model, labor supply and the growth rate of technology are independent of monetary policy when endogenous fertility and human capital are considered simultaneously. The nominal interest rate i negatively affects economic growth through its negative effect on human capital accumulation.

With a CIA constraint on R&D only (i.e., $\eta_r > 0, \eta_c = \eta_m = 0$), equations (32) and (33) become

$$s_x = \rho\left(\frac{1}{\xi} + \frac{1}{\varphi}\right)\left(\frac{1+\eta_r i}{\gamma + \eta_r i}\right), \tag{35}$$

$$s_r = \frac{\rho}{\xi} - \rho\left(\frac{1}{\xi} + \frac{1}{\varphi}\right)\left(\frac{1+\eta_r i}{\gamma + \eta_r i}\right), \tag{36}$$

which imply that the production labor share s_x is increasing in the nominal interest rate i, whereas the R&D labor share s_r is decreasing in it.[13] Intuitively, when the CIA constraint is applied only to R&D, an increase in i raises the R&D cost, which reallocates labor from R&D to production. Then the nominal interest rate i has a negative effect on the growth rate of technology $g_Z = \varphi s_r \ln z$ due to its negative effect on s_r. The negative effect of i on g_Z is consistent with previous studies such as Chu and Cozzi (2014) and Huang *et al.* (2021).

Moreover, to explore the impact of i on the growth rate of human capital g_h, we first examine its effect on the fertility rate n. Equation (34), the key function determining the equilibrium fertility rate, implies that households choose the fertility rate based on the tradeoff between the marginal utility of fertility and its marginal cost including (a) the dilution of financial assets per capita, (b) the dilution of real-money balance per capita, (c) foregone wages, and (d) the dilution of human capital per capita. Intuitively, a CIA constraint on R&D raises the R&D cost, requiring a higher revenue to R&D to satisfy the free-entry condition to research (equation (16)), thereby increases the asset per capita. Hence, raising i reduces the fertility rate n by enhancing the financial assets-diluting effect of fertility, captured by $(\gamma-1)/[\gamma(\varphi s_r + \rho)]$. Moreover, a higher R&D cost decreases the financing demand of R&D sector and thereby the real money balance held by households. As a result, a higher i raises the fertility rate n by weakening the real-money-diluting effect, captured by $\varphi s_r \eta_r(\gamma-1)/[\gamma(1+\eta_r i)(\varphi s_r+\rho)]$. In addition, a higher R&D cost reduces the demand for R&D labor and thereby the wage rate w_t, which leads to a decline in the foregone-wage effect, captured by $(\theta\gamma s_x)^{-1}$, and also a decline in the human-capital-diluting effect, captured by $(\xi\gamma s_x)^{-1}$; therefore, a higher nominal interest rate i in turn results in an increase in the fertility rate n. To summarize, a higher nominal interest rate i leads to one negative effect and three positive effects on fertility.

To further examine the overall effect of i on n, we substitute (35) and (36) into (34) and differentiate it with respect to i, which yields

$$\frac{\partial(\alpha/n)}{\partial i} = \frac{\eta_r}{\gamma\rho(1+\varphi/\xi)}\left[1 - \frac{\varphi\rho(\gamma-1)\eta_r}{\xi(1+\eta_r i)^2} - \frac{\varphi(1/\theta+1/\xi)(\gamma-1)}{(1+\eta_r i)^2}\right]. \tag{37}$$

Therefore, we have

$$\frac{\partial(\alpha/n)}{\partial i} = 0 \Leftrightarrow i = i^* \equiv \frac{\sqrt{\varphi(\gamma-1)(\rho\eta_r/\xi + 1/\theta + 1/\xi)} - 1}{\eta_r} > 0. \tag{38}$$

where the last inequality uses the condition $s_r > 0$. Moreover, it can be shown that $\partial^2(\alpha/n)/\partial i^2 >$

[13]Notice that as γ approaches one, the equilibrium labor allocations (35)-(36) become independent of i. This is because as the markup ratio equals unity, the monopolistic profits disappear, implying that entrepreneurs would have no incentives to do R&D. As a result, the effect of monetary policy becomes absent when only R&D is cash-constrained.

0, which indicates that the nominal interest rate i has an inverted U-shaped effect on the fertility rate n. Due to the negative relationship between fertility and human capital accumulation, the growth rate of human capital g_h is decreasing in i for $i < i^*$ and increasing in i for $i > i^*$.

In summary, for $i < i^*$, the effect of i on g_h is strictly negative, which reinforces the negative effect of i on g_Z and thereby leads to an overall negative impact of i on the economic growth rate g_c. In contrast, for $i > i^*$, the effect of i on g_h becomes positive. If this positive-growth effect is always dominated by its impact on g_z, the overall growth effect would be monotonically negative. Alternatively, there exists a threshold value for i above which the positive effect of i on g_h dominates its negative effect on g_z, yielding an overall U-shaped impact of i on g_c.

The above result stands in stark contrast to Chu *et al.* (2019b), in which an increase in the nominal interest rate i reduces the growth rate of human capital g_h and the growth rate of technology g_z, thereby leading to an overall negative impact on economic growth g_c. This is because in their model, the growth rate technology g_z is determined by the growth rate of human capital g_h, given the increasing-complexity effect of technology on R&D productivity. Thus, endogenous human capital accumulation only generates an additional negative growth effect, strengthening the negative effect of i on g_c. However, in our model that endogenizes both fertility and human capital accumulation, these two growth engines (i.e., innovation from R&D and human capital accumulation from fertility) become independently determined. We have seen that our model can not only capture a negative effect of i on g_h, but also a positive effect of i on g_h when i is sufficiently large. Accordingly, endogenous human capital accumulation may strengthen the negative effect of i on g_c as in Chu *et al.* (2019b), but it may also offset the negative effect of i on g_z, generating more diverse results than Chu *et al.* (2019b). In our model, endogenous fertility is the determinant of the human-capital-accumulation growth engine. Similar to the previous subsection, the human-capital-accumulation growth engine will shut down in the case of exogenous fertility, and consequently innovation becomes the sole growth engine. In other words, with exogenous fertility and the CIA constraint on R&D, the nominal interest rate and economic growth are negatively correlated, which is consistent with the finding in Chu and Cozzi (2014).

5.3 CIA constraint on manufacturing

In this subsection, we proceed to investigate the case in which the CIA constraint is only imposed on manufacturing. Accordingly, we obtain the following result.

Proposition 4. *In the presence of a CIA constraint on manufacturing only (i.e., $\eta_m > 0, \eta_c = \eta_r = 0$), a higher nominal interest rate i increases the growth rate of technology g_Z and decreases the growth rate of human capital g_h. Moreover, if the step size γ is sufficiently large, the overall effect of i on the economic growth rate g_c is positive. Otherwise, there exists a threshold value of i^+ above (below) which human capital accumulation dominates (is dominated by) R&D-based innovation, resulting in a negative (positive) effect of i on g_c.*

Proof. Proven in the text. □

Similar to the previous analysis, with the CIA constraint on manufacturing only, equations

(32) and (33) are rewritten as

$$s_x = \rho\left(\frac{1}{\xi}+\frac{1}{\varphi}\right)\frac{1}{(1+\eta_m i)(\gamma-1)+1}, \tag{39}$$

$$s_r = \frac{\rho}{\xi} - \rho\left(\frac{1}{\xi}+\frac{1}{\varphi}\right)\frac{1}{(1+\eta_m i)(\gamma-1)+1}. \tag{40}$$

Equations (39) and (40) imply that when manufacturing is subject to CIA constraint, a higher nominal interest rate i reduces the labor share of production s_x but increases the labor share of R&D s_r, leading to an increase in the growth rate of technology g_Z. This result also aligns with the counterpart in existing studies (such as Chu and Cozzi 2014). The intuition behind this result is as follows. Due to the CIA constraint on manufacturing, a higher nominal interest rate raises the production cost, reallocating labor from production to R&D. Therefore, entrepreneurs' incentives for investing in R&D become higher, which raises the innovation rate accordingly.

Next, we investigate how the nominal interest rate impacts fertility. Substituting (39) and (40) into (34) yields

$$\begin{aligned}\frac{\alpha}{n} &= \frac{1}{\gamma(1+\eta_m i)}\left[\frac{\frac{1}{\varphi}+\frac{1}{\theta}+\frac{1}{\xi}}{\rho(\frac{1}{\xi}+\frac{1}{\varphi})}+\eta_m\right]+\frac{\gamma-1}{\gamma}\frac{\frac{1}{\varphi}+\frac{1}{\theta}+\frac{1}{\xi}}{\rho(\frac{1}{\xi}+\frac{1}{\varphi})},\\ &= \frac{1}{\gamma(1+\eta_m i)}\Gamma+\frac{\gamma-1}{\gamma}(\Gamma-\eta_m),\end{aligned} \tag{41}$$

where $\Gamma \equiv (1/\varphi+1/\theta+1/\xi)/[\rho(1/\xi+1/\varphi)]+\eta_m > 0$. Equation (41) implies $\partial(\alpha/n)/\partial i < 0$. That is, when the CIA constraint is applied to manufacturing, an increase in the nominal interest rate i lowers the marginal cost of fertility (i.e., the RHS of (41)) by weakening the asset-diluting effect, the real-money-diluting effect, the foregone-wage effect and the human-capital-diluting effect of fertility together. The intuition is as follows. First, imposing a CIA constraint on manufacturing raises the cost of manufacturing and then reduces the monopolistic profits, decreasing the asset per capita a_t and in turn weakening the asset-diluting effect of fertility. Second, with an increase in the manufacturing cost, the financing demand for production decreases, leading to a decline in the real money balance held by households, which in turn weakens the real-money-diluting effect. Moreover, a higher manufacturing cost reduces the wage rate w_t by decreasing the demand for manufacturing labor, resulting in a lower foregone-wage effect and human-capital-diluting effect of fertility. Therefore, a higher nominal interest rate raises the fertility rate and then retards the growth rate of human capital g_h.

Recall that long-run economic growth is determined by the interaction of two engines (i.e., human capital accumulation and R&D-based innovation). To assess the overall growth effect, we first substitute (40) to (22) and differentiate it with respect to i to obtain

$$\frac{\partial g_Z}{\partial i} = \rho\varphi\ln\gamma\left(\frac{1}{\xi}+\frac{1}{\varphi}\right)\frac{(\gamma-1)\eta_m}{[(1+\eta_m i)(\gamma-1)+1]^2}. \tag{42}$$

Then differentiating (25) with respect to i yields

$$\frac{\partial g_h}{\partial i} = -\left(1+\frac{\xi}{\theta}\right)\frac{\partial n}{\partial i} = -\left(1+\frac{\xi}{\theta}\right)\frac{\alpha\eta_m\gamma\Gamma}{[\Gamma+(\gamma-1)(1+\eta_m i)(\Gamma-\eta_m)]^2}, \tag{43}$$

where the second equality uses $\partial n/\partial i = -(n^2/\alpha)[\partial(\alpha/n)/\partial i]$. If the negative effect of human capital accumulation dominates the positive effect of technological progress, we have

$$\left|\frac{\partial g_h}{\partial i}\right| > \frac{\partial g_Z}{\partial i} \Leftrightarrow \left[\frac{\Gamma + (\gamma-1)(1+\eta_m i)(\Gamma - \eta_m)}{(1+\eta_m i)(\gamma-1)+1}\right]^2 < \Theta, \tag{44}$$

where $\Theta \equiv [\alpha\gamma\Gamma(1+\xi/\theta)]/[\rho\varphi(\ln\gamma)(\gamma-1)(1/\xi+1/\varphi)] > 0$. From equation (44), if $\gamma/[\ln\gamma(\gamma-1)] \leq \Phi$, where $\Phi \equiv [\varphi(1/\varphi+1/\theta+1/\xi)^2]/[\alpha\rho\Gamma(1/\xi+1/\varphi)(1+\xi/\theta)]$, we have

$$\left|\frac{\partial g_h}{\partial i}\right| > \frac{\partial g_Z}{\partial i} \Leftrightarrow i < i^+ \equiv \frac{1}{\eta_m}\left[\frac{\Gamma - \sqrt{\Theta}}{(\sqrt{\Theta} - \Gamma + \eta_m)(\gamma-1)} - 1\right] < 0, \tag{45}$$

which implies that the overall impact of i on economic growth would be positive. That is, when the step size γ is sufficiently large, the positive effect of technological progress is so strong that it completely dominates the negative effect of human capital accumulation, leading to an overall positive effect of i on the economic growth rate g_c.

Conversely, if $\gamma/[\ln\gamma(\gamma-1)] > \Phi$, we have

$$\left|\frac{\partial g_h}{\partial i}\right| > \frac{\partial g_Z}{\partial i} \Leftrightarrow i > i^+ \equiv \frac{1}{\eta_m}\left[\frac{\Gamma - \sqrt{\Theta}}{(\sqrt{\Theta} - \Gamma + \eta_m)(\gamma-1)} - 1\right]. \tag{46}$$

In this case, the positive effect of technological progress dominates the negative effect of human capital accumulation if the following condition is satisfied:

$$\left|\frac{\partial g_h}{\partial i}\right| < \frac{\partial g_Z}{\partial i} \Leftrightarrow i < i^+. \tag{47}$$

In other words, the overall growth effect of i is negative for $i > i^+$, whereas it is positive for $i < i^+$.

In summary, endogenizing both human capital accumulation and fertility allows for diverse growth effects of monetary policy when a CIA constraint is only on manufacturing, which complements the results in Chu *et al.* (2019b). These diverse effects will not arise in the case of exogenous fertility, since a higher nominal interest rate can only generate a positive growth effect by affecting the growth rate of technology g_z.

Before closing this section, we summarize the comparative statics effects in Table 1 for ease of comparison among the above cases. On the one hand, a higher nominal interest rate can generate a positive, negative or no impact on the growth rate of technology, depending on the presence of the cash constraint. On the other hand, the effect of the nominal interest rate on the growth rate of human capital can be negative or U-shaped under different CIA constraints. Due to the complicated interaction of technological progress and human capital accumulation, the long-run relationship between economic growth and monetary policy is analytically ambiguous in our DGE model. In the next section, we will further explore the growth effects of monetary policy using a quantitative analysis.

Table 1: Comparative statics results

Different CIA constraints	s_x	s_r	n	g_h	g_Z	g_c
CIA constraint on consumption	-	-	↑	↓	-	↓
CIA constraint on R&D	↑	↓	∩	∪	↓	↓ ∪
CIA constraint on manufacturing	↓	↑	↑	↓	↑	↓↑

6 Numerical analysis

In this section, we provide a quantitative analysis to explore how monetary policy affects economic growth in our theoretical framework. In Subsection 6.1, we calibrate our model to the US economy and examine the effects of inflation (and the nominal interest rate) on the fertility rate, human capital accumulation, technological progress, and economic growth, respectively, in the presence of CIA constraints on all sectors. Next, in Subsection 6.2, we consider three special cases in which the CIA constraint is only imposed on one sector. Then we perform robustness checks on our quantitative results in Subsection 6.3. Finally, we conduct a welfare analysis in Subsection 6.4.

6.1 Calibration and benchmark results

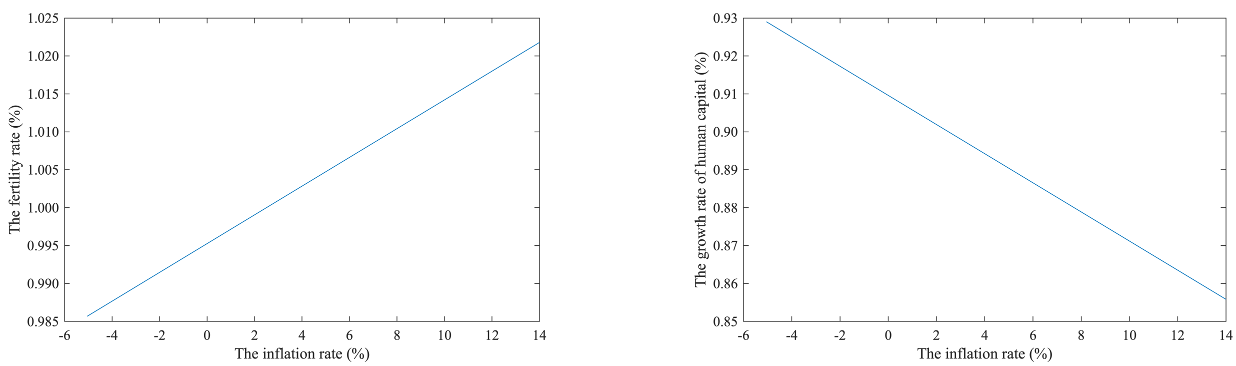
To perform this numerical analysis, the strategy is to assign steady-state values to the following set of parameters $\{\rho, \alpha, \theta, \delta, \xi, \gamma, \varphi, \eta_c, \eta_r, \eta_m, i\}$. We first pin down the values of six parameters that are commonly used in the literature. We set the discount rate ρ and the step size γ to conventional values of 0.02 and 1.034, respectively. In addition, the fertility-preference parameter α is also set to a conventional value of 1. Following Chu *et al.* (2013), the depreciation rate of human capital is set at 0.055. Following Zheng *et al.* (2021), we set the degree of the CIA constraint on consumption η_c to 0.17 to match the ratio of M1-consumption in the US. We also follow Huang *et al.* (2021) to set the degree of the CIA constraint on manufacturing η_m to 0.01, for the fact that the investment (production)-cash flow sensitivity has declined dramatically and is probably almost zero for the US and most OECD countries.

Then we calibrate the parameters $\{\theta, \xi, \varphi, \eta_r\}$ jointly to match the following four empirical moments: (1) R&D intensity; (2) the arrival rate of innovation; (3) the population growth rate; and (4) the equilibrium economic growth. For (1), we follow Chu *et al.* (2013) to consider an equilibrium R&D share of GDP at 0.03. For (2), as in Acemoglu and Akcigit (2012), the arrival rate of innovation is set at 1/3, which indicates a three-year average interval between consecutive innovations. Given the parameter value of γ, the arrival rate of innovation implies that the growth rate of innovation is $g_z = 1.1\%$. For (3) and (4), we use the conventional values of the population growth rate (1%) and the economic growth rate (2%). Finally, the steady-state value of i is calibrated by targeting the average inflation rate in the US, which is about 2.5% over the last two decades. Table 2 summarizes these targeted moments and calibrated parameter values.

The benchmark parameters enable us to quantify the impacts of inflation (and the nominal interest rate) on fertility, human capital accumulation, innovation and economic growth, respectively. Figure 1a shows that the fertility rate is increasing in the inflation rate, which is in line with the empirical evidence provided by He (2018a). Figure 1b displays that the growth rate of human capital is monotonically decreasing in the inflation rate.[14] Moreover, Figure 2a shows that the growth rate of technology is also decreasing in the inflation rate. This result indicates that the negative effect on innovation arising from the CIA constraint on R&D dominates the positive effect arising from the CIA constraint on manufacturing. Since human capital accumulation and technological progress are both decreasing in the inflation rate, the relationship between economic growth and inflation is monotonically negative, as displayed in Figure 2b.[15] The long-run negative correlation between inflation and economic growth is consistent with some existing empirical findings such as Vaona (2012), Barro (2013), and Chu *et al.* (2014). When the inflation rate rises from -5.05% (i.e. $i = 0$) to 15.08% (i.e., $i = 0.2$), the growth rate of human capital falls from 0.929% to 0.852% and the growth rate of innovation reduces from 1.137% to 1.044%, respectively. As a result, the equilibrium economic growth declines from 2.066% to 1.895%.

Table 2: Targeted moments and parameter values

Parameters				Target moments	
α	1	η_c	0.17	$R\&D/GDP$	0.03
ρ	0.02	η_m	0.01	The growth rate of innovation	1.1%
δ	0.055	η_r	0.4434	Population growth rate	1%
γ	1.034	θ	0.1015	Economic growth rate	2%
φ	56.97	ξ	0.1043	Average inflation rate	2.5%
i	0.075				



(a) (b)

Fig. 1. (a) Inflation and fertility; (b) Inflation and human capital accumulation

[14] He (2018b) assumes that human capital investments are subject to CIA constraints and shows that an increase in the nominal interest rate negatively impacts the accumulation of human capital. Although we do not assume that human capital investment is subject to CIA constraints in this study, a higher nominal interest rate still has a negative effect on human capital accumulation.

[15] In Appendix A.5 with CIA constraints only on consumption and manufacturing, we perform a counterfactual exercise in which inflation generates a non-monotonic impact on economic growth.

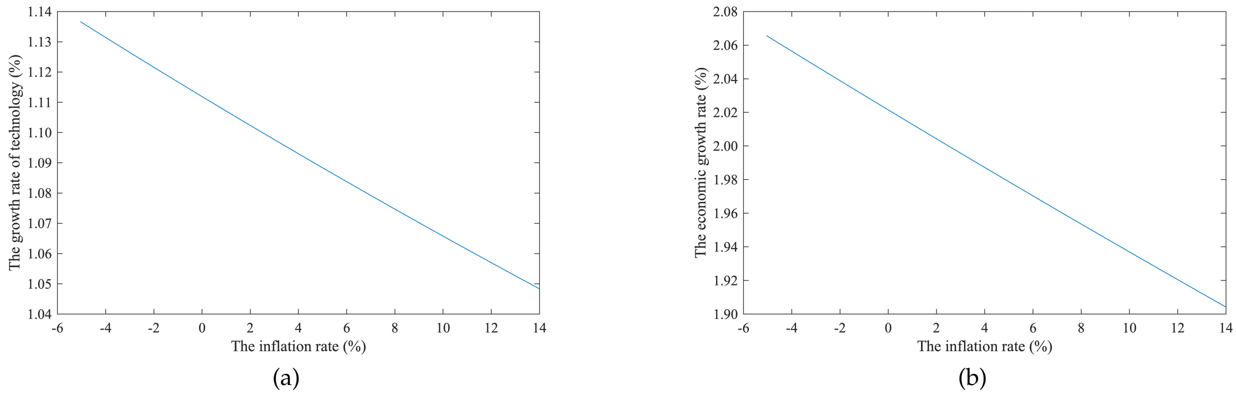


Fig. 2. (a) Inflation and innovation; (b) Inflation and economic growth

6.2 Special cases

In order to clarify the role of each CIA constraint in the inflation-growth relationship, we explore three special cases in this subsection. First, we examine the case of CIA constraint on consumption only (i.e., $\eta_r = \eta_m = 0$) to correspond to Proposition 2. Second, we perform an exercise with CIA constraint on R&D only (i.e., $\eta_c = \eta_m = 0$) to verify the effects of inflation on human capital accumulation and economic growth through a quantitative analysis. Applying the benchmark parameter values in equation (38), we can calculate the threshold value of $i^* = 11.61$, which implies that the inequality $i < i^*$ holds for the entire range of $i \in [0, 1]$. Thus, in the case of CIA constraint on R&D, similar to those situations on consumption and manufacturing, inflation also negatively impacts human capital accumulation. In this case, combining the negative effect of i on innovation, the economic growth rate is monotonically decreasing in i. Finally, the exercise of CIA constraint on manufacturing only (i.e., $\eta_c = \eta_r = 0$) is implemented to verify the overall growth effect of inflation. Based on the benchmark parameter values, it can shown that the condition $\gamma/[\ln \gamma(\gamma - 1)] > \Phi$ is satisfied. Hence, the threshold value i^+ is calculated to be -3040.17, indicating that the inequality $i > i^+$ holds for the entire range of $i \in [0, 1]$. As a result, the human-capital-accumulation channel dominates the technological-progress channel, implying that the overall growth effect of i is negative in this case.

We elaborate the results of each special case as follows. We first consider the special case of CIA constraint on consumption only. Given that other calibrated parameters remain unchanged as in the benchmark, we reduce the strength of CIA constraints on R&D η_r and manufacturing η_m to zero. Figure 3 depicts the impacts of inflation on fertility and human capital accumulation. As in Proposition 2, a higher inflation rate (and the nominal interest rate) has a positive effect on the fertility rate, whereas it has a negative effect on the growth rate of human capital. Specifically, when the inflation rate rises from -5.05% to 15.08%, the growth rate of human capital reduces by 0.068%, and the decline in human capital accumulation caused by the CIA constraint on consumption accounts for roughly 87% of the change in the benchmark case, which suggests that the CIA constraint on consumption is the crucial channel for monetary policy to affect human capital accumulation. The intuition is as follows. Since the CIA constraint directly influences households' tradeoff between consumption and production of children, the inflationary impact on fertility would be transmitted to human capital accumulation through the quality-quantity tradeoff. Figure 4 shows that inflation does not affect innovation, which is consistent with Proposition 2. Thus, human capital accumulation is the single channel through which monetary policy affects economic growth in this case.

Next, we consider the special case of CIA constraint on R&D only by reducing the strength of CIA constraint on consumption η_c and manufacturing η_m to zero while preserving other parameter values in the benchmark. Figures 5 and 6 depict the impacts of inflation on the interested variables. Figure 5b describes a negative relationship between inflation and human capital accumulation. Furthermore, when the inflation rate rises from -5.05% to 15.08%, the growth rate of human capital only decreases by 0.005%. In line with Proposition 3, the growth rate of innovation is decreasing in inflation as shown in Figure 6a. Consequently, the overall growth effect becomes unambiguous in the current case. Under the interaction of human capital accumulation and technological progress, the economic growth rate strictly decreases in inflation, as display in Figure 6b.[16] Moreover, when the inflation rate rises from -5.05% to 15.08%, the growth rate of technology declines by 0.095% and the economic growth rate declines by 0.1%. Therefore, unlike the previous case with the CIA constraint on consumption, in the current case with the CIA constraint on R&D, innovation becomes the main channel through which inflation affects economic growth whereas human capital accumulation plays a relatively minor role.

Third, we examine the last case of CIA constraint on manufacturing only in which the strengths of CIA constraints on consumption and R&D are both set to zero. The impacts of inflation on fertility, human capital accumulation, technological progress, and economic growth are displayed in Figures 7 and 8. As shown in Figure 7, the pattern of fertility and human capital accumulation is similar to previous cases. However, the growth rate of technology is increasing in the inflation rate as in Figure 8a. The negative effect of inflation on human capital accumulation and the positive effect on innovation correspond to the results in Proposition 4. Furthermore, the overall negative growth effect, as shown in Figure 8b, indicates that the negative effect arising from human capital accumulation dominates the positive effect arising from R&D-based innovation.[17] Balancing the opposing effects from these two engines results in a small contribution of the CIA constraint on manufacturing (by only 0.88%) to the negative growth effect of monetary policy.

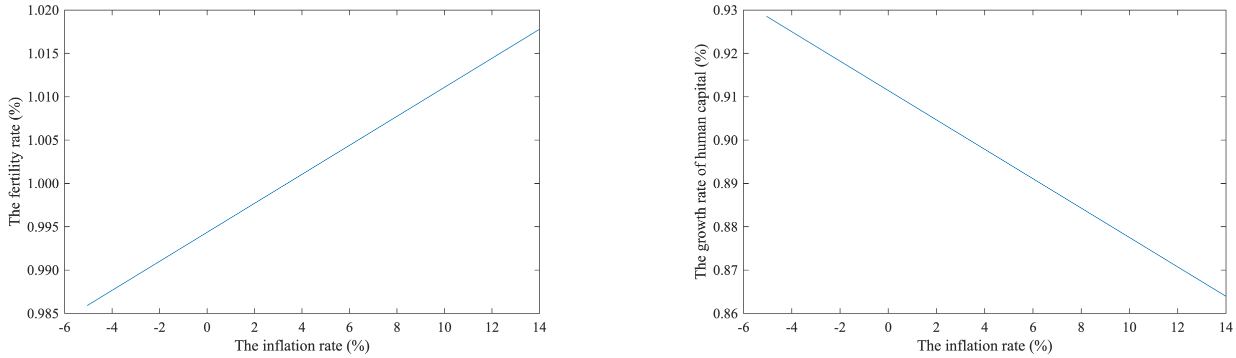
6.3 Robustness

To check the robustness of our quantitative results, we perform two exercises by (1) reducing the CIA constraint on manufacturing to zero, and (2) considering an alternative value of the growth rate of innovation g_Z, respectively.

To capture the fact that the CIA constraint on manufacturing is almost zero in the US, we first consider the case in which CIA constraints are only imposed on consumption and R&D. The results are displayed in Figure 9 and Figure 10 . It is shown that the benchmark estimation results are robust to the counterpart with a CIA constraint on manufacturing η_m. Recalling that

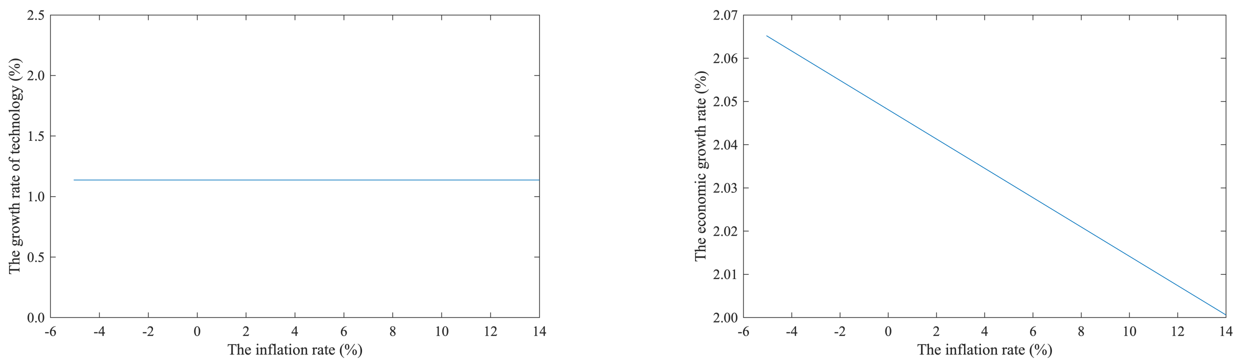
[16]In Appendix A.3 with the CIA constraint only on R&D, we show a counterfactual exercise in which inflation generates a U-shaped effect on economic growth.

[17]In Appendix A.4 with the CIA constraint only on manufacturing, we perform an additional exercise in which inflation generates a positive effect on economic growth.



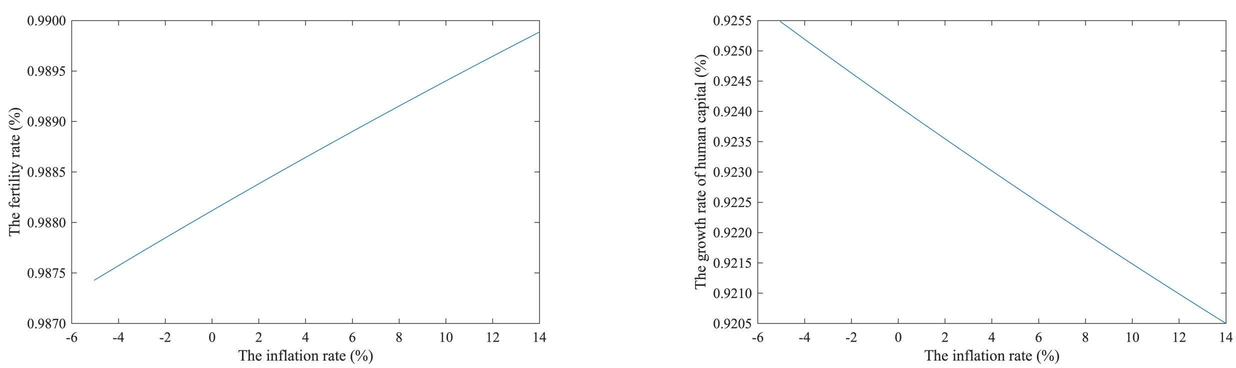
(a) (b)

Fig. 3. (a) Inflation and fertility ($\eta_r = \eta_m = 0$);
(b) Inflation and human capital accumulation ($\eta_r = \eta_m = 0$);



(a) (b)

Fig. 4. (a) Inflation and innovation ($\eta_r = \eta_m = 0$);
(b) Inflation and economic growth($\eta_r = \eta_m = 0$)



(a) (b)

Fig. 5. (a) Inflation and fertility ($\eta_c = \eta_m = 0$);
(b) Inflation and human capital accumulation ($\eta_c = \eta_m = 0$);

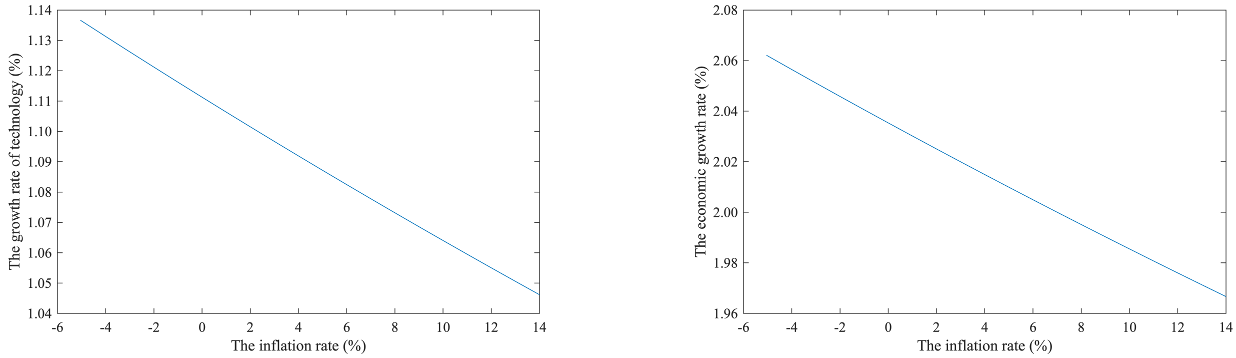


Fig. 6. (a) Inflation and innovation ($\eta_c = \eta_m = 0$);
(b) Inflation and economic growth($\eta_c = \eta_m = 0$)

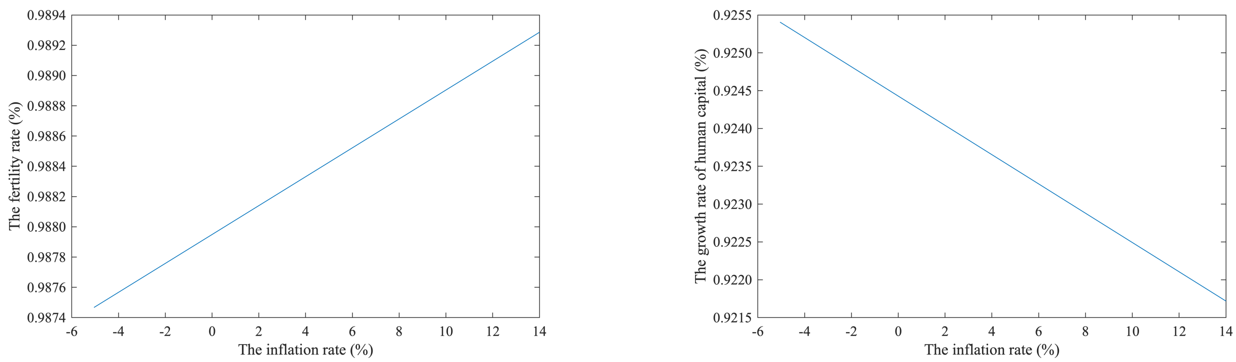


Fig. 7. (a) Inflation and fertility ($\eta_c = \eta_r = 0$);
(b) Inflation and human capital accumulation ($\eta_c = \eta_r = 0$);

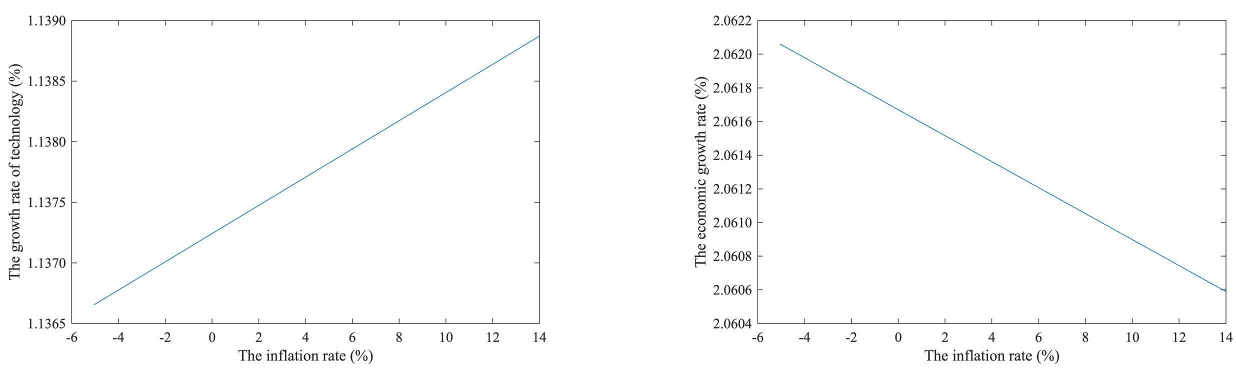


Fig. 8. (a) Inflation and innovation ($\eta_c = \eta_r = 0$);
(b) Inflation and economic growth($\eta_c = \eta_r = 0$)

the analysis in Subsection 5.3, the presence of a CIA constraint on manufacturing retards human capital accumulation but promotes technological progress. Therefore, when the CIA constraint on manufacturing becomes absent, the negative effect of inflation on human capital accumulation becomes weaker whereas its negative effect on innovation becomes larger. However, these opposing effects almost offset each other so that the decline in the economic growth growth in this case (i.e., -0.168%) is similar to the benchmark case (i.e., -0.177%).

Next, we examine the robustness of numerical results under $g_Z = 0.8\%$, where the fraction of R&D contributes to long-run economic growth is approximately 40%, as suggested by Chu and Cozzi (2014). The results regarding the impacts of inflation on fertility, human capital accumulation, innovation, and economic growth are depicted in Figure 11 and Figure 12, respectively. Similar to the benchmark case, the fertility rate increases with the inflation rate, whereas the growth rate of human capital, innovation and consumption per capita all decrease with it, indicating that our quantitative results are robust to the alternative value of g_Z. Moreover, the decline in economic growth in this case (i.e., -0.097%) is smaller than in the benchmark case (i.e., -0.177%).

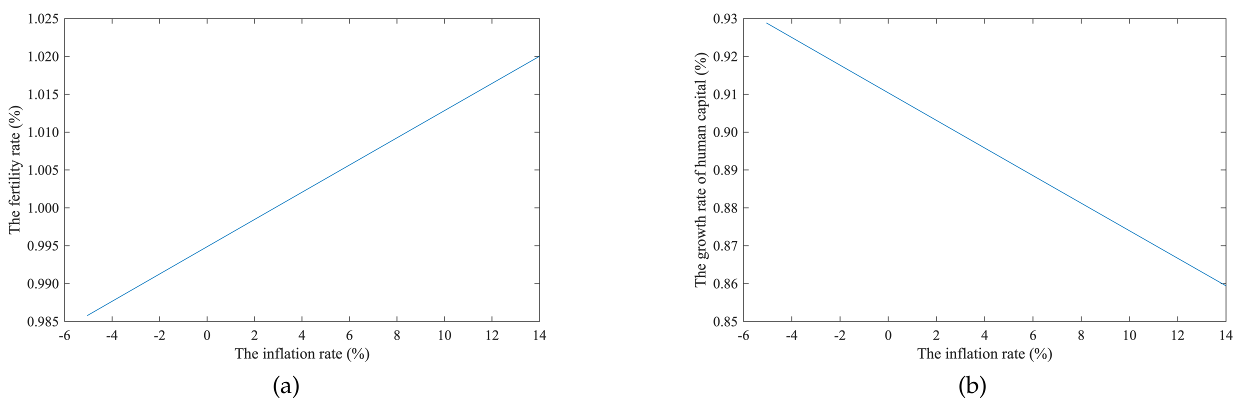


Fig. 9. (a) Inflation and fertility ($\eta_m = 0$);
(b) Inflation and human capital accumulation ($\eta_m = 0$);

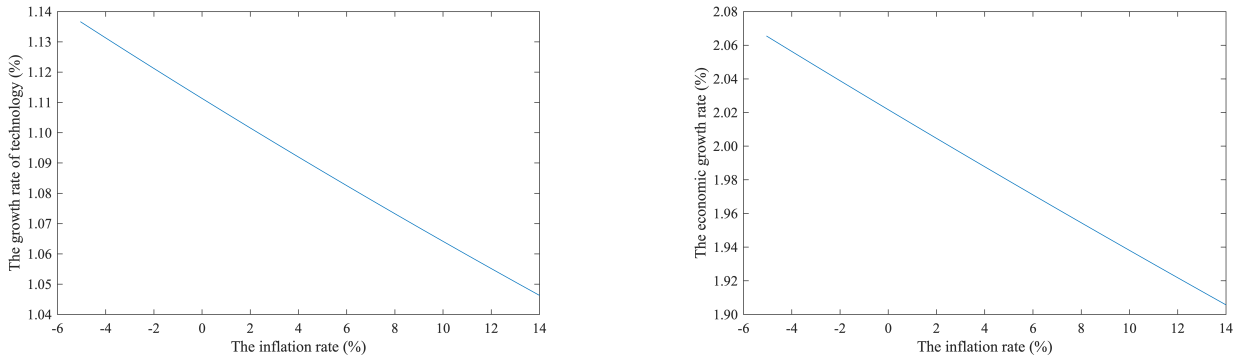
6.4 Welfare analysis

In this subsection, we explore the welfare effects of monetary policy and analyze the optimality of Friedman rule. First, we derive the steady-state welfare function by imposing the balanced growth condition on (1), which is given by

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c}{\rho} + \alpha \ln n\right), \tag{48}$$

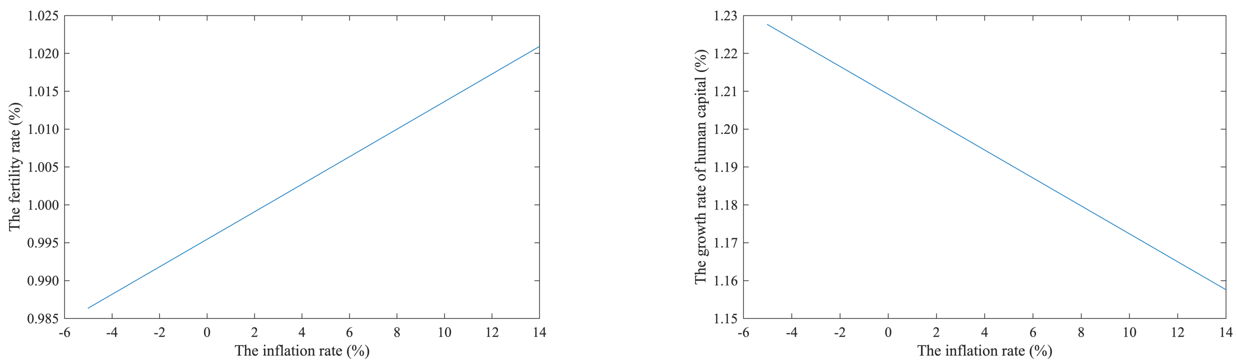
where c_0 is the steady-state level of consumption along the BGP. Combining $c_t = Y_t/N_t$ and (19), c_0 can be expressed as:

$$c_0 = Z_0 h_0 s_x, \tag{49}$$



(a) (b)

Fig. 10. (a) Inflation and innovation ($\eta_m = 0$);
(b) Inflation and economic growth($\eta_m = 0$)



(a) (b)

Fig. 11. (a) Inflation and fertility ($g_Z = 0.008$);
(b) Inflation and human capital accumulation ($g_Z = 0.008$);

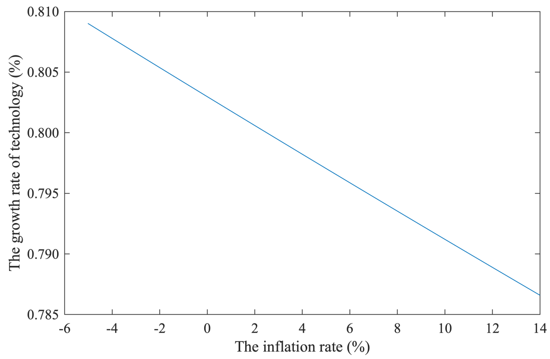
where Z_0 and h_0 denote the initial level of technology and human capital per capita, respectively. Using the equilibrium growth rate of consumption per capita given by (26) and the resource condition given by (4), and dropping the exogenous terms, we obtain

$$U = \frac{1}{\rho}\left(\ln s_x + \frac{\varphi \ln z}{\rho} s_r + \frac{\xi e - n}{\rho} + \alpha \ln n\right), \tag{50}$$

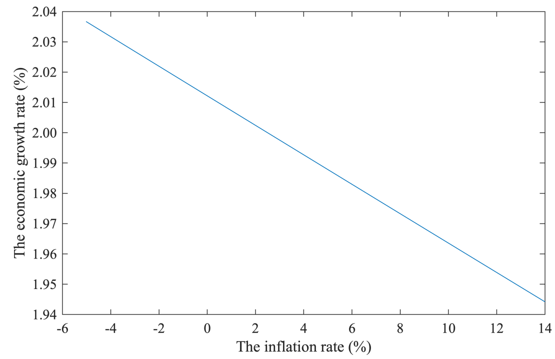
where e is defined as $e \equiv e_0/h_0$. Differentiating (50) with respect to i yields:

$$\frac{\partial U}{\partial i} = \frac{1}{\rho}\left(\underbrace{\frac{\partial \ln s_x}{\partial i}}_{>0} + \frac{\varphi \ln z}{\rho}\underbrace{\frac{\partial s_r}{\partial i}}_{<0} + \frac{\xi}{\rho}\underbrace{\frac{\partial e}{\partial i}}_{<0} - \frac{1}{\rho}\underbrace{\frac{\partial n}{\partial i}}_{>0} + \alpha\underbrace{\frac{\partial \ln n}{\partial i}}_{>0}\right), \tag{51}$$

$$= \frac{1}{\rho}\left[\frac{\partial \ln s_x}{\partial i} + \frac{\varphi \ln z}{\rho}\frac{\partial s_r}{\partial i} - \frac{1}{\rho}\left(1 + \frac{\xi}{\theta}\right)\frac{\partial n}{\partial i} + \alpha\frac{\partial \ln n}{\partial i}\right]. \tag{52}$$



(a)

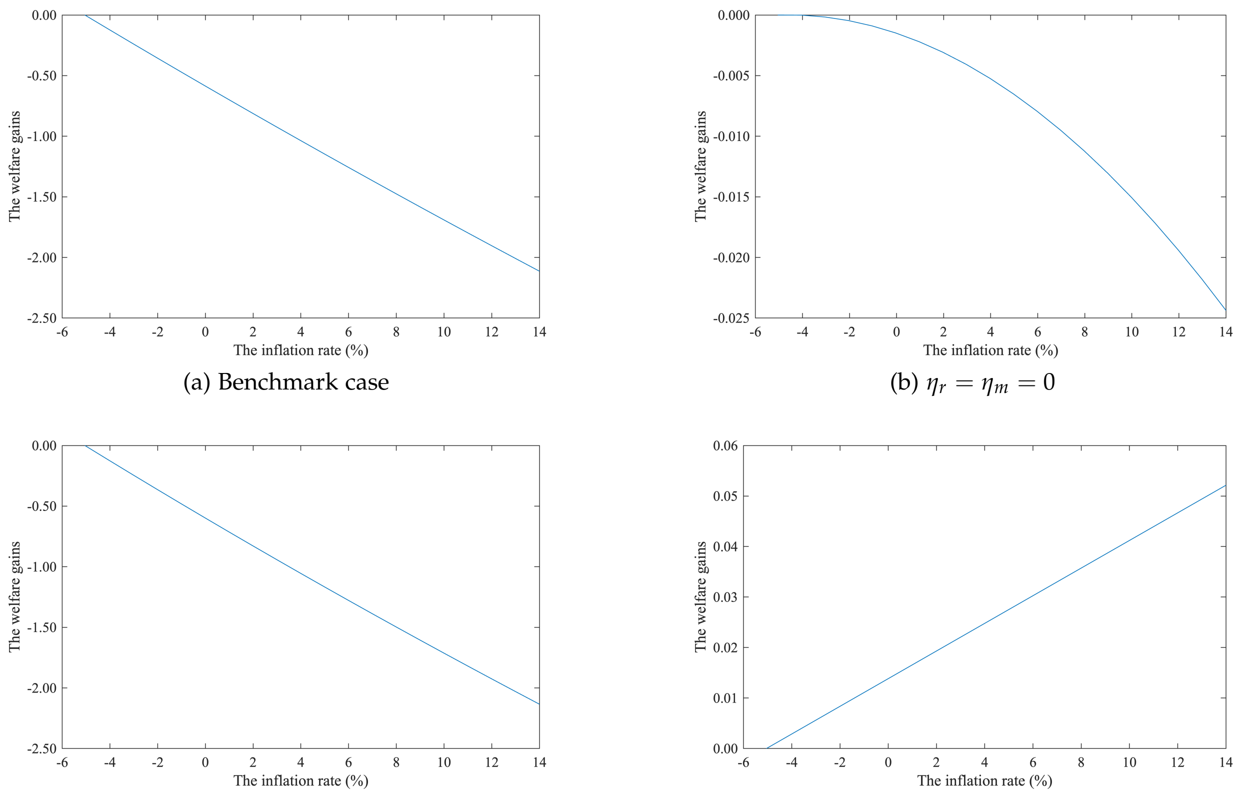


(b)

Fig. 12. (a) Inflation and innovation ($g_Z = 0.008$);
(b) Inflation and economic growth($g_Z = 0.008$)

Based on the benchmark results, a higher nominal interest rate has the following effects on social welfare. First, it increases the share of human capital allocated to production s_x and thereby increases the initial level of consumption per capita, which has a positive effect on welfare. However, it negatively impacts social welfare due to its negative-growth effect. Specifically, On the one hand, a higher nominal interest rate i decreases the R&D share of human capital s_r and then declines the growth rate of technology, which leads to a negative impact on social welfare. On the other hand, it reduces social welfare through a lower growth rate of human capital due to its negative effect on human-capital investment e. At the same time, a higher i reduces the growth rate of human capital through a higher fertility rate n, resulting in an indirect negative effect on welfare. In addition to the above welfare effects, the increase in fertility n exhibits a direct positive effect on welfare since it increases the households' utility level. As in Chu *et al.* (2019b), the additional long-run growth effect arising from endogenous human capital accumulation amplifies the negative effect of monetary policy on welfare. However, the welfare effect caused by population growth, a vital factor affecting human capital accumulation and households' utility, has been neglected in the literature.

It is analytically difficult to clarify the overall effect of inflation on social welfare. Therefore, in the subsequent analysis, we use the benchmark parametrization in the previous analysis to quantify the overall welfare effect of monetary policy and how a single CIA constraint affects social welfare. The results are displayed in Figure 13. Figure 13a depicts a negative relationship between welfare gains and inflation in the benchmark case, which shows that the optimal interest rate is zero. In other words, the Friedman rule is optimal in the benchmark case. Intuitively, there are two positive welfare effects of higher inflation: the first effect stems from a rise in the household's initial consumption level and the second effect stems from an increase in population growth, both of which lead to a higher utility level. However, these two positive effects are fully dominated by the negative effect that arises from the decline in economic growth. As a result, the overall welfare effect is generally negative. Figure 13b and 13c, which feature the single CIA constraint cases, indicate a similar pattern as in the benchmark case. That is, the negative welfare effect of inflation still holds, and the Friedman rule is also optimal in the presence of a CIA constraint on only consumption or R&D. However, the welfare effect of inflation differs in the presence of CIA constraint on manufacturing only, as shown in Figure 13d. In the presence of CIA constraint on manufacturing, a higher nominal interest rate leads human capital to shift from production to R&D and thereby decreases the share of human capital allocated to production; thus, the welfare effect arising from the initial level of households' consumption changes from being positive to negative. Nevertheless, the overall welfare effect overturns (to be positive) in this case, as the negative welfare effect caused by economic growth becomes weaker due to those two offsetting growth engines, leaving the positive welfare effect of population in the dominant position.



(a) Benchmark case

(b) $\eta_r = \eta_m = 0$

(c) $\eta_c = \eta_m = 0$

(d) $\eta_c = \eta_r = 0$

Fig. 13. Inflation and the social welfare

7 Conclusion

In this study, we explore the growth and welfare implications of monetary policy in a Schumpeterian economy featuring two independent growth engines: human capital accumulation and technological progress. In contrast to previous studies, the novel contribution of this study is to incorporate endogenous fertility into the monetary Schumpeteiran growth framework, which makes inflation operate through the growth engines differently and generates diverse inflation-growth relationships. Moreover, we consider the roles of vaiours CIA constraints in this model.

We first theoretically analyze the growth effects of monetary policy when a CIA constraint is applied to consumption, R&D and manufacturing, respectively. The long-run relationship between economic growth and monetary policy is analytically ambiguous in our DGE model due to the complicated interaction of technological progress and human capital accumulation. By applying the aggregate data of US, we conduct a quantitative analysis to further explore such a relationship. Combining the theoretical and quantitative analysis, we find that a higher nominal interest rate stifles economic growth by lowering human capital accumulation when consumption is CIA-constrained. When R&D is CIA-constrained, raising the nominal interest rate hinders economic growth since it negatively impact both engines of growth (technological progress and human capital accumulation). When manufacturing is CIA-constrained, the nominal interest rate has a negative effect on human capital accumulation and a positive effect on technological progress. The overall growth effect is still negative since the positive effect on innovation is completely dominated. Furthermore, when all three constraints are taken into account simultaneously, we find a negative relationship between the nominal interest rate and human capital accumulation, technological progress and the equilibrium growth rate in the benchmark calibrated economy. Finally, we analyze the welfare implication of monetary policy including the utilities from consumption and the number of children. We find that in the benchmark case the welfare effect of monetary policy is negative and thereby the Friedman rule is optimal.

Facing the realistic case of *zero* or even *negative* population growth in many industrialized economies, the consideration of endogenous fertility choice in our study tends to enrich the analysis of policy implications on population growth, economic growth, and social welfare. Dramatic changes in demographic structures in many countries over the past few decades have inspired plenty of studies exploring mechanisms through which demographic changes affect economic growth (for example, Connolly and Peretto 2003; Strulik *et al.* 2013; Brunnschweiler *et al.* 2021). In addition, other demographic change forces, such as mortality, migration, are also worthwhile examining. Extending the current analysis by taking into consideration these crucial issues will be promising directions for future research.

A Appendix

A.1 Household's dynamic optimization

The household's Hamiltonian function is

$$\begin{aligned} H_t = & \ln c_t + \alpha \ln n_t + \mu_t \left[\left(r_t - n_t \right) a_t + w_t l_t - c_t - \left(\pi_t + n_t \right) m_t + i_t b_t + \tau_t \right] \\ & + \varepsilon_t \left\{ \xi \left[h_t \left(1 - n_t / \theta \right) - l_t \right] - \left(n_t + \delta \right) h_t \right\} + \vartheta_t \left(m_t - b_t - \eta_c c_t \right), \end{aligned}$$

where μ_t is the co-state variable associated with the law of motion in (2), ε_t is the co-state variable associated with the law of motion in (5), and ϑ_t is the multiplier on the CIA constraints in (3). Differentiating the Hamiltonian function, we can obtain the following first-order conditions:

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - \mu_t - \eta_c \vartheta_t = 0, \tag{A.1}$$

$$\frac{\partial H_t}{\partial a_t} = \mu_t \left(r_t - n_t \right) = \rho \mu_t - \dot{\mu}_t, \tag{A.2}$$

$$\frac{\partial H_t}{\partial m_t} = -\mu_t \left(\pi_t + n_t \right) + \vartheta_t = \rho \mu_t - \dot{\mu}_t, \tag{A.3}$$

$$\frac{\partial H_t}{\partial l_t} = \mu_t w_t - \xi \varepsilon_t = 0, \tag{A.4}$$

$$\frac{\partial H_t}{\partial h_t} = \varepsilon_t \xi \left(1 - \frac{n_t}{\theta} \right) - \left(n_t + \delta \right) \varepsilon_t = \rho \varepsilon_t - \dot{\varepsilon}_t, \tag{A.5}$$

$$\frac{\partial H_t}{\partial n_t} = \frac{\alpha}{n_t} - \mu_t \left(a_t + m_t \right) - \varepsilon_t \left(\xi \frac{h_t}{\theta} + h_t \right) = 0, \tag{A.6}$$

$$\frac{\partial H_t}{\partial b_t} = \mu_t i_t - \vartheta_t = 0. \tag{A.7}$$

Combining (A.1) and (A.7) yields

$$\frac{1}{c_t} = \mu_t \left(1 + \eta_c i_t \right). \tag{A.8}$$

Moreover, equation (A.2) implies the intertemporal optimality condition in (7). Substituting (A.4) and (A.8) into (A.6) yields the optimal condition for fertility choice in (8). Log-differentiating (A.4) with respect to t yields

$$\frac{\dot{\mu}_t}{\mu_t} + \frac{\dot{w}_t}{w_t} = \frac{\dot{\varepsilon}_t}{\varepsilon_t}. \tag{A.9}$$

Then combining (A.9) with (A.2), (A.4), and (A.5) yields (9). Finally, combining (A.2), (A.3) and (A.7) yields $\vartheta_t = \mu_t (r_t + \pi_t) = \mu_t i_t$, implying the Fisher equation such that $i_t = r_t + \pi_t$.

A.2 Proof of Proposition 1

In this proof, we examine the dynamics of this model by the time paths of $s_{x,t}$ and $s_{r,t}$, which are transformed variables defined as $s_{x,t} \equiv L_{x,t} / (h_t N_t)$ and $s_{r,t} \equiv L_{r,t} / (h_t N_t)$, respectively, in the case of a constant interest rate i. Taking the log of $s_{x,t}$ and differentiating it with respect to t yields

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \frac{\dot{L}_{x,t}}{L_{x,t}} - \frac{\dot{h}_t}{h_t} - \frac{\dot{N}_t}{N_t}. \tag{A.10}$$

Similarly, taking the log of $Y_t = c_t N_t$ and $Y_t = Z_t L_{x,t}$ and then differentiating with respect to t yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} + n_t = \frac{\dot{Z}_t}{Z_t} + \frac{\dot{L}_{x,t}}{L_{x,t}}. \tag{A.11}$$

Combining (A.11) and (24) yields

$$\frac{\dot{L}_{x,t}}{L_{x,t}} = \xi\left(1 - \frac{n_t}{\theta}\right) - \rho - \delta. \tag{A.12}$$

Substituting (A.12) and (5) into (A.10) yields

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \xi\left(1 - \frac{n_t}{\theta} - \frac{e_t}{h_t}\right) - \rho \quad = \xi(s_{x,t} + s_{r,t}) - \rho. \tag{A.13}$$

Combining (13), (14) and (16) yields

$$\frac{s_{r,t}}{s_{x,t}} = \frac{(1+\eta_m i)\gamma\lambda_t v_t}{(1+\eta_r i)Y_t} \Leftrightarrow \frac{\Pi_t}{v_t} = \left(\frac{1+\eta_m i}{1+\eta_r i}\right)(\gamma - 1)\varphi s_{x,t}, \tag{A.14}$$

where $\lambda_t = \varphi s_{r,t}$ as shown in (19). Differentiating the log of (A.14) with respect to t yields

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{v}_t}{v_t}. \tag{A.15}$$

Substituting (7) and (15) into (A.15) yields

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \left(\frac{1+\eta_m i}{1+\eta_r i}\right)(\gamma - 1)\varphi s_{x,t} - \varphi s_{r,t} - \rho. \tag{A.16}$$

Equating (A.13) and (A.16) yields

$$s_{r,t} = \left[\frac{(1+\eta_m i)(\gamma-1)\varphi}{(1+\eta_r i)(\xi+\varphi)} - \frac{\xi}{\xi+\varphi}\right] s_{x,t}. \tag{A.17}$$

Finally, we substitute (A.17) into (A.13) to obtain

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \left\{\frac{\varphi\xi\left[(1+\eta_m i)(\gamma-1) + (1+\eta_r i)\right]}{(\xi+\varphi)(1+\eta_r i)}\right\} s_{x,t} - \rho. \tag{A.18}$$

Therefore, the dynamics of $s_{x,t}$ is characterized by saddle-point stability in the sense that $s_{x,t}$ jumps immediately to its unique steady-state value given by $s_x = \rho\,(1/\xi + 1/\varphi)\,(1+\eta_r i)/[(1+\eta_m i)(\gamma-1) + (1+\eta_r i)]$. Similarly, equation (A.17) shows that $s_{r,t}$ jumps to its steady-state value as well. The stationarity of $s_{r,t}$ implies that the arrival rate of innovation λ_t is also stationary.

Moreover, equation (34) shows that given a constant nominal interest rate i, n_t is stationary when $s_{x,t} = s_x$ and $s_{r,t} = s_r$. Finally, $e_t/h_t = 1 - n_t/\theta - \rho/\xi$ is stationary as well.

A.3 Additional numerical analysis with CIA on R&D

From Proposition 3, our model implies that in the presence of a CIA constraint on R&D, human capital accumulation can be a U-shaped function of inflation; namely, inflation exhibits a U-shaped effect on economic growth rather than a monotonically decreasing one. In this subsection, we conduct a counterfactual exercise by considering a new set of parameter values to correspond to this theoretical implication. Here, we consider the special case where the CIA constraint is only imposed on R&D (i.e, $\eta_c = \eta_m = 0$). While keeping other parameter values unchanged, we set $\alpha = 10, \gamma = 1.0022$, $\rho = 0.001$, $\theta = 5$,$\eta_r = 1$. The impacts of inflation on fertility and human capital accumulation are summarized in Figures 14, which suggests an inverted-U relationship between fertility and inflation whereas a U-shaped relationship between human capital accumulation and inflation, respectively. The growth rate of technology is still decreasing in inflation, as shown in Figure 15a. Furthermore, Figure 15b implies that in the case of CIA constraint only on R&D, at low levels of inflation, the negative effect on human capital accumulation reinforces the negative effect on innovation, leading to a negative growth effect. Nevertheless, as inflation rises to a high level (6.09%), the effect of inflation on human capital accumulation turns to be positive and it tends to dominate the negative effect on innovation. As a result, in line with the implication of Proposition 3, the overall effect of inflation on economic growth becomes U-shaped. In addition, comparing Figure 15b with Figure 14b, we can see that the threshold value for i that overturns the effect of inflation on g_c is greater than the counterpart on g_h (12.09%).

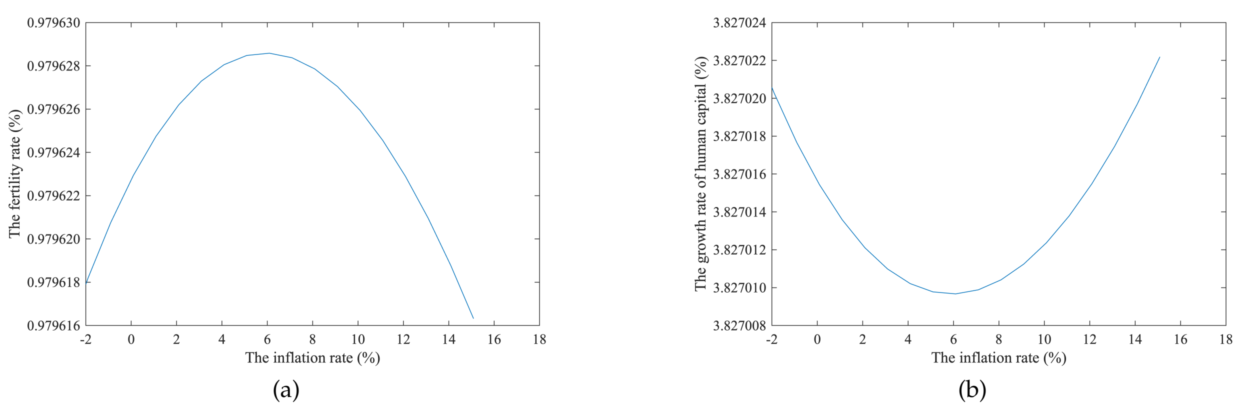
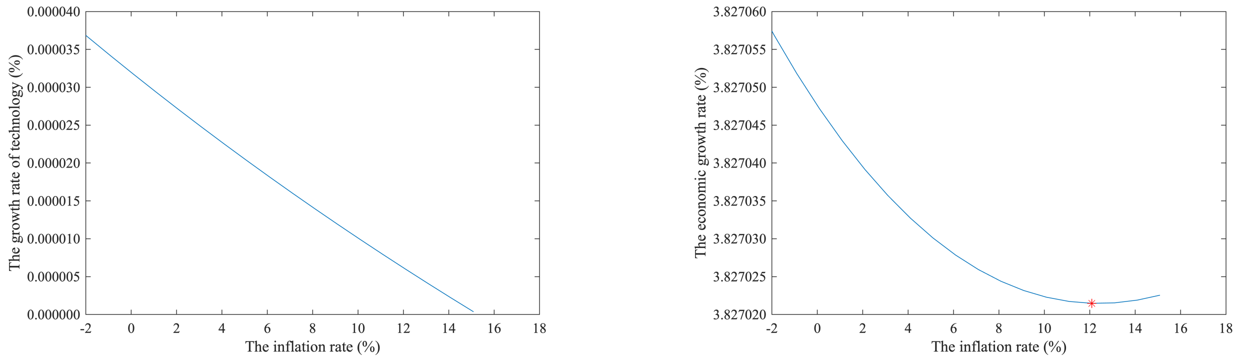


Fig. 14. (a) Inflation and fertility ($\alpha = 10, \gamma = 1.0022$, $\rho = 0.001$, $\theta = 5$, $\eta_r = 1$); (b) Inflation and human capital accumulation ($\alpha = 10, \gamma = 1.0022$, $\rho = 0.001$, $\theta = 5$, $\eta_r = 1$)

A.4 Additional numerical analysis with CIA on manufacturing

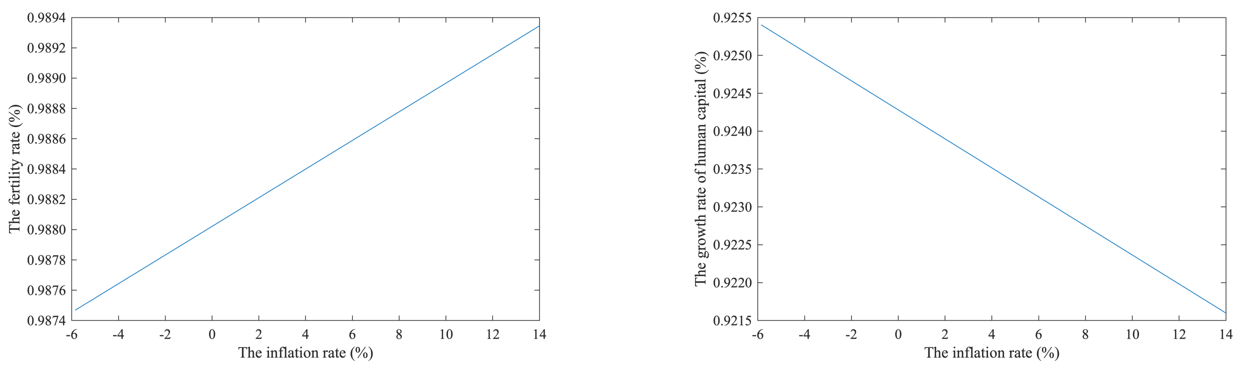
Proposition 4 indicates that when the CIA constraint is only imposed on manufacturing (i.e., $\eta_c = \eta_r = 0$), the positive inflation-innovation effect would entirely dominate the nega-



(a) (b)

Fig. 15. (a) Inflation and innovation ($\alpha = 10, \gamma = 1.0022, \rho = 0.001, \theta = 5, \eta_r = 1$);
(b) Inflation and economic growth ($\alpha = 10, \gamma = 1.0022, \rho = 0.001, \theta = 5, \eta_r = 1$)

tive inflation-human capital effect under a sufficiently large step size γ, leading to a positive impact of i on economic growth. To correspond to this theoretical implication, we conduct an extra exercise by considering an alternative value of γ (i.e., 1.0445) while other parameter values are preserved. The relationship between inflation and interested variables are shown in Figure 16 and 17. Similar to the benchmark case, human capital accumulation decreases with inflation whereas innovation increases with it. Moreover, the positive relationship between economic growth and inflation, as depicted in Figure 17b, is consistent with Proposition 4.



(a) (b)

Fig. 16. (a) Inflation and fertility ($\gamma = 1.0445$);
(b) Inflation and human capital accumulation ($\gamma = 1.0445$)

A.5 Additional numerical analysis with CIA on consumption and manufacturing

From Proposition 4 and Figure 17b, imposing a CIA constraint on manufacturing makes it possible for our model to capture an overall non-monotonic relationship between inflation and economic growth, which has been documented in existing empirical studies (e.g., López-Villavicencio and Mignon 2011; Eggoh and Khan 2014; Hu *et al.* 2021). Thus, to verify that our

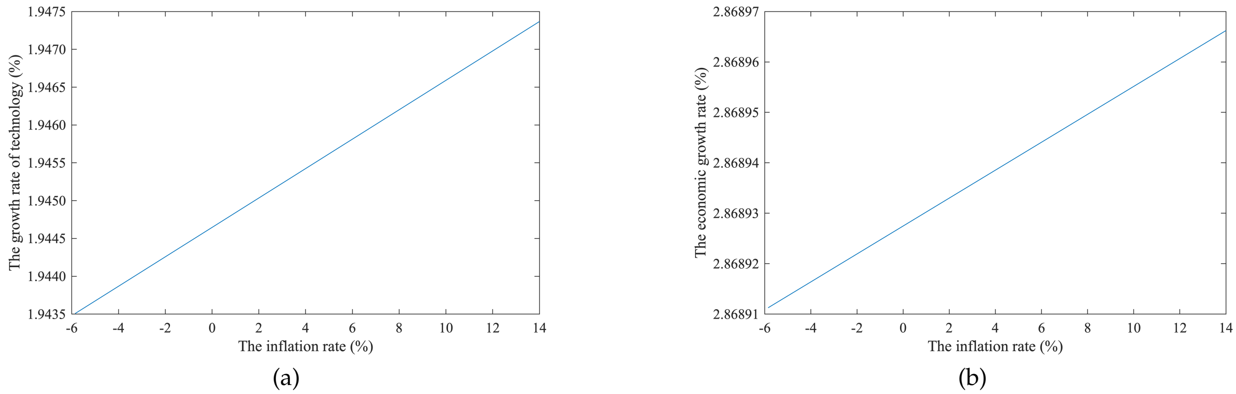
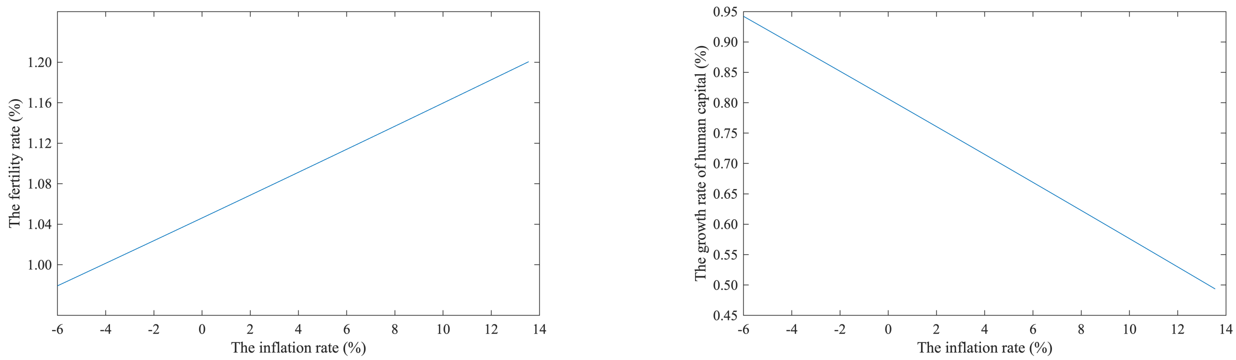


Fig. 17. (a) Inflation and innovation ($\gamma = 1.0445$);
(b) Inflation and economic growth ($\gamma = 1.0445$)

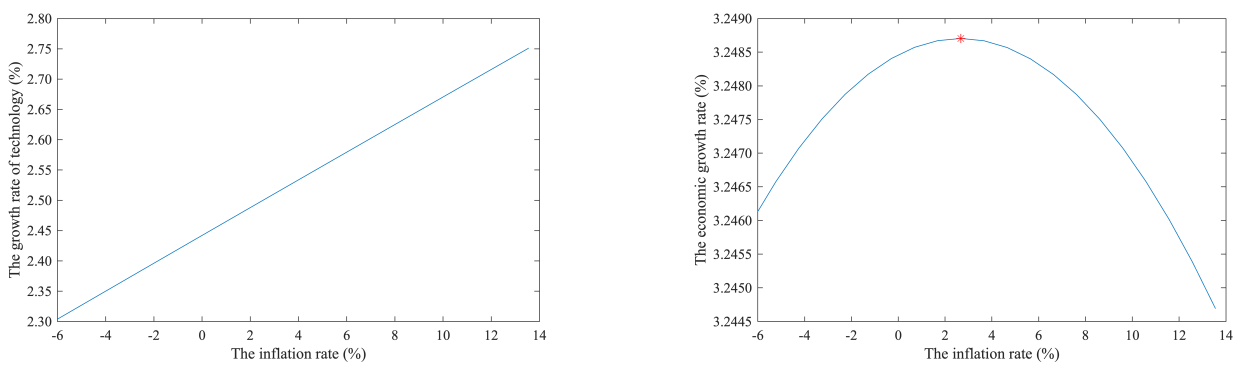
model is also flexible to generate an inverted U-shaped relationship between inflation and economic growth, a counterfactual exercise on alternative values of η_r, η_m, and γ is conducted. We first reduce η_r to zero to eliminate the negative growth effect caused by the CIA constraint on R&D. Then we set the step size γ to 1.04845 to enlarge the positive-growth effect arising from technological progress with the aid of CIA constraint on manufacturing. Lastly, we follow Arawatari *et al.* (2018) to set the strength of CIA constraint on manufacturing $\eta_m = 1$ to amplify the growth effect. Figures 18 and 19 summarize the impacts of inflation on the interested variables.

Figure 18 shows that both the monotonically increasing relationship between inflation and fertility and the monotonically decreasing relationship between inflation and human capital accumulation continue to hold. Nevertheless, the impact of inflation on technological progress alters and thereby economic growth becomes an inverted-U function of inflation, as shown in Figure 19. Intuitively, when the CIA constraint on R&D is absent, the impact of inflation on innovation is solely determined by the CIA constraint on manufacturing, leading to a positive correlation between innovation and inflation, as displayed in Figure 19a. In addition, as shown in Figure 19b, if the strength of the CIA constraint on manufacturing and the step size are raised to sufficiently larger values, the positive-growth effect from technological progress tends to initially dominate the negative-growth effect from human capital accumulation, but this domination reverses when the inflation rate exceeds the threshold value at 2.67%. That is, the growth-maximizing inflation rate is approximately 2.67%, which is close to the empirical estimation of López-Villavicencio and Mignon (2011) (i.e., 2.7%).



(a) (b)

Fig. 18. (a) Inflation and fertility ($\eta_r = 0, \eta_m = 1, \gamma = 1.04845$);
(b) Inflation and human capital accumulation ($\eta_r = 0, \eta_m = 1, \gamma = 1.04845$)



(a) (b)

Fig. 19. (a) Inflation and innovation ($\eta_r = 0, \eta_m = 1, \gamma = 1.04845$);
(b) Inflation and economic growth ($\eta_r = 0, \eta_m = 1, \gamma = 1.04845$)

References

ACEMOGLU, D. and AKCIGIT, U. (2012). Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association*, **10** (1), 1–42.

AFFUSO, E., ISTIAK, K. and SWOFFORD, J. (2022). Interest rates, house prices, fertility, and the macroeconomy. *Journal of Risk and Financial Management*, **15** (9), 403.

ARAWATARI, R., HORI, T. and MINO, K. (2018). On the nonlinear relationship between inflation and growth: A theoretical exposition. *Journal of Monetary Economics*, **94**, 79–93.

BARRO, R. J. (2013). Inflation and economic growth. *Annals of Economics & Finance*, **14** (1), 85–109.

BATES, T. W., KAHLE, K. M. and STULZ, R. M. (2009). Why do u.s. firms hold so much more cash than they used to? *The Journal of Finance*, **64** (5), 1985–2021.

BLACKBURN, K., HUNG, V. T. Y. and POZZOLO, A. F. (2000). Research, development and human capital accumulation. *Journal of Macroeconomics*, **22** (2), 189–206.

BRONIATOWSKA, P. (2019). Population ageing and inflation. *Journal of Population Ageing*, **12** (2), 179–193.

BROWN, J. R., MARTINSSON, G. and PETERSEN, B. C. (2012). Do financing constraints matter for r&d? *European Economic Review*, **56** (8), 1512–1529.

— and PETERSEN, B. C. (2015). Which investments do firms protect? liquidity management and real adjustments when access to finance falls sharply. *Journal of Financial Intermediation*, **24** (4), 441–465.

BRUNNSCHWEILER, C. N., PERETTO, P. F. and VALENTE, S. (2021). Wealth creation, wealth dilution and demography. *Journal of Monetary Economics*, **117**, 441–459.

BULLARD, J., GARRIGA, C. and WALLER, C. J. (2012). Demographics, redistribution, and optimal inflation. *Review*, **94** (6).

CHANG, W.-Y., CHEN, Y.-A. and CHANG, J.-J. (2013). Growth and welfare effects of monetary policy with endogenous fertility. *Journal of Macroeconomics*, **35**, 117–130.

CHU, A. C. and COZZI, G. (2014). R&d and economic growth in a cash-in-advance economy. *International Economic Review*, **55** (2), 507–524.

—, —, LAI, C.-C. and LIAO, C.-H. (2015). Inflation, r&d and growth in an open economy. *Journal of International Economics*, **96** (2), 360–374.

—, — and LIAO, C.-H. (2013). Endogenous fertility and human capital in a schumpeterian growth model. *Journal of Population Economics*, **26** (1), 181–202.

—, Kan, K., Lai, C.-C. and Liao, C.-H. (2014). Money, random matching and endogenous growth: A quantitative analysis. *Journal of Economic Dynamics and Control*, **41**, 173–187.

— and Lai, C.-C. (2013). Money and the welfare cost of inflation in an r&d growth model. *Journal of Money, Credit and Banking*, **45** (1), 233–249.

—, — and Liao, C.-H. (2019a). A tale of two growth engines: Interactive effects of monetary policy and intellectual property rights. *Journal of Money, Credit and Banking*, **51** (7), 2029–2052.

—, Ning, L. and Zhu, D. (2019b). Human capital and innovation in a monetary schumpeterian growth model. *Macroeconomic Dynamics*, **23** (5), 1875–1894.

Connolly, M. and Peretto, P. F. (2003). Industry and the family: Two engines of growth. *Journal of Economic Growth*, **8** (1), 115–148.

Cozzi, G., Giordani, P. E. and Zamparelli, L. (2007). The refoundation of the symmetric equilibrium in schumpeterian growth models. *Journal of Economic Theory*, **136** (1), 788–797.

Dotsey, M. and Sarte, P. D. (2000). Inflation uncertainty and growth in a cash-in-advance economy. *Journal of Monetary Economics*, **45** (3), 631–655.

Eggoh, J. C. and Khan, M. (2014). On the nonlinear relationship between inflation and economic growth. *Research in Economics*, **68** (2), 133–143.

Galor, O. (2005). Chapter 4 from stagnation to growth: Unified growth theory. In P. Aghion and S. N. Durlauf (eds.), *Handbook of Economic Growth*, vol. 1, Elsevier, pp. 171–293.

Gil, P. M. and Iglésias, G. (2020). Endogenous growth and real effects of monetary policy: R&d and physical capital complementarities. *Journal of Money, Credit and Banking*, **52** (5), 1147–1197.

Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of growth. *The Review of Economic Studies*, **58** (1), 43–61.

Hall, B. H. and Lerner, J. (2010). Chapter 14 - the financing of r&d and innovation. In B. H. Hall and N. Rosenberg (eds.), *Handbook of the Economics of Innovation*, *Handbook of The Economics of Innovation, Vol. 1*, vol. 1, North-Holland, pp. 609–639.

He, Q. (2018a). Inflation and fertility in a schumpeterian growth model: Theory and evidence. *International Review of Economics & Finance*, **58**, 113–126.

— (2018b). Inflation and innovation with a cash-in-advance constraint on human capital accumulation. *Economics Letters*, **171**, 14–18.

—, Luo, Y., Nie, J. and fu Zou, H. (2023). Money, growth, and welfare in a schumpeterian model with the spirit of capitalism. *Review of Economic Dynamics*, **47**, 346–372.

Hu, R., Yang, Y. and Zheng, Z. (2021). Inflation, endogenous quality increment, and economic growth. *Mathematical Social Sciences*, **114**, 72–86.

Huang, C.-Y., Chang, J.-J. and Ji, L. (2021). Inflation, market structure, and innovation-driven growth with distinct cash constraints. *Oxford Economic Papers*, **73** (3), 1270–1303.

—, Yang, Y. and Cheng, C.-C. (2017). The growth and welfare analysis of patent and monetary policies in a schumpeterian economy. *International Review of Economics & Finance*, **52**, 409–426.

Juselius, M. and Takáts, E. (2015). Can demography affect inflation and monetary policy?

— and Takáts, E. (2016). The age-structure–inflation puzzle. *SSRN Electronic Journal*.

— and Takáts, E. (2021). Inflation and demography through time. *Journal of Economic Dynamics and Control*, **128**, 104136.

Laincz, C. A. and Peretto, P. F. (2006). Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth*, **11** (3), 263–288.

Li, H., Zhang, J. and Zhu, Y. (2008). The quantity-quality trade-off of children in a developing country: Identification using chinese twins. *Demography*, **45** (1), 223–243.

Lin, J. Y., Miao, J. and Wang, P. (2020). Convergence, financial development, and policy analysis. *Economic Theory*, **69** (3), 523–568.

Liu, J.-T., Tsou, M.-W. and Wang, P. (2008). Differential cash constraints, financial leverage and the demand for money: Evidence from a complete panel of taiwanese firms. *Journal of Macroeconomics*, **30** (1), 523–542.

López-Villavicencio, A. and Mignon, V. (2011). On the impact of inflation on output growth: Does the level of inflation matter? *Journal of Macroeconomics*, **33** (3), 455–464.

Lucas, R. E. (1980). Equilibrium in a pure currency economy. *Economic Inquiry*, **18** (2), 203–220.

Mao, S.-Z., Huang, C.-Y. and Chang, J.-J. (2019). Growth effects and welfare costs in an innovation-driven growth model of money and banking. *Journal of Macroeconomics*, **62**, 103049.

Marquis, M. H. and Reffett, K. L. (1994). New technology spillovers into the payment system. *The Economic Journal*, **104** (426), 1123–1138.

Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, **3** (4), 283–311.

Razin, A. and Ben-Zion, U. (1975). An intergenerational model of population growth. *The American Economic Review*, **65** (5), 923–933.

Strulik, H. (2005). The role of human capital and population growth in r&d-based models of economic growth. *Review of International Economics*, **13** (1), 129–145.

—, Prettner, K. and Prskawetz, A. (2013). The past and future of knowledge-based growth. *Journal of Economic Growth*, **18** (4), 411–437.

— and Weisdorf, J. (2008). Population, food, and knowledge: A simple unified growth theory. *Journal of Economic Growth*, **13** (3), 195–216.

Vaona, A. (2012). Inflation and growth in the long run: A new keynesian theory and further semiparametric evidence. *Macroeconomic Dynamics*, **16** (1), 94–132.

Yip, C. K. and Zhang, J. (1997). A simple endogenous growth model with endogenous fertility: Indeterminacy and uniqueness. *Journal of Population Economics*, **10** (1), 97–110.

Zeng, J. (2003). Reexamining the interaction between innovation and capital accumulation. *Journal of Macroeconomics*, **25** (4), 541–560.

Zheng, Z., Huang, C.-Y. and Yang, Y. (2021). Inflation and growth: A non-monotonic relationship in an innovation-driven economy. *Macroeconomic Dynamics*, **25** (5), 1199–1226.