Environmental Regulation Stringency and Allocation between R&D and Physical Capital: A Two-Engine Growth Model*

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Abstract

Many studies have identified the negative effect of environmental regulation on capital accumulation and the positive effect on innovation, but this observed capital-innovation tradeoff due to environmental regulation lacks theoretical underpinning. We fill this gap by developing a unified two-engine endogenous growth model with environmental regulation, and show that a stringent environmental policy (in terms of pollution tax) leads to a *sectoral reallocation* from dirty inputs to clean final-good sectors, which increases the demand for R&D and activates the innovation engine. The capital engine depends on the elasticity of substitution between polluting and capital inputs. If both are sufficiently complementary, capital accumulation slows down. Another novelty of our model is that the contrasting responses of the two growth engines can lead to an inverted-U relation between overall GDP growth and environmental taxation. Our calibration shows that a well-designed environmental regulation can achieve a "double dividend": both improving the environment and enhancing economic growth and social welfare. Our empirical analysis provides macro-level evidence to justify our model prediction by using cross-country panel data.

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1 Introduction

Many empirical studies have identified the negative effect of environmental regulation on physical capital accumulation (Nelson et al., 1993; Gray and Shadbegian, 2003) and the positive effect on technology innovation (Jaffe and Palmer, 1997; Popp, 2006), both of which are important growth engines (Young, 1995; Barro and Sala-i-Martin, 2004). Understanding the mechanisms behind this capital-innovation tradeoff is crucial for public policy making to maintain a balance between economic growth and environmental quality. However, theoretic studies thus far have focused only on either capital-driven growth (Xepapades and de Zeeuw, 1999; Rauscher, 2009) or R&D-driven growth (Smulders and Di Maria, 2012; Acemoglu et al., 2012) to separately explain one of the empirical findings. In this paper we develop a unified two-engine endogenous growth model with environmental policy that can explain the observed capital-innovation tradeoff due to environmental regulation.

Our two-engine endogenous model introduces environmental taxes into the Iwaisako and Futagami (2013) framework that combines the Romer-type product variety model and AK-type capital accumulation model. In this framework, environmental regulations differentially affect R&D activities and capital accumulation, both jointly determining the overall GDP/output growth.[1] We carry out a complete theoretical (both analytical and numerical) analysis of environmental regulations on both engines of growth. On top of that, to confirm precisely our model prediction at the macro-level, we also conduct an empirical estimation of the effects of carbon tax on the growth rates of TFP and capital stock using cross-country data from the Carbon Pricing Dashboard of the World Bank and Penn World Table (PWT),[2] showing that the tradeoff between the two engines of growth induced by the environmental stringency policy indeed exists.

We analytically show that a stringent environmental policy, in terms of increasing the pollution tax, leads to *sectoral reallocation* from the dirty sector to clean ones; the resources for the production of dirty inputs decline and are reassigned to other sectors. Specifically, the resources for final goods increase, and an expansionary final-good market induces higher demand for R&D. As the resources shift to the R&D sector, the environmental regulation starts the R&D engine, boosting innovation growth. By contrast, the resources used in the production of capital may either increase or decrease, depending on the elasticity of substitution between the polluting and capital inputs. If both are sufficiently complementary,[3] capital accumulation slows down and hence the other engine (capital

[1]Note also that some macroeconomic studies, for example, Howitt and Aghion (1998), Iwaisako and Futagami (2013) and Chu et al. (2019) have built distinct two-engine models for various purposes; however, environmental issues are abstracted from their analyses.

[2]Most of the existing empirical studies cited above focus on micro-industry level data to test the effects of environmental regulations on capital accumulation and innovation growth.

[3]The literature on the complementary relationship between polluting inputs and capital inputs dates back to the

growth) decreases accordingly, resulting in the tradeoff between innovation and capital.

The contrasting responses of the two growth engines can more interestingly lead to an inverted-U relationship between overall GDP growth and environmental taxation. This result implies that environmental taxation can stimulate or impede GDP growth, depending on the status quo tax level. In particular, there is a threshold tax level below which raising the environmental tax rate stimulates growth and above which raising the tax rate instead stifles growth. Our calibration shows that a well-designed environmental regulation can achieve "green growth" that exerts a double-dividend in terms of not only improving the environment (the *environmental green dividend*), but also enhancing economic growth and social welfare (the *non-environmental blue dividend*).[4] This result is very different from the conventional double-dividend hypothesis of environmental taxation.

Our model also sheds light on practical policy issues. Our calibration results show that an environmental regulation is more likely to obtain the non-environmental blue dividend in terms of enhancing growth and improving welfare if capital and polluting inputs are less complementary. However, an environmental regulation is less likely to obtain the non-environmental blue dividend, if (i) the capital sector is more productive than the R&D sector and (ii) the goods market is less competitive. These two conditions imply that in developing countries environmental regulations are less likely to result in non-environmental blue dividends, because in these countries capital accumulation is the main driving force of growth, and markets in less developed non-OECD countries are in general less competitive than in developed OECD countries.[5,6]

1.1 Literature Review

This study contributes to the literature on environmental regulation and physical capital formation. Nelson et al. (1993) show that air pollution regulations discourage new investment in capital, resulting in downsizing of the aggregate capital stock in US electric utilities. Gray and Shadbegian (2003) point out that more stringent air and water regulations have a significant impact on US paper mills' investment decisions, discouraging firms from investing in the pollution-related production of capital. Hamamoto (2006) finds that a stringent environmental policy in Japan induces manufacturing firms

seminar work by Berndt and Wood (1975), who show that energy and capital are two input factors that strongly complement one another in US manufacturing. In more recent studies, Kim and Heo (2013) show that fuel and capital tend to be complementary in the manufacturing sectors of OECD countries since 1980, and Tovar and Iglesias (2013) find a long-run complementarity between energy and capital in 8 industries in the UK during 1970-2006.

[4] Green growth proposes that well-designed environmental policies can foster innovation that underpins sustained growth, while natural assets continue to provide environmental resources on which people's well-being relies (OECD, 2011). See Smulders et al. (2014) for a relevant discussion on green growth.

[5] See the 2008 OECD Indicators of Product Market Regulation.

[6] This result is somehow consistent with that of Shapiro (2016) in the sense that the CO2 regulation is more likely to decrease welfare in poor countries.

to modernize their equipment and therefore decreases the average age of capital, but it still results in downsizing of the aggregate capital stock. Furthermore, both partial and general equilibrium models have been developed to explain the negative effect of environmental regulation on capital accumulation. Xepapades and de Zeeuw (1999) note that increasing the emission tax leads firms to reduce the average age of their capital stock (modernization effect), and this restructuring further decreases total capital stock (downsizing effect). They restrict their analysis to a partial equilibrium without sustained growth and sector allocation. In a general equilibrium model, Rauscher (2009) presents that increasing the environmental standard lowers both conventional and green R&D, given that both grow at the same rate along the balanced-growth-path equilibrium. Unlike his model, our two-engine model allows the growth rates of innovation and capital to be determined separately.

This study also contributes to the literature on environmental regulation and innovation. The so-called Porter hypothesis (Porter, 1991), stating that strict environmental regulation may generate potential innovation-stimulating effects, turns the empirical investigation from capital to R&D (Jaffe et al., 1995). Environmental regulations increase both environment-related and overall R&D expenditures and patenting activities.[7] Jaffe and Palmer (1997) find a positive link between the stringency of environmental regulation and total R&D expenditures in the US manufacturing industries: an increase of 0.15% in R&D expenditures leads to an increase of 1% in the pollution abatement cost. Hamamoto (2006) and Yang et al. (2012) find similar effects on overall R&D spending in Japan and Taiwan, respectively. Popp (2006) shows that environmental regulations on sulphur dioxide (SO2) in the US and on nitrogen dioxide in Germany and Japan lead to a significant increase in the number of not only environment-related patents, but also overall patents. Rubashkina et al. (2015) also state that environmental regulations lead to an increase in patent applications in manufacturing sectors of 17 European countries.[8] From the theoretical perspective, the model of Smulders and Di Maria (2012) shows that a stricter environmental policy induces green resource/energy-efficient technologies, but crowds out brown innovations, resulting in a reduction in the overall rate of innovation. In their framework it is not possible for environmental regulations to generate the non-environmental blue dividend, but our model can. By developing a two-sector model of directed technical change, Acemoglu et al. (2012) propose that technologies that are biased towards the dirty industry sector will lead a

[7]See Popp (2002) and Brunnermeier and Cohen (2003) for the positive effects of environmental regulatory stringency on environment-related R&D and patents granted in the US. The reader can refer to Popp et al. (2010) for a comprehensive survey.

[8]These findings support the Porter hypothesis in the sense that more stringent environmental policies force profit-maximizing firms to engage in innovations more aggressively. The innovations, as Porter and van der Linde (1995) stress, are more than environment-related technological changes and can take various forms, including a product's or service's design, the segments it serves, how it is produced, how it is marketed, and how it is supported.

laissez-faire equilibrium to a natural disaster, and the provision of an appropriate subsidy to clean research can help redirect research to clean innovation and prevent such a disaster from taking place. Furthermore, Acemoglu et al. (2016) develop a model to shed light on the competition between clean and dirty technologies. It shows that if dirty technologies are more advanced, then the transition to clean technology can be difficult. Carbon taxes and research subsidies would help the transition, although the process can be slow. Our model differ from these interesting studies by focusing on the interaction between the observed capital-innovation tradeoff and environmental regulation in a unified framework with separately determined twin growth engines.

Finally, this study closely relates to existing studies on the double-dividend effects of environmental tax reforms. The double-dividend hypothesis proposes that raising environmental taxes and using the revenues to cut other distortionary taxes may improve welfare on both environmental (green dividend) and non-environmental (blue dividend) ground. The weak form of double-dividend indicates that a tax switch to environmental taxes from other distortionary taxes (i.e., recycling the environmental tax revenue to finance other distortionary taxes) yields higher non-environmental welfare than the counterpart from lump-sum taxes. The strong form of double-dividend indicates that a tax reform from switching distortionary taxes (on either labor or capital) to pollution taxes gives rise to both environmental and other non-environmental (in either employment or output/growth) dividends in contrast to the case without such a tax reform. There is broad support for the weak form of the double-dividend, but the strong form is debatable.[9] Karydas and Zhang (2019) construct a dynamic general equilibrium model and show that environmental tax reforms can exert the innovation growth dividend by economizing on the use of environment-related factors. Different than these tax reforms (tax switch), our model provides a novel channel through which the double-dividend can be obtained via an effective resource reallocation between the capital-producing sector and the R&D sector. In our calibration, a well-designed environmental regulation can induce the engine of innovation growth, which partially replaces the relatively dirty engine of capital growth and therefore enhances overall economic growth and improves social welfare.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 characterizes the decentralized equilibrium. Section 4 derives the comparative dynamics of the effect of environmental tax. Section 5 calibrates the model to the US economy and performs a quantitative analysis. Section 6 presents an empirical analysis. Section 7 concludes.

[9]This depends, in general, on the relative magnitude between the positive revenue recycling effect and the negative tax interdependence effect. Environmental taxes result in a high excess tax burden and hence are more likely to exacerbate, rather than alleviate, pre-existing tax distortions (i.e., the tax-interdependence effect). See Bovenberg and de Mooij (1994), Gould (1995), de Mooij (1999), or Schöeb (2003) for the failure or validity of the strong double-dividend.

2 The Model

We follow Iwaisako and Futagami (2013) to build a two-engine endogenous growth framework that features both the Romer-type variety-expansion and AK-type capital accumulation models. To analyze the effect of environmental regulation, our model considers (a) polluting inputs (namely, pollutants) in the intermediate-good manufacturing sector along with capital inputs using a constant elasticity of substitution (CES) production function; (b) a pollutant-producing sector for dirty goods; and (c) a disutility for households incurred from pollution. To control pollution, the government imposes an environmental tax on the use of polluting inputs and spends on abatement.[10]

2.1 Households

There is no population growth in the economy. Suppose that the economy admits a unit continuum of identical households, whose lifetime utility is given by

$$U = \int_0^\infty \exp(-\rho t)\left(\ln c_t - \psi \ln s_t\right) dt, \tag{1}$$

where $\rho > 0$ is the discount rate. Each household derives utility from final-good consumption c_t and incurs disutility from aggregate pollution s_t with the weight $\psi > 0$ relative to its consumption. As an externality, an individual household takes the aggregate pollution s_t as given, while in equilibrium it is determined by the firms' use of polluting inputs (see Section 2.2.2) and the government's abatement (see Section 2.2.6).

Each household is endowed with one unit of time for labor. Thus, the law of motion for the household's total assets is

$$\dot{a}_t = r_t a_t + w_t - c_t - \mu_t, \tag{2}$$

where a_t is the real value of the household's assets, r_t is the interest rate, w_t is the wage rate, and μ_t is a lump-sum tax imposed by the government. Households are assumed to own a balanced portfolio of all firms. Thus, the standard dynamic optimization implies the usual Euler equation

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

[10]Environmental taxation can be imposed on either the user of polluting inputs (in terms of intermediate-good firms' expenditure on polluting inputs) or the producer of polluting inputs (in terms of pollutant-producing firms' revenue). In this analysis we only focus on environmental taxation on intermediate-good firms' polluting-input expenditure. Our qualitative results continue to hold if environmental taxation is imposed on the producer instead of on the user.

2.2 Firms

There are five production activities in the economy: production of final goods, intermediate goods, R&D innovation, capital, and polluting inputs.

2.2.1 Final Goods

Final goods y_t are produced competitively by using production labor $l_{y,t}$ and a continuum of intermediate goods $x_t(j)$ for $j \in [0, n_t]$, according to a standard Cobb-Douglas aggregator given by

$$y_t = l_{y,t}^{1-\alpha} \int_0^{n_t} x_t(j)^\alpha dj, \tag{4}$$

where n_t is the number of varieties for intermediate goods.[11] Assume that there is free entry into the final-good sector. This assumption together with (4) yields the conditional demand functions for production labor and intermediate goods as follows

$$w_t = (1-\alpha) y_t / l_{y,t}, \tag{5}$$

$$p_t(j) = \alpha \left[l_{y,t} / x_t(j) \right]^{1-\alpha}, \tag{6}$$

where $p_t(j)$ is the price of $x_t(j)$ relative to the final good.

2.2.2 Intermediate Goods

For each variety, intermediate goods are manufactured by a monopolist that uses capital $k_t(j)$ and polluting $e_t(j)$ inputs, according to the following CES production function

$$x_t(j) = \left[\gamma k_t(j)^\sigma + (1-\gamma) e_t(j)^\sigma \right]^{\frac{1}{\sigma}}, \tag{7}$$

where γ $(1-\gamma)$ is the share parameter of the capital (polluting) input. The term $\sigma \equiv (\epsilon - 1)/\epsilon$, where $\epsilon \in (0, \infty)$, measures the elasticity of substitution between capital and polluting inputs. Capital and polluting inputs are relatively complementary (substitutable) when $-\infty < \sigma < 0$ $(0 < \sigma < 1)$.[12] By using production data to estimate the elasticities of substitution between capital and polluting inputs, recent empirical studies (such as Kim and Heo, 2013, Tovar and Iglesias, 2013, and Liu and Shumway,

[11] Our qualitative results hold if knowledge spillovers exist in the final-good production as assumed in the capital and emission input productions. The current setting in (4) maintains analytical simplicity without loss of generality. We thank the referee for raising this point.

[12] To shed light on the importance of the substitution elasticity between capital and dirty inputs, capital itself is not presumed to be either polluting or non-polluting. The influence of capital on the environment depends on how it is complementable with polluting inputs. For example, a fossil fuel-driven engine of an automobile will be dirty capital, whereas a solar-driven engine that substitutes fossil fuel with solar power is clean capital.

2016) show that these factors are complementary in a variety of industries in OECD countries. With these observations, we focus on the case where $\sigma \in (-\infty, 0)$.

Let $q_{k,t}$ and $q_{e,t}$ be the factor prices of capital and polluting inputs, respectively. The government levies a pollution tax at the rate of τ_t on the use of dirty goods, which reflects the strength of the environmental regulation.[13] Thus, the monopolist subject to (6) and (7) chooses $k_t(j)$ and $e_t(j)$ to maximize profit

$$\pi_{x,t}(j) = p_t(j)x_t(j) - q_{k,t}k_t(j) - (1+\tau_t)q_{e,t}e_t(j). \tag{8}$$

The first-order conditions for the maximization problem are given by

$$\alpha p_t(j)x_t(j)\left[\frac{\gamma k_t(j)^\sigma}{\gamma k_t(j)^\sigma + (1-\gamma)e_t(j)^\sigma}\right] = q_{k,t}k_t(j), \tag{9}$$

$$\alpha p_t(j)x_t(j)\left[\frac{(1-\gamma)e_t(j)^\sigma}{\gamma k_t(j)^\sigma + (1-\gamma)e_t(j)^\sigma}\right] = (1+\tau_t)q_{e,t}e_t(j). \tag{10}$$

By combining (9) with (10), we have

$$\frac{\gamma k_t(j)^{\sigma-1}}{(1-\gamma)e_t(j)^{\sigma-1}} = \frac{q_{k,t}}{(1+\tau_t)q_{e,t}}, \tag{11}$$

indicating that the condition for profit maximization is pinned down where the marginal rate of technical substitution (MRTS) is equal to the ratio of (after-tax) factor-input prices. Moreover, substituting (9) and (10) into the monopolist's profit yields

$$\pi_{x,t}(j) = (1-\alpha)p_t(j)x_t(j), \tag{12}$$

implying that the monopolist charges a price with the unconstrained markup $1/\alpha$ over the marginal cost. Hence, the monopolistic profit $\pi_{x,t}(j)$ is decreasing in α.

2.2.3 Inventions and R&D

The value of the invented variety is denoted by $v_{n,t}$. The familiar no-arbitrage condition for the asset value is

$$r_t v_{n,t} = \pi_{x,t} + \dot{v}_{n,t}, \tag{13}$$

[13] For ease of analytical tractability, in the baseline model an ad valorem tax is imposed upon the expenditure on dirty goods. Speck (2008) reports that ad valorem taxes have been used in the UK and Germany since the 1990s to discourage the usage of electricity. In the US, unit taxes are the common form of pollution taxation (Krutilla et al., 1995), while ad valorem taxes are also used in some states, particularly for reducing gasoline expenditures (Wood-Doughty et al., 2011). In Appendix H, we consider a unit tax as the form of environmental taxation and prove the equivalence between these two forms of taxation in terms of the effects on labor allocation, growth, and welfare.

which implies that the return on this asset $r_t v_{n,t}$ equals the sum of the flow payoffs as a monopolist $\pi_{x,t}$ and the capital gain $\dot{v}_{n,t}$.

New innovations for each variety are invented by a unit continuum of R&D firms indexed by $\iota \in [0,1]$. Each of these firms employs R&D labor $l_{r,t}(\iota)$ for creating inventions. The profit of the ι-th R&D firm is given by

$$\pi_{r,t}(\iota) = v_{n,t}\dot{n}_t(\iota) - w_t l_{r,t}(\iota), \tag{14}$$

where

$$\dot{n}_t(\iota) = \varphi n_t l_{r,t}(\iota). \tag{15}$$

Given the productivity parameter of R&D φ, (15) indicates that the number of inventions produced by firm ι depends not only on R&D labor $l_{r,t}(\iota)$, but also on the existing number of varieties n_t which captures a positive R&D externality. In equilibrium, the number of inventions created at the aggregate level equals the counterpart at the firm level for each variety; namely, $\dot{n}_t = \dot{n}_t(\iota)$. Thus, free entry into the R&D sector implies the following zero-expected-profit condition

$$\varphi n_t v_{n,t} = w_t. \tag{16}$$

This condition allows us to pin down the labor allocation to R&D.

2.2.4 Capital Input

The value of one unit of capital is denoted by $v_{k,t}$. The no-arbitrage condition for the capital asset is

$$r_t v_{k,t} = q_{k,t} + \dot{v}_{k,t}. \tag{17}$$

Again, this equation implies that the return on this asset $r_t v_{k,t}$ equals the sum of the rental price of capital $q_{k,t}$ and the capital gain $\dot{v}_{k,t}$.

Capital goods for each variety are produced by a unit continuum of capital-producing firms indexed by $\nu \in [0,1]$. Each of these firms employs capital-producing labor $l_{k,t}(\nu)$ for the production. The profit of the ν-th capital-producing firm is

$$\pi_{k,t}(\nu) = v_{k,t}\dot{K}_t(\nu) - w_t l_{k,t}(\nu), \tag{18}$$

where

$$\dot{K}_t(\nu) = \phi A_{k,t} l_{k,t}(\nu), \tag{19}$$

is the amount of capital goods produced by firm ν. The term $\phi A_{k,t}$ determines the effectiveness of capital production at time t. In line with Romer (1986), Iwaisako and Futagami (2013), and Chu et al. (2019), we assume that $A_{k,t} = K_t$ such that this effectiveness is increasing in the accumulated stock of capital. This setting introduces knowledge spillovers from past production experiences to capture the usual capital externality as in the AK model, which enables the growth of physical capital to be sustainable.[14] This setting, as will be shown, makes variety expansion and capital accumulation both engines of growth, since the growth of physical capital is determined independently of the growth in the number of intermediate goods.

In equilibrium the amount of capital goods created at the aggregate level equals the counterpart at the firm level for each variety; namely, $\dot{K}_t = \dot{K}_t(\nu)$. Thus, free entry into the capital-producing sector implies the following zero-expected-profit condition

$$\phi K_t v_{k,t} = w_t. \tag{20}$$

This equation pins down the labor allocated to capital accumulation.

2.2.5 Polluting Input

A unit continuum of firms indexed by $\kappa \in [0, 1]$ engages in the production of pollutants that are used as inputs (e.g., fuels, energy, and chemical materials) to manufacture intermediate goods. The dirty inputs, when used in the manufacturing process, generate emissions that harm environmental quality through increasing the level of aggregate pollution. The production function of the κ-th firm for dirty goods is given by

$$E_t(\kappa) = \delta A_{e,t} l_{e,t}(\kappa), \tag{21}$$

where $E_t(\kappa)$ is the amount of polluting inputs (or pollutants), $l_{e,t}(\kappa)$ denotes the amount of labor devoted to producing the polluting inputs, and $\delta A_{e,t}$ is the effectiveness of labor in producing polluting inputs.

To guarantee balanced growth, we assume that the effectiveness in the pollutant-producing sector is determined by the same knowledge spillovers that determine the effectiveness in the capital-producing sector; i.e., $A_{e,t} = A_{k,t} = K_t$. On this occasion, the production of capital goods and that of dirty

[14] This setting can be alternatively interpreted as follows. Suppose that the production function of investment goods in (19) involves a learning-by-doing effect represented by $A_{k,t} = \int_{-\infty}^{t} e^{-\phi\vartheta} y_{k,\vartheta} d\vartheta$, implying that the capital labor productivity $A_{k,t}$ depends on a weighted sum of past capital outputs $y_{k,\vartheta}$. Differentiating this equation yields $\dot{A}_{k,t} = \phi y_{k,t} = \phi \dot{K}_t$; that is $A_{k,t} = \phi K_t$. Assuming that each firm fully internalizes this learning-by-doing effect implies the substitution of $A_{k,t} = \phi K_t$ into (19), which leads to $\dot{K}_t = \phi K_t l_{k,t}$.

goods share an identical technology. Thus, the profit of the κ-th pollutant-producing firm is

$$\pi_{e,t}(\kappa) = q_{e,t}E_t(\kappa) - w_t l_{e,t}(\kappa) = q_{e,t}\delta K_t l_{e,t}(\kappa) - w_t l_{e,t}(\kappa). \tag{22}$$

In equilibrium the amount of dirty goods created at the aggregate level equals the counterpart at the firm level for each variety; namely, $E_t = E_t(\kappa)$.[15] Under perfect competition, free entry into the pollutant-producing sector implies the following zero-expected-profit condition

$$q_{e,t}\delta K_t = w_t. \tag{23}$$

This equation pins down the labor allocation to pollutant production.

2.3 Government

The government represents the environmental authority that, on the one hand, imposes a pollution tax on the use of dirty inputs and, on the other hand, engages in abatement activity by hiring pollution-abatement labor $l_{b,t}$. To focus on our point, the abatement is assumed to be entirely produced by the government according to the following technology[16] [17]

$$B_t = A_t l_{b,t}, \tag{24}$$

Moreover, we use $A_t \equiv K_t$ (which later equals $n_t k_t$ in the symmetric equilibrium) to capture Smulders' (1995) insight into which abatement services are produced according to the same technology as the private physical capital K_t.[18]

With the abatement output B_t, the aggregate pollution can be expressed as

$$s_t = \frac{E_t}{B_t},$$

indicating that the level of pollution increases with the polluting (emission) input E_t, but decreases

[15] The aggregate pollution is assumed to be a linear aggregation of all firms' emissions. See Ligthart and van der Ploeg (1994) and Michel and Rotillon (1995) for the same assumption.

[16] This assumption allows us to avoid the incentive problem of private firms investing in abatement technology (see, for instance, Requate and Unold, 2003). This assumption is also in line with the facts reported in Hatzipanayotou et al. (2005): the share of public abatement expenditure in total abatement expenditure is large in the US in regard to water pollution (i.e., 66%) and in the Netherlands in the case of air pollution (i.e., 55%).

[17] To shed light on the role of technology spillovers in abatement activities, (24) can be alternatively specified as $B_t = \exp(\xi l_{r,t}) A_t l_{b,t}$, where $\exp(\xi l_{r,t})$ characterizes the knowledge spillovers from private R&D efforts that originate from the private technology stock n_t. Our results are robust to this alternative specification.

[18] This formulation implies that, on the balanced-growth path, the public abatement output B_t grows at the same rate as physical capital K_t. See Chu and Lai (2014) for an analogous setting.

with the abatement output B_t.[19] This setting prevents aggregate pollution s_t from growing to infinity; otherwise, an *environmental disaster*, which exceeds the level that human beings can tolerate, will occur.[20]

In addition to the pollution tax rate τ_t, the government imposes a lump-sum tax μ_t to finance its abatement expenditure and endogenously meet its balanced budget

$$\mu_t + \tau_t q_{e,t} E_t = w_t l_{b,t}. \tag{25}$$

For simplicity, we assume that the abatement spending is maintained at a constant share b of the GDP; i.e., $b = w_t l_{b,t}/Y_t$.

3 General Equilibrium

In this section we define a balanced-growth-path (BGP) general equilibrium for the competitive economy and accordingly characterize the allocation of resources.

3.1 Competitive Dynamic Equilibrium

A competitive dynamic equilibrium (CDE) is defined as a tuple of allocations $[c_t, y_t, x_t(j), k_t(j), e_t(j), l_{y,t}, l_{r,t}, l_{k,t}, l_{e,t}]_{t=0}^{\infty}$ and a sequence of prices $[p_t(j), r_t, w_t, q_{k,t}, q_{e,t}, v_{n,t}, v_{k,t}]_{t=0}^{\infty}$ and policies $[\tau_t]_{t=0}^{\infty}$ such that

- the households choose $[c_t]$ to maximize their lifetime utility (1), taking $[r_t, w_t, \mu_t]$ as given;
- the final-good manufacturers produce $[y_t]$ and choose $[x_t(j), l_{y,t}]$ to maximize their profits taking $[w_t, p_t(j)]$ as given;
- the intermediate-good monopolist in industry $j \in [0, n_t]$ produces $[x_t(j)]$ and chooses $[k_t(j), e_t(j)]$ to maximize profits taking $[q_{k,t}, q_{e,t}]$ as given;
- the R&D firms choose $[l_{r,t}]$ to maximize profits taking $[w_t, v_{n,t}]$ as given;
- the capital-producing firms choose $[l_{k,t}]$ to maximize profits taking $[w_t, v_{k,t}]$ as given;
- the polluting input-producing firms choose $[l_{e,t}]$ to maximize profits taking $[w_t, q_{e,t}]$ as given;
- the government meets its flow budget constraint by adjusting the lump-sum tax μ_t;

[19] As in Elbasha and Roe (1996) and Chang et al. (2009), aggregate pollution s_t is simply viewed as a flow instead of a stock. The reasons are two-fold. First, as Bovenberg and Smulder (1995) and Fullerton and Kim (2008) stress, "sustainable development" requires that pollution is constant in the long run and does not exceed the maximum absorption capacity. In our model, $s_t = E_t/B_t$ is also constant in the steady-state equilibrium. Second, our analysis focuses on the steady-state effects of environmental regulation. The steady-state results are robust, regardless of whether pollution is modeled as a flow or a stock, given that the flow value is proportional to the stock value in the steady state. Consequently, considering this simplified assumption can rule out the possibility of transitional dynamics without loss of generality.

[20] See Smulders (1995) for a comprehensive discussion. See also the recent environmental endogenous growth literature, such as Acemoglu et al. (2012) and Chu et al. (2016), for a similar argument.

given a stationary path of the environmental policy (i.e., $\tau_t = \tau$), all markets clear – that is

- the final-good market clears such that $c_t = y_t$;
- the labor market clears such that $l_{y,t} + l_{r,t} + l_{k,t} + l_{e,t} + l_{b,t} = 1$;
- the capital-good market clears such that $K_t = \int_0^{n_t} k_t(j)dj$;
- the dirty-good market clears such that $E_t = \int_0^{n_t} e_t(j)dj$;
- the values of intangible and tangible assets add up to the households' assets value such that $v_{n,t}n_t + v_{k,t}K_t = a_t$.

3.2 Characterization of the BGP Equilibrium

We characterize the BGP equilibrium before we head to our analysis.

Lemma 1. *There exists a non-degenerate, unique, and determinate BGP equilibrium along which all variables grow at a constant (possibly zero) rate.*

Proof. All proofs are relegated to the Appendix. □

In Appendix A we first prove that the unique equilibrium allocation of labor exists and is stationary at all times while the steady-state equilibrium labor allocation does not admit a closed-form solution. Accordingly, we further ensure that the steady-state growth rates of innovation, capital, and overall output (GDP) also exist. The steady state is locally determinate, given that the economy jumps to an equilibrium labor allocation along the balanced-growth path. This implies that there are no transitional dynamics in our model.

The determination of the equilibrium labor allocation can be summarized in Lemma 2 below.

Lemma 2. *In the BGP equilibrium the steady-state labor allocation is given by*

$$l_r = \alpha l_y - \frac{\rho}{\varphi}, \tag{26}$$

$$l_k = \frac{1}{1 + \frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^\sigma} \frac{\alpha^2}{1-\alpha} l_y - \frac{\rho}{\phi}, \tag{27}$$

$$l_e = \frac{\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^\sigma}{1 + \frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^\sigma} \frac{\alpha^2}{1-\alpha} \frac{l_y}{1+\tau}, \tag{28}$$

$$l_y = \frac{1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}}{1 + \alpha + \frac{b}{1-\alpha} + \frac{\alpha^2}{1-\alpha}\left[\frac{1}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^\sigma} + \frac{\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^\sigma}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^\sigma}\frac{1}{1+\tau}\right]}, \tag{29}$$

$$\frac{e}{k} = \frac{\delta\alpha^2(1-\gamma)\left(\frac{e}{k}\right)^\sigma\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right)}{(1+b)(1+\tau)\gamma + [(1-\alpha^2+b)(1+\tau) + \alpha^2](1-\gamma)\left(\frac{e}{k}\right)^\sigma}. \tag{30}$$

From (30), we can solve the steady-state e/k recursively. Substituting the steady-state e/k into (29) then determines l_y in the steady state. With the two solved variables, we can further use (26)-(28) to obtain the steady-state l_r, l_k and l_e. These labor allocations are all a function of the environmental policy τ.

Let $\eta \equiv e/k$. It is clear from Lemma 2 that the emission-capital ratio $\eta \equiv e/k$ plays a crucial role in governing the labor allocation in the BGP equilibrium. Thus, we start with the effect of τ on η, which enables us to establish the following lemma.

Lemma 3. *The steady-state ratio between the polluting and capital inputs $\eta \equiv e/k$ is decreasing in the pollution tax rate τ.*

Lemma 3 conveys an intuitive result that a tightening of environmental policy results in a lower factor ratio of the polluting input to the capital input. A higher pollution tax, as shown in (12), raises the relative price of the polluting input to capital input, which induces firms to substitute the capital input for the polluting input. Thus, e/k decreases with the environmental tax τ. Note that this result holds true regardless of the extent of the substitution elasticity between the capital and polluting inputs.

Once the steady-state ratio between the polluting and capital inputs ($\eta \equiv e/k$) and the labor allocation (l_y, l_r, l_k and l_e) are determined, it is easy to further obtain the growth rate in the BGP equilibrium. Focusing on the symmetric equilibrium where $x_t(j) = x_t$ for all $j \in (0, n_t)$, equation (4) becomes $y_t = l_y^{1-\alpha} n_t x_t^{\alpha} = (n_t l_y)^{1-\alpha}[\gamma(n_t k_t)^{\sigma} + (1-\gamma)(n_t e)^{\sigma}]^{(\alpha/\sigma)}$, where by definition $n_t k_t = K_t$ and $n_t e_t = E_t = \delta K_t l_e$ from (21). Since l_y and l_e stay constant along BGP, differentiating y_t with respect to t yields the GDP (total output) growth rate, denoted by g, such that

$$g = \alpha g_K + (1-\alpha) g_n, \tag{31}$$

which is a weighted sum of the two engines: the growth of innovation (varieties) g_n (obtained from (19)) and the growth of capital g_K (obtained from (15)).[21]

[21] Scale effects in our model can be removed by the *fully endogenous* solution as in Cozzi (2017). Assume that total labor equals l_t and that R&D (capital) externality of spillovers is a function of the ratio of the number of varieties (amount of capital) to labor l_t. Hence, the (aggregate) production functions for varieties, capital, pollution input, and abatement are modified to $\dot{n}_t = \varphi(n_t/l_t)l_{r,t}$, $\dot{K}_t = \phi(K_t/l_t)l_{k,t}$, $E_t = \delta(K_t/l_t)l_{e,t}$, and $B_t = (K_t/l_t)l_{b,t}$, respectively. Focusing on symmetry, equation (4) becomes $y_t = (n_t l_y)^{1-\alpha} K_t^{\alpha}[\gamma + (1-\gamma)\eta^{\sigma}]^{(\alpha/\sigma)}$, where $\eta \equiv e/k$. In the BGP equilibrium the output growth rate is then given by $g = (1-\alpha)(\dot{n}_t/n_t) + \alpha(\dot{K}_t/K_t) = (1-\alpha)\varphi(l_r/l) + \alpha\phi(l_k/l)$. This solution implies that our current model features $l = 1$, and thereby scale effects on growth can be removed by the normalization procedure on total labor available for supply.

4 Effects of the Pollution Tax

We are ready to examine the effects of the environmental regulation (i.e., the pollution tax τ) on labor allocation (l_y, l_r, l_k and l_e), economic growth (g_n, g_K, and g), and social welfare (W).

4.1 Labor Allocation

Based on Lemmas 1-3, we establish the following proposition.

Proposition 1. *In response to an increase in the pollution tax τ,*

(i) *the labor devoted to the production of polluting inputs l_e decreases, while the labor devoted to the production of final goods l_y and R&D l_r increases;*

(ii) *the labor devoted to the production of capital inputs l_k decreases if the polluting input and the capital input are sufficiently complementary; i.e., $\sigma < \bar{\sigma}$, where $\bar{\sigma}$ is a threshold value solved by*

$$\sigma = -\frac{\frac{1}{1+\tau}\left(\frac{\alpha^2}{1-\alpha}\right)\left[(1-\sigma)(1+b)\gamma + \left(1-\alpha^2+b+\frac{\alpha^2}{1+\tau}\right)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma}\right]}{\left(1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\frac{1}{1+\tau}\right)\left[(1+b)\gamma+(1-\alpha^2+b)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma}\right]}. \tag{32}$$

The existence of $\bar{\sigma}$ is ensured by the Bolzano theorem, as shown in Appendix D.

A more stringent environmental regulation (higher τ), as shown in Lemma 3, decreases the relative demand for the dirty goods e/k, which unambiguously reduces the labor devoted to the dirty goods' production l_e. The effect of a higher τ on the capital-good production labor l_k, however, is mixed. There are two opposite effects that govern the impact of the pollution tax on l_k. *The relative price effect* indicates that a higher pollution tax increases the relative price of the polluting input to the capital input. Thus, the intermediate-good firms decrease the demand for the polluting input, but increase the demand for the capital input. *The technical complementarity effect*, however, indicates that a decrease in the polluting input may lower the marginal product of capital as both are "technically complementary."[22] If $\sigma < \bar{\sigma}$, then the substitution elasticity between the capital and polluting inputs $\epsilon(= 1/(1-\sigma))$ is substantially low such that the relative price effect attenuates while the technical complementarity effect amplifies. As a result, the latter effect dominates the former one, resulting in a decrease in l_k. In this case, more stringent environmental regulation is unfavorable to the labor demand for not only dirty goods, but also capital goods, leading to a reduction in the labor devoted to the capital-good production.

[22] From (7), we derive: $\frac{\partial(\partial x_t/\partial k_t)}{\partial e_t} = (1-\sigma)\gamma(1-\gamma)x_t^{1-2\sigma}(k_t e_t)^{\sigma-1} > 0$. The technical complementarity effect, *ceteris paribus*, becomes larger if the value of σ is higher.

A higher pollution tax gives rise to sectoral reallocation, shifting resource/labor from the dirty sector to other sectors. Such a *sectoral reallocation effect* then increases the final-good production labor l_y. An increase in l_y expands the final-good market, which induces higher demand for an intermediate variety and increases the profit of intermediate-good firms, $(1-\alpha)y_t/n_t$. This in turn increases the returns to R&D and hence the R&D labor l_r, as shown in (26).

4.2 Economic Growth

With steady-state labor allocation, the growth rates of innovation g_n and capital g_K are determined by (15) and (19), and accordingly the GDP growth rate g is obtained by (31). Specifically, we have

$$\begin{aligned} g = \alpha\phi l_k + (1-\alpha)\varphi l_r = & \alpha\phi \left\{ \frac{\frac{\alpha^2}{1-\alpha}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)}{\left(1+\alpha+\frac{b}{1-\alpha}\right)\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\right]+\frac{\alpha^2}{1-\alpha}\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\frac{1}{1+\tau}\right]} \right\} + \\ & (1-\alpha)\varphi \left\{ \frac{\alpha\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)}{1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\left[\frac{1}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}+\frac{\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}\frac{1}{1+\tau}\right]} \right\} - \rho, \end{aligned} \tag{33}$$

where e/k is solved by (30). Note that $l_k \geqq 0$ and $l_r \geqq 0$ guarantee a non-negative growth rate for variety and capital, respectively. If $l_k = 0$, then the model degenerates to an R&D-based variety-expansion model. By contrast, if $l_r = 0$, then the model turns out to become a capital-based AK model.

To sharpen the empirical findings on the distinctive R&D and capital effects, we first examine the effects of stringent environmental regulation on the innovation and capital growth rates separately.

Proposition 2. *In response to the pollution tax τ,*

(i) *the innovation growth rate g_n unambiguously rises;*

(ii) *whereas the capital growth rate g_K falls, provided that the polluting input sufficiently complements the capital inputs (i.e., the condition $\sigma < \bar{\sigma}$ in (32) holds).*

Stringent environmental regulation shifts the resource away from the production of polluting goods to the final-good production. As shown in Proposition 1(i), this induces higher demand for an intermediate variety and increases the R&D labor l_r. Therefore, (15) indicates that the R&D (variety) growth rate $g_n \equiv \dot{n}_t/n_t = \varphi l_r$ increases. This result implies that a stringent environmental policy can help achieve "green growth" by fostering the speed of innovation, which supports the empirical findings of Brunnermeier and Cohen (2003) and Popp (2006), and by inducing more efforts on R&D

l_r, which supports Jaffe and Palmer's (1997) finding. It is somehow also consistent with the Porter hypothesis.

Proposition 2(ii) indicates that a higher pollution tax has an ambiguous effect on the other growth engine – capital accumulation. Stringent environmental regulation, as shown in Proposition 1(ii), decreases the labor demand for not only dirty goods l_e, but also capital goods l_k, if the polluting input sufficiently complements the capital input (i.e., $\sigma < \bar{\sigma}$). The decrease in labor devoted to the production of capital goods retards the capital growth rate $g_K \equiv \dot{K}_t / K_t = \phi l_k$, as shown in (19). This result is consistent with empirical findings that environmental regulations are unfavorable to overall capital accumulation (Nelson et al., 1993), especially when the formation of capital is highly associated with (complements) the use of dirty goods (Gray and Shadbegian, 2003).

We next turn to the effect of the pollution tax on the steady-state overall output (GDP) growth rate. Define a threshold value of the relative productivity of R&D to capital as

$$\overline{(\varphi/\phi)} \equiv \frac{(1+\tau)^2 \sigma \frac{\eta_\tau}{\eta} \left[1 + \alpha + \frac{b}{1-\alpha} + \frac{\alpha^2}{1-\alpha} \frac{1}{1+\tau}\right] - \frac{\alpha^2}{1-\alpha}}{(1-\alpha)\left[(1+\tau)\sigma\tau \frac{\eta_\tau}{\eta} + \left(1 + \frac{1-\gamma}{\gamma} \eta^\sigma\right)\right]}, \tag{34}$$

where $\eta_\tau = \partial \eta / \partial \tau$. Under the threshold, we arrive at the following proposition.

Proposition 3. *Under the condition $\sigma < \bar{\sigma}$, an increase in the pollution tax τ has a positive (negative) effect on the steady-state GDP growth rate g if the relative productivity of R&D to capital is substantially high (low); namely, $\varphi/\phi > (<) \overline{(\varphi/\phi)}$. The ambiguity leads to an inverted U-shaped relation between the pollution tax and the overall GDP growth.*

Proposition 3 shows that given the fact that polluting inputs are necessary complements to the use of capital goods, a stringent environmental policy can still enhance, rather than reduce, the overall GDP growth, provided that the relative productivity of R&D to capital is substantially high.

If the dirty input e and capital input k are relatively complementary in production ($\sigma < \bar{\sigma}$), then a stringent environmental policy leads to sectoral reallocation from the dirty sector to other sectors. Sectoral reallocation, on the one hand, decreases demand for the pollutant-producing labor l_e and the capital-producing labor l_k. On the other hand, the R&D labor l_r and the final-good-producing labor l_y increase, given that both sectors are more environmentally friendly. As a consequence, the growth of capital g_K decreases, whereas the growth of innovation g_n increases. In the two-engine growth model, the overall effect of τ on the GDP growth g depends on the relative magnitude of the two effects. If the relative productivity of R&D to capital is high enough (i.e., $\varphi/\phi > \overline{(\varphi/\phi)}$), then such sectoral reallocation is favorable to the growth of innovation. Thus, the growth of innovation becomes the main growth engine, which offsets the negative capital growth effect and enhances the overall GDP growth rate g. In the opposite case, with relatively low productivity of R&D, overall GDP growth is biased toward the capital engine. The sectoral reallocation caused by the environmental regulation thus undermines the main driving force of growth. Because the negative capital growth effect dominates, the overall GDP growth rate g decreases with the pollution tax.

A particularly important implication is that the ambiguous effects above generate an inverted-U relation between GDP growth g and pollution tax τ. The non-monotonic relation implies a *threshold effect*, indicating that the *status quo tax level* is crucial to the overall GDP growth impact of an environmental policy. If the *initial tax levels* are lower (higher) than the threshold, then raising the environmental tax rate has a positive (negative) effect on the overall GDP growth (this threshold effect will be clearer in our numerical analysis in Section 6.2).

The conventional notion indicates that regulations aimed at improving environmental quality are expected to mitigate pollution at the cost of reducing economic growth. Our analysis suggests that environmental regulations can result in a "double-dividend" in terms of reducing pollution and boosting economic growth. The win-win outcome breaks the tradeoff between economic growth and environmental quality. Of particular note, in our model the double-dividend is achieved by effectively reallocating the resource between the pollutant/capital-producing sector and the innovation sector.[23] The double-dividend result is in contradiction to that obtained from single engine growth models, such as in Rauscher (2009) and Smulders and Di Maria (2012), that refer to an unambiguous negative growth effect. Our channel is also quite different from the conventional double-dividend hypothesis of environmental tax reforms, which stresses that increasing taxes on polluting activities can improve economic efficiency from the use of environmental tax revenues to reduce other distortionary taxes, such as income taxes.

4.3 Social Welfare

In welfare analysis the social planner takes into account the aggregate pollution $s_t = E_t/B_t$, while individual households/firms take it as given. Let $t = 0$ be an arbitrary starting date. Imposing the BGP equilibrium on the lifetime utility in (1) yields the steady-state level of social welfare, denoted

[23] The theoretical result in the current model is based on a common assumption in the literature that labor is homogenous across sectors. Nevertheless, if R&D labor is "partially" sector-specific, then one may expect that the sectoral reallocation effect (shifting from dirty inputs to the clean final-good sector, which increases the demand for R&D and activates the innovation engine) will become smaller. Due to a less pronounced trade-off between innovation growth and capital growth, a stringent environmental regulation is less likely to generate a growth dividend. We are grateful to an anonymous referee for bringing this point to our attention.

by W, as follows

$$W=\frac{1}{\rho}\left[\ln c_0+\frac{1}{\rho}\left(g-\psi g_e+\psi g_b\right)-\psi\ln\left(\frac{E_0}{B_0}\right)\right],$$

where $c_0=y_0=l_y^{1-\alpha}n_0x^{\alpha}$, $E_0=\delta K_0l_e$, $B_0=A_0l_b$, and g_e and g_b denote the steady-state growth rate of pollution and that of abatement technology, respectively. By dropping the exogenous terms and substituting the output growth rate g in the above expression, we have

$$W=\frac{1}{\rho}\left\{\underbrace{(1-\alpha)\ln l_y+\frac{\alpha}{\sigma}\ln\left[\gamma+(1-\gamma)\left(\delta l_e\right)^{\sigma}\right]}_{\substack{\text{consumption effect}\\ (+)\text{ or }(-)}}+\frac{1}{\rho}\underbrace{\left[\alpha\phi l_k+(1-\alpha)\varphi l_r\right]}_{\substack{\text{growth effect}\\ (+)\text{ or }(-)}}+\underbrace{\psi\left[\ln(l_e)^{-1}+\ln l_b\right]}_{\substack{\text{pollution effect}\\ (+)}}\right\}.$$

The welfare effect of the environmental tax can be decomposed into three parts. First, $(1-\alpha)\ln l_y+(\alpha/\sigma)\ln\left[\gamma+(1-\gamma)\left(\delta l_e\right)^{\sigma}\right]$ captures the *level effect of consumption* on welfare. From Proposition 1, an increase in the pollution tax τ increases l_y, but decreases l_e, giving rise to an ambiguous effect on the level of consumption. Second, $[\alpha\phi l_k+(1-\alpha)\varphi l_r]/\rho$ captures the *growth effect* on welfare. From Propositions 1-3, a more stringent environmental regulation also has an ambiguous effect on the growth rate of total output, depending on the relative magnitude of the growth effect of two engines — R&D and capital. Given that the polluting and capital inputs are substantially complementary, increasing τ increases l_r, but decreases l_k, which delivers an ambiguous growth effect on welfare. Third, $\psi[\ln(l_e)^{-1}+\ln l_b]$ captures the *pollution effect* on welfare. A higher pollution tax τ, on the one hand, decreases l_e and hence the aggregate pollution s_t and, on the other hand, increases l_b, which raises the abatement knowledge and hence reduces pollution. Thus, the pollution effect unambiguously increases welfare.

Given that the pollution tax unambiguously reduces aggregate pollution, the relation between welfare and the environmental tax can be positive, if the following two conditions hold: (i) when the resource is reallocated from the dirty sector to other sectors, the increase in the labor devoted to the final-good production l_y outweighs the decrease in the labor devoted to the production of polluting inputs l_e, which ensures a positive consumption effect; and (ii) substantially high productivity of R&D relative to capital guarantees a positive GDP growth effect. This implies that through a well-designed environmental regulation, the (relatively clean) engine of innovation growth can partially replace the (relatively dirty) engine of capital growth to maintain or even enhance the overall economic growth. Such "green" growth will not only reduce pollution, but will also improve social welfare.

Remark 1 (Flexible Labor Supply): To make our point more striking, in the baseline model we focus on the labor allocation across sectors by assuming a fixed labor supply. Our main results qualitatively hold even in the presence of flexible labor supply. Intuitively, a stricter environmental regulation decreases the total demand for labor, resulting in a reduction in the equilibrium amount of labor as labor is elastically supplied. Nonetheless, labor allocation remains the same, as shown in Appendix G. As the amount of labor supplied shrinks, the GDP growth rate may become lower, but social welfare may be higher due to an increase in leisure.

5 Quantitative Analysis

In this section we calibrate the model to the US economy to numerically evaluate the effects of the pollution tax on the steady-state labor allocation, the growth rates of innovation, capital and output, aggregate pollution, and social welfare, respectively.

5.1 Calibration

The structural parameters $\{\rho, \gamma, b, \tau, \sigma, \delta, \alpha, \varphi, \phi, \psi\}$ are calibrated to the US data to provide a numerical characterization of the steady-state equilibrium. The benchmark parameter values are summarized in Table 1.

We first follow Grossmann et al. (2013) to choose a conventional value of 0.02 for the discount rate ρ. We calibrate the value of the share of capital in the intermediate-good production γ by using the estimate in Berndt and Wood (1975), which is approximately 1/2. As a benchmark, government spending over GDP is 20%, and the abatement over government spending is 6%, implying that the share of output used in abatement spending against pollution, b, is set to 1.2%. We obtain an estimate from Acemoglu et al. (2012) to set the weight of pollution relative to consumption ψ to 0.1443. Furthermore, since the empirical analysis in Section 6 shows that the US is not in the list of the countries that implemented a carbon tax, the pollution tax τ is set to 0 in the benchmark. Accordingly, we increase τ from 0 to 1 to strengthen the effect of environmental policy.

To calibrate the value of the Cobb-Douglas parameter α in final-good production, we follow Yang (2021) to assume that R&D investment only contributes to a fraction of the US long-run economic growth and set the fraction to be around 0.4.[24] With this fraction, we further calculate $\alpha = 0.5503$ using the formula for the variety growth rate $g_n = (g - \alpha g_K)/(1 - \alpha)$, given that the overall GDP growth rate is $g = 2.06\%$ and the capital growth rate is $g_K = 3.07\%$ from Chu et al. (2019). To calibrate the R&D productivity parameter φ and the capital productivity parameter ϕ, we use a

[24] Yang (2021) explores the growth and welfare effects of patent protection in a similar two-engine growth model with R&D and capital accumulation.

Table 1: Parameter values in the benchmark.

ρ	γ	b	τ	σ	δ	α	φ	ϕ	ψ
0.02	0.5	0.012	0	-0.5	503.1647	0.5503	0.0839	0.1451	0.1443

standard value of 15.56% for the percentage of capital investment as GDP in addition to the values of g and g_K; this implies that $\varphi = 0.0839$, $\phi = 0.1451$, and $(e/k)^\sigma = 0.1785$ in the benchmark case.

As for the degree of complementarity between capital and the polluting inputs σ, we use the elasticity of substitution between these two inputs ϵ to estimate it. The reasonable range of substitution elasticity between polluting inputs and capital inputs in the existing literature is between 0.6 (Liu and Shumway, 2016) and 0.75 (Kim and Heo, 2013). Therefore, we choose the average value of $\epsilon = 0.67$ (i.e., $\sigma = -0.5$) as the benchmark value.[25] Consequently, using (30) and the value of $(e/k)^\sigma$ yields the calibrated value of the productivity parameter $\delta = 503.1647$ in the production of the polluting inputs.

5.2 Numerical Results

We start with the effects of the pollution tax τ on the steady-state labor allocation. Figure 1 shows that an increase in τ decreases the labor devoted to the production of polluting inputs l_e, but it increases the manufacturing labor l_y and the R&D labor l_r due to the sectoral reallocation effect, as predicted in Proposition 1. The capital-producing labor l_k decreases with the pollution tax, given that the calibrated value of σ is smaller than the threshold value $\bar{\sigma}$ implied by (32) (which is around -0.43 for positive values of τ); namely, the complementarity between the polluting e and capital k inputs in the intermediate-good production is strong. It is also intuitive that a strengthening of environmental policy results in the allocation of more labor to engage in pollution abatement l_b.[26]

We next turn to the growth effects of τ on innovation, capital, and output. Figures 1(f)–1(h) display these growth effects, and Table 2 documents the growth rates for a range of τ from 0 to 1. As shown in Proposition 2, there is a tradeoff between the two growth engines – innovation and capital – in the presence of a stringent environmental regulation, which is consistent with the empirical evidence. As τ increases from 0 to 1, the innovation growth rate g_n rises from 0.8235% to 0.9035%, whereas the capital growth rate g_K declines from 3.0704% to 3.0131%. In particular, the effects of τ

[25] Energy intensity, defined as the amount of energy required per unit of output, has been decreasing in many industries due to advances in technology and increased efficiency measures. This reduction in energy intensity implies that the degree of complementarity σ between polluting and capital inputs may change over time. To capture this effect, Subsection 5.3.1 considers the case with a lower (absolute) degree of σ.

[26] In our model the pollution-abatement labor positively relates to the manufacturing labor l_y, given a constant share of abatement in output, b (see Appendix B). As the pollution tax increases the manufacturing labor, this favors the production of final goods Y and hence induces the government to allocate more resources to abatement.

on innovation dominate those on capital accumulation for low levels of τ, but this domination reverses as the environmental policy becomes increasingly strict (i.e., for high levels of τ). As a result, the overall GDP (output) growth g has an inverted-U relation with the environmental tax τ given that the calibrated value for the relative productivity of R&D to capital φ/ϕ is first higher, but becomes lower than the critical value $\overline{(\varphi/\phi)}$ as τ rises (recall that $\overline{(\varphi/\phi)}$ is a function of τ in (34)).

Figure 1(h) shows that the growth-maximizing pollution tax rate (i.e., the threshold value) is $\tau = 0.4983$. *The threshold effect* implies that the *status quo tax level* is crucial to the overall GDP growth impact of an environmental policy. If the *status quo* tax levels are lower than the threshold $\tau = 0.4983$, then raising the environmental tax rate has a positive effect on the overall GDP growth. By contrast, it has a negative effect on the GDP growth when the *status quo* taxes have already been substantially high (i.e., larger than $\tau = 0.4983$). Moreover, Figure 1(i) shows that the steady-state pollution s unambiguously decreases. This is because a higher τ, on the one hand, discourages firms from using labor in the dirty-good production l_e, which lowers the amount of pollutants E and, on the other hand, induces the government to engage in more abatement B, which also reduces the aggregate pollution. Given that aggregate pollution s is decreasing in τ, the threshold effect is also decisive for the existence of a double-dividend. Our numerical analysis predicts that as long as the environmental policy has not been too stringent already (the *status quo* tax rates are lower than $\tau = 0.4983$), a stringent environmental regulation can lower steady-state pollution without a loss of GDP growth, exerting a double-dividend in terms of reducing pollution s and boosting growth g.

We now discuss the effects of the pollution tax τ on social welfare W, according to the decomposition in Section 4.3. First, as τ rises, the consumption effect is negative, because a large decrease in l_e reduces the market scale. Second, the overall growth effect is hump-shaped due to the opposing impacts of l_r and l_k, which are favorable to innovation growth, but unfavorable to capital growth. Third, the pollution effect is unambiguously positive, as noted above. It turns out from Figure 1(j) that this positive pollution effect dominates the other two effects, leading the welfare level to monotonically increase in τ. In the benchmark, an increase in τ from 0 to 1 leads to a welfare improvement of 4.56% increase in consumption equivalence.[27]

We summarize the main numerical results above in the following.

Result 1. *In the presence of a stringent environmental regulation,*

(i) *the growth of innovation rises;*

(ii) *the growth of capital falls as* $\sigma < \bar{\sigma}$*;*

[27] This welfare improvement is expressed as the usual equivalent variation in consumption flow.

(iii) *the tradeoff between two growth engines – innovation and capital – leads to the growth-maximizing pollution tax rate of* $\tau = 0.4983$*;*

(iv) *the threshold effect indicates that if the status quo tax is lower than* $\tau = 0.4983$*, then there exists a double-dividend in terms of not only improving the environment (green dividend), but also raising growth and welfare (blue dividend).*

Table 2: Effects of increasing the pollution tax on labor allocation, growth, pollution, and welfare.

τ	0	0.2	0.4	0.6	0.8	1
l_y	0.6115	0.6164	0.6204	0.6237	0.6264	0.6289
l_r	0.0982	0.1009	0.1030	0.1048	0.1064	0.1077
l_k	0.2116	0.2111	0.2103	0.2095	0.2086	0.2077
l_e	0.0624	0.0552	0.0497	0.0454	0.0419	0.0390
g_n	0.8235%	0.8462%	0.8643%	0.8794%	0.8923%	0.9035%
g_K	3.0704%	3.0626%	3.0518%	3.0400%	3.0265%	3.0131%
g	2.0600%	2.0659%	2.0681%	2.0681%	2.0667%	2.0644%
s	1923.25	1687.85	1511.20	1373.01	1261.48	1169.29
W	59.8735	60.6179	61.1469	61.5340	61.8226	62.0399

Finally, two interesting remarks are noted for welfare analysis.

Remark 2 (Environmental Consciousness): The welfare effect of environmental taxation relates to the weight of *external* pollution ψ relative to internal consumption, which captures the extent of households' environmental consciousness. Figure 2 shows that if households are less environmentally conscious, say, a lower $\psi = 0.05$ compared with the benchmark value 0.1443, then the positive effect stemming from a reduction in pollution no longer always outweighs the effects of consumption and growth. Specifically, welfare increases with the pollution tax only for a relatively low τ (for $\tau \leq 0.3131$); over the threshold tax, welfare decreases with the pollution tax given that the negative consumption and growth effects turn out to become dominating.

Remark 3 (Pollution Abatement): The welfare effect of environmental taxation also relates to the share b of abatement expenditure relative to GDP, which captures the level of abatement activities.[28] Figure 3 shows that when the abatement spending ratio declines, say, to a lower $b = 0.8\%$ as compared to the benchmark value 1.2%, the level of aggregate pollution s rises significantly in magnitude over the entire range of pollution tax due to less employment in abatement service. This results in considerable downsizing in the welfare level even if a higher tax is still welfare-improving.

[28] Grubb et al. (2021) show that in a recent version of the Dynamic Integrated Climate-Economy (DICE) model (Nordhaus, 2017), abatement expenditure is subject to a peak of 0.8% of GDP. Hence, we vary the abatement spending ratio b to 0.8%, 0.5%, and 0.1%, respectively, to investigate the changes in the effects of environmental taxation on aggregate pollution and the welfare.

5.3 Sensitivity

We perform three sensitivity checks to investigate the role played by the complementarity between the capital and polluting inputs σ, the relative productivity of R&D to capital φ/ϕ, and the market imperfection $1/\alpha$, respectively.

5.3.1 Complementarity between Capital and Polluting Inputs σ

In addition to the benchmark value $\sigma = -0.5$, we consider two different degrees of complementarity between the capital and polluting inputs of $\sigma = -0.7$ and $\sigma = -0.3$ to investigate the role of σ in labor allocation, growth, and welfare.

The main results of the benchmark model are in general robust in the case where capital and the polluting inputs are more complementary ($\sigma = -0.7$, and hence $\epsilon = 0.59$). Table 3 shows that a higher pollution tax τ increases l_r and l_y, but decreases l_k and l_e, thereby enhancing innovation growth g_n and retarding capital growth g_K. A stronger complementarity between capital and polluting inputs (lower σ and ϵ) weakens the relative price effect and reinforces the technical complementarity effect (see Footnote 14). Thus, in contrast to the benchmark case, the negative capital growth effect strictly dominates the positive innovation growth effect such that overall GDP growth g becomes monotonically decreasing in the pollution tax τ. Nevertheless, social welfare unambiguously increases with τ, because the pollution tax reduces the aggregate pollution s substantially. Therefore, the growth dividend fails while the welfare dividend still exists.

Table 3: Effects of increasing the pollution tax on labor allocation, growth, and welfare: $\sigma = -0.7$.

τ	0	0.2	0.4	0.6	0.8	1
l_y	0.6115	0.6150	0.6178	0.6202	0.6223	0.6241
l_r	0.0982	0.1000	0.1016	0.1029	0.1041	0.1051
l_k	0.2311	0.2301	0.2290	0.2280	0.2269	0.2258
l_e	0.0429	0.0385	0.0351	0.0324	0.0302	0.0283
g_n	0.8235%	0.8393%	0.8523%	0.8634%	0.8730%	0.8816%
g_K	3.3526%	3.3385%	3.3233%	3.3077%	3.2921%	3.2767%
g	2.2153%	2.2146%	2.2121%	2.2085%	2.2043%	2.1996%
s	1323.69	1180.63	1071.56	985.08	914.47	885.49
W	60.2634	60.8749	61.3224	61.6593	61.9177	62.1181

Table 4 shows the other case where the capital and polluting inputs are less complementary ($\sigma = -0.3$, and hence $\epsilon = 0.77$). A weaker complementarity between capital and polluting inputs amplifies the relative price effect and attenuates the technical complementarity effect. Once the relative price effect dominates (is dominated by) the technical complementarity effect, l_k and hence g_K increase

Table 4: Effects of increasing the pollution tax on labor allocation, growth, and welfare: $\sigma = -0.3$.

τ	0	0.2	0.4	0.6	0.8	1
l_y	0.6115	0.6191	0.6250	0.2972	0.6337	0.6370
l_r	0.0982	0.1023	0.1055	0.1082	0.1103	0.1122
l_k	0.1763	0.1770	0.1772	0.1772	0.1769	0.1765
l_e	0.0977	0.0850	0.0756	0.0682	0.0622	0.0573
g_n	0.8235%	0.8584%	0.8855%	0.9074%	0.9256%	0.9411%
g_K	2.5584%	2.5688%	2.5718%	2.5705%	2.5665%	2.5608%
g	1.7782%	1.7996%	1.8135%	1.8226%	1.8286%	1.8325%
s	3011.21	2589.75	2279.71	2041.21	1851.56	1696.81
W	59.2362	60.2169	60.8896	61.3648	61.7063	61.9534

(decrease) with the pollution tax. The turning point, as shown in Table 4, is approximately at $\tau = 0.42$. Since the technical complementarity effect becomes smaller, a stricter environmental regulation is more likely to increase, rather than decrease, capital growth. Once the environmental regulation favors both engines of growth, a rise in the pollution tax monotonically increases the overall GDP growth rate g, as shown in Table 4. The positive growth effect, together with a reduction in the aggregate pollution s, leads welfare W to increase unambiguously.

Result 2. *Environmental regulation is more likely to obtain the non-environmental blue dividend in terms of enhancing growth and improving welfare, if capital and polluting inputs are less complementary.*

5.3.2 Relative Productivity of R&D to Capital φ/ϕ

The next sensitivity examination is to investigate the role played by the relative productivity of R&D compared to capital φ/ϕ in the effects of the environmental taxation, particularly on growth and welfare. We follow Iwaisako and Futagami (2013) and Yang (2021) to consider different values of φ/ϕ by altering ϕ while maintaining φ. For interesting cases, we vary ϕ to consider lower values of φ/ϕ, say, 0.3, 0.1, and 0.03, compared to the benchmark value $\varphi/\phi = 0.578$. These cases imply that, relative to innovation, the growth engine of capital contributes more substantially to overall GDP growth in the economies, such as in many developing countries.

Figure 4 shows that in the case where $\varphi/\phi = 0.3$, the growth and welfare effects of τ are similar to those in the benchmark. The GDP growth g has an inverted-U relation with the pollution tax where the growth-maximizing tax rate is $\tau = 0.1567$, while welfare W is monotonically increasing in τ. Note that the threshold of $\tau = 0.1567$ is smaller than $\tau = 0.4983$ in the benchmark, implying that a higher pollution tax is less likely to enhance economic growth.

The effects, however, are quite different from those in the benchmark, when the relative productivity of R&D to capital is further reduced to $\varphi/\phi = 0.1$ or $\varphi/\phi = 0.03$. In these cases, capital becomes a more important growth engine in governing overall GDP growth. Since the negative capital growth effect outweighs the positive innovation growth effect, the overall growth rate now unambiguously decreases with the pollution tax τ. Figure 4 further indicates that in the case where $\varphi/\phi = 0.1$, this negative growth effect, together with a reduction in consumption, tends to overwhelm the pollution effect for a wider range of taxes; i.e., $\tau > 0.2076$), inducing W to take an inverted U-shape with respect to τ. Moreover, in the case where $\varphi/\phi = 0.03$, the negative effects of the capital growth and overall GDP growth become substantially strong, leading W to strictly decrease with τ.

These results enable us to establish the following result.

Result 3. *Environmental regulation is less likely to obtain the non-environmental blue dividend in terms of enhancing growth and improving welfare, if capital, relative to innovation, is a more important driving force of the overall GDP growth.*

Since in many developing and Asian countries capital accumulation is an important growth engine, Result 3 potentially points out that stringent environmental regulation in these countries is more likely to give rise to a loss of GDP growth and is less likely to result in the double-dividend.

5.3.3 Market Imperfection $1/\alpha$

We finally investigate the role of market imperfection in the growth effects of pollution taxation by varying the markup from the benchmark value of $1/\alpha = 1.8172$ to 1.3, 1.5, 1.7, and 2. As shown in Figure 5, increasing τ still stimulates innovation growth g_n as in the benchmark, but the impacts on capital growth g_K and output growth g vary for different degrees of price markup.

When the market is more competitive (a lower price markup $1/\alpha = 1.3$), a profit-maximizing intermediate-good firm tends to set a lower price by expanding its output level $x_t(j)$, as shown in (6). This amplifies the relative price effect, but attenuates the technical complementarity effect. Faced with a higher pollution tax, a fall in the demand for polluting inputs leads the firm to substantially expand its demand for capital inputs in order to support the expansion in output, which amplifies the relative price effect. However, the technical complementarity effect attenuates, since a higher pollution tax only results in a limited reduction in polluting inputs under output expansion (and thus input demands), which gives rise to a slight decrease in capital inputs. Overall, the relative price effect dominates the technical complementarity effect. Therefore, in contradiction to the benchmark, the capital growth rate unambiguously increases, rather than decreases, with the pollution tax in the case where the markup is lower; i.e., $1/\alpha = 1.3$. Figure 5 shows that the positive capital growth effect g_K, together with a rise in g_n, refers to a monotonic increase in overall GDP growth g.

Figure 5 further indicates that the relative price effect (technical complementarity effect) of pollution taxation becomes weaker and weaker (stronger and stronger) as the price markup is increasing. In the parametrization the capital growth rate is monotonically decreasing in τ when the price markup is above $1/\alpha = 1.7$. It is interesting to note that because the magnitude of the relative price effect decreases with the degree of price markup, the GDP growth-maximizing pollution tax will be lower; e.g., $\tau = 0.3778$ in the case where the markup is relatively high $1/\alpha = 2$. In other words, if the market is less competitive, then stricter environmental regulation is more likely to reduce economic growth. The 2008 OECD Indicators of Product Market Regulation reveal that OECD countries in general are more competitive than less developed non-OECD countries. With the observation, this sensitivity analysis confirms our argument that a stringent environmental regulation in developing countries is more likely to give rise to a loss of GDP growth and is less likely to result in the double-dividend.[29]

To summarize, we have the following result.

Result 4. *If the goods market is less competitive, then stringent environmental regulation is less likely to generate a growth dividend.*

One point should be noted before we move to the empirical analysis. To make our point more striking, we isolate the double-dividend effect of tax reforms by assuming a non-distortionary lump-sum tax, instead of a distortionary income tax, to finance government expenditures in the baseline model. If the government levies a distortionary income tax to meet the government's budget constraint (equation 25), then increasing the environmental tax implies a lower income tax. Per the double-dividend hypothesis, one can expect that replacing distortionary taxes with environmental taxes will yield a double dividend by not only improving the environment quality, but also reducing the non-environmental costs of the tax system (see Bovenberg and Goulder, 2002). As a result, a stringent environmental regulation is more likely to increase overall growth and thereby welfare.[30]

6 Empirical Analysis

In this section we conduct empirical analysis using country-level data to examine the growth effects of carbon tax on TFP and capital stock. The data are collected from the Carbon Pricing Dashboard

[29] Existing studies demonstrate that a higher degree of patent protection reinforces firms' ability to charge a higher markup (e.g., Iwaisako and Futagami, 2013, Chu et al., 2019, and Yang, 2021). Hence, in this numerical exercise, stronger patent protection leads to a less competitive market, implying that stringent environmental regulation is less likely to generate a growth dividend, as shown in Result 4.

[30] The detailed analysis is available upon request.

of the World Bank and Penn World Table (PWT). By merging the data obtained from these two data sources, we construct variables for the annual growth rates of TFP and capital stock, as well as the average price of carbon tax (in US dollars per ton of carbon dioxide equivalent). Our sample includes 178 countries covering a 24-year period from 1995 to 2018. A detailed description of the data sources and the definitions of variables can be found in Appendix I.1.

Table 5 presents the summary statistics of the sample. After excluding observations with missing data in the variables to be applied in the regressions, there are 2,688 and 4,094 observations involved in the regressions for the growth rate of TFP and that of capital stock, respectively.

Table 5: Summary statistics of the data for 178 countries

Variable	Obs	Mean	Std. dev.	Min	Max
TFP	2,668	0.717	4.171	-44.954	45.532
Capital Stock	4,094	3.696	3.535	-3.756	45.329
L.Carbon Tax (avg)	4,094	0.134	0.648	0.000	4.948
L.Carbon Tax (High)	4,094	0.140	0.680	0.000	5.135
L.Carbon Tax (Low)	4,094	0.123	0.593	0.000	4.948

Notes: *TFP* and *Capital Stock* denote the annual growth rate of TFP and capital stock in percent terms. *Carbon Tax* is the natural logarithm of the price per ton of carbon dioxide equivalent (tCO2e), which is transformed by $(\ln(1 + Carbon\ Tax))$.

Among the 178 countries, the average annual growth rates of TFP and capital stock are 0.717% and 3.696%, respectively. In our sample there are 19 countries that had implemented carbon tax.[31] The average logged price of carbon tax is 0.134. The highest and lowest logged prices of carbon tax are marginally distinct from the average prices for which the average values are 0.140 and 0.123, respectively.

Our theoretical model predicts that strengthening environmental policy stimulates innovation growth and stifles capital growth. Therefore, the main purpose of this empirical analysis is to explore the respective impacts of carbon tax on the growth of TFP and capital stock. The empirical model adopts a general policy analysis framework; that is, an extension of the difference-in-difference (DID) method for policy evaluation using panel data. This type of regression models is widely used to infer causal relationships between outcomes and government policies, initiatives, or interventions (Slaughter 2001; Beny and Cook 2009; Papaioannou 2021). The results from those models are persuasive since unobserved heterogeneity is eliminated via country-fixed effects, and aggregate time effects are taken into account through a complete set of year dummies. The panel data facilitate the estimation for the partial effects of carbon tax with the staggered interventions since countries with a carbon

[31] The list of countries involved in the sample is in Appendix I.2.

tax implemented the policy in different years. In particular, we estimate the panel regression models specified below:

$$TFPGR_{i,t} = \beta_0 + \beta_1 CTax_{i,t-1} + \zeta_t + \chi_i + u_{i,t}, \tag{35}$$

$$CSGR_{i,t} = \lambda_0 + \lambda_1 CTax_{i,t-1} + \zeta_t + \chi_i + v_{i,t}, \tag{36}$$

where $TFPGR_{i,t}$ and $CSGR_{i,t}$ are the growth rates of TFP and capital stock, respectively. The subscript i represents the country and t represents the year. In both models, the time fixed effects and country fixed effects are captured by ζ_t and χ_i, respectively. $CTax_{i,t-1}$ is the one-year lagged independent variables of the natural logs of the prices of carbon tax.

Table 6 reports the results from estimating equations (35) and (36). Panels (1-), (2-), and (3-) present the results from the pooled OLS, year-fixed effects, and country- and year-fixed effects estimations, respectively.

Table 6: Estimation results for the growth of TFP and capital stock

	(1-a)	(1-b)	(2-a)	(2-b)	(3-a)	(3-b)
	TFP	*Capital Stock*	*TFP*	*Capital Stock*	*TFP*	*Capital Stock*
L.Carbon Tax (avg)	0.005	-0.666***	0.075	-0.675***	0.343***	-0.319***
	(0.066)	(0.087)	(0.077)	(0.087)	(0.128)	(0.122)
Constant	0.716***	3.786***	1.154**	3.527***	1.125**	3.502***
	(0.141)	(0.204)	(0.566)	(0.351)	(0.544)	(0.261)
Year FE	No	No	Yes	Yes	Yes	Yes
Country FE	No	No	No	No	Yes	Yes
Observations	2668	4094	2668	4094	2668	4094
R2/Within R2	0.000	0.015	0.053	0.035	0.061	0.046

Notes: Robust standard errors clustered at country level are in parentheses. Specifications: (1-) are pooled OLS; (2-) are with year-fixed effects; and (3-) are with country- and year- fixed effects. ***, **, and * denote significance at 1%, 5%, and 10%, respectively.

Table 6 indicates that after including the country- and year- fixed effects in the models, the lagged average carbon tax has significant effects on the growth of TFP and growth of capital stock. As shown in column (3-a), if there is a 1% increase in the average carbon tax, then the annual growth of TFP will rise by 0.0034 percentage points. In contrast, the results in column (3-b) show that the average carbon tax has a significantly negative impact on the growth of capital stock, that is, a 1% increase in carbon tax will lead to an approximately 0.0032 percentage point decrease in the annual growth in capital stock. These empirical results regarding the impacts of carbon tax on the growth of TFP and capital stock are consistent with the theoretical predictions by our model.

Three robustness tests are also run to confirm the results of Table 6 on the estimations. First, according to the Carbon Pricing Dashboard, among the countries that implemented a carbon tax, some of them imposed two different tax rates on different types of fossil fuels. For robustness check, in addition to the average price of the carbon tax as the independent variable in the regressions, we re-estimate the regressions using two different measures of the carbon tax, which are the highest and lowest prices among the carbon taxes imposed in country i at year t. Second, in order to avoid potential short-run noise from other random events that occurred, we run the regressions again by converting all the variables into three-year averages. Third, since we employ the carbon tax in our regression models, concerns may arise about the limitations of using just one carbon pricing initiative as the independent variable. To ensure the robustness of our analysis, we also incorporate a broader indicator of environmental policy stringency, namely the Environmental Policy Stringency Index provided by OECD, as the independent variable in the regression models. The results from the robustness tests are consistent with the results presented in Table 6. Details of the estimation results can be found in Appendix I.3.

7 Concluding Remarks

This paper presents the macroeconomic effects of environmental taxation in a growth model that features two growth engines of capital and innovation. Analytically, we show that increasing the pollution tax leads to a sectoral reallocation from the dirty sector to other sectors (including manufacturing and R&D). Provided that the polluting and capital inputs are sufficiently complementary, innovation growth increases while capital growth decreases. This thus provides a convincing explanation for the tradeoff between capital and innovation as observed by our empirical result and also studies in the literature. Due to this tradeoff, environmental taxation has an inverted-U relation with the overall GDP growth. These results suggest that an environmental regulation can result in a double-dividend in terms of reducing pollution (environmental green dividend) and increasing growth and welfare (non-environmental blue dividend). By means of an appropriate sectoral resource reallocation, an environmental regulation can give rise to such a double-dividend.

We quantitatively show a threshold effect of environmental taxation: if the *status quo* tax rate is less than 0.4983, then raising the environmental tax rate can increase overall GDP growth; it, however, impedes GDP growth if the existing tax rate has been higher than this threshold. Moreover, the calibration results indicate that an environmental regulation is more likely to obtain the non-environmental blue dividend in growth and welfare; if the innovation sector, relative to the capital sector, is more productive, then the capital and polluting inputs are less complementary, or the goods market is more competitive. Given these conditions, the double-dividend can exist under a wide range of calibrated parameter values in the US. By contrast, a stringent environmental regulation is less likely to generate a non-environmental dividend in growth and welfare in developing countries.

There are two potential dimensions to extend the present study. First, the current model can be extended to consider that two engines (i.e., R&D and physical capital) are asymmetric not only in their productivity, but also the necessity of skills. In this case, the R&D sector requires only skilled workforce, whereas the capital-good sector uses both skilled and unskilled workforces. This specification implies that labor in the R&D sector is partially sector-specific. Incorporating such labor heterogeneity into our theoretic framework implies that environmental policy leads to reallocation across sectors through skills, affecting the policy implications for employment and growth (see Metcalf and Stock, 2020). Second, our study examines the importance of considering the capital-innovation tradeoff in the analysis of environmental-policy effects on growth and welfare. While we focus on implications on growth and welfare, the effects of environmental policy on other key macroeconomic variables, for example, income inequality (see Aloi and Tournemaine, 2013), are worth examining. We leave these crucial issues as interesting directions for future research.

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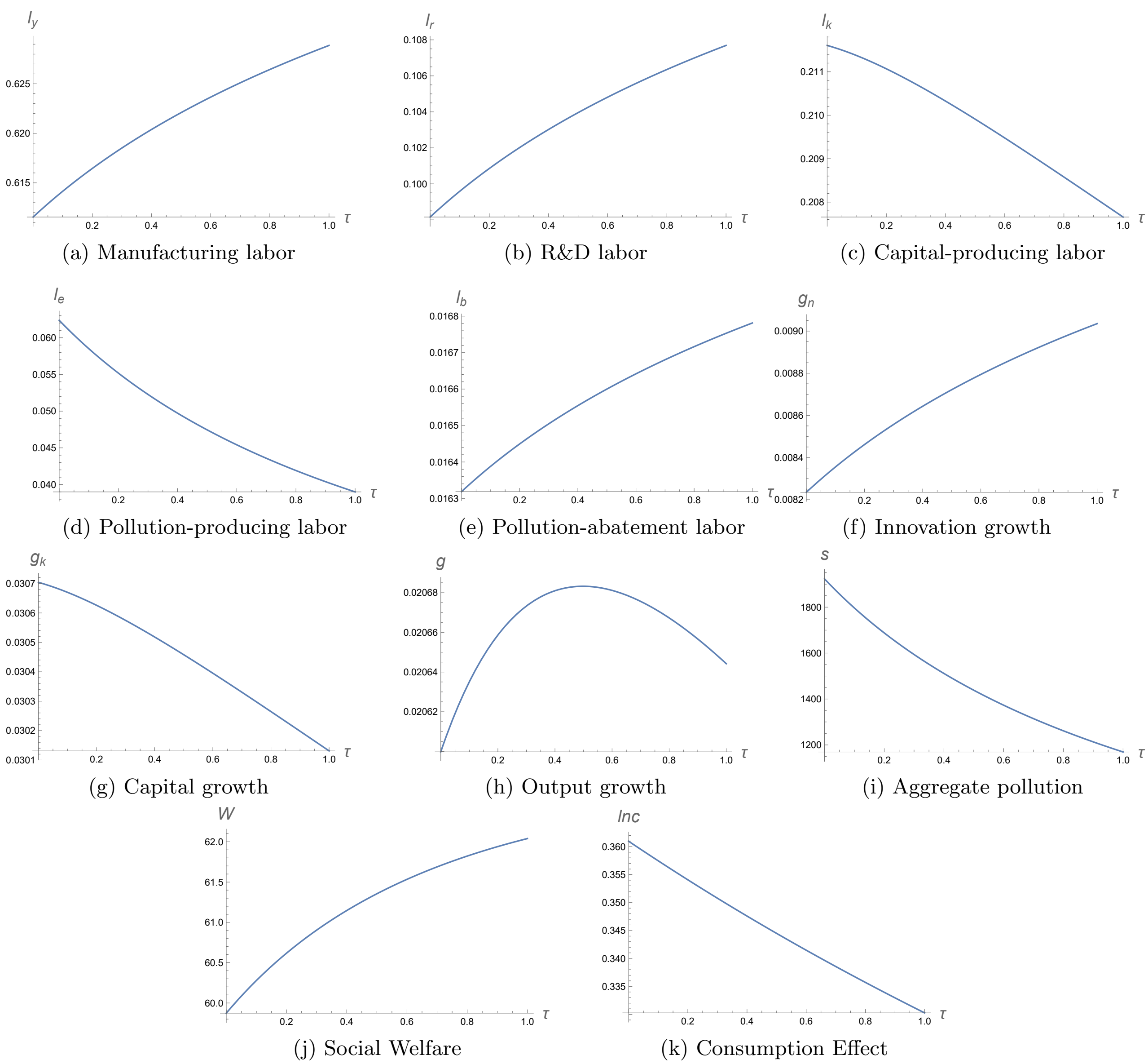


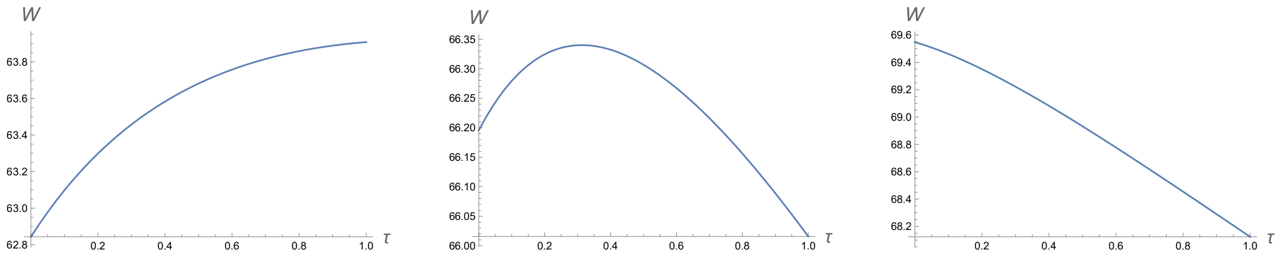
(a) Manufacturing labor (b) R&D labor (c) Capital-producing labor

(d) Pollution-producing labor (e) Pollution-abatement labor (f) Innovation growth

(g) Capital growth (h) Output growth (i) Aggregate pollution

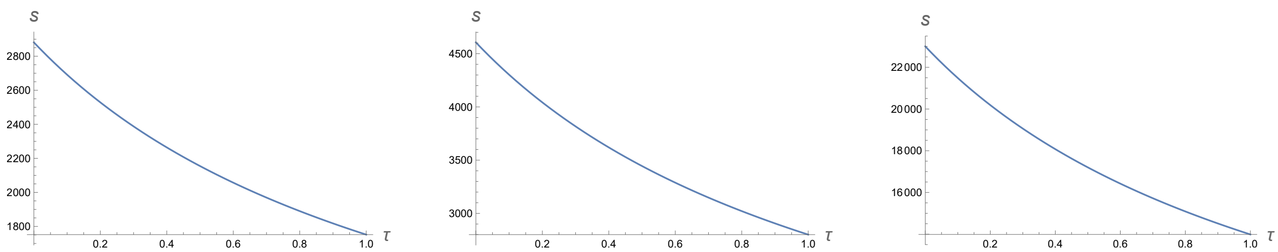
(j) Social Welfare (k) Consumption Effect

Figure 1: Effects of τ on labor, growth, pollution, welfare, and consumption.



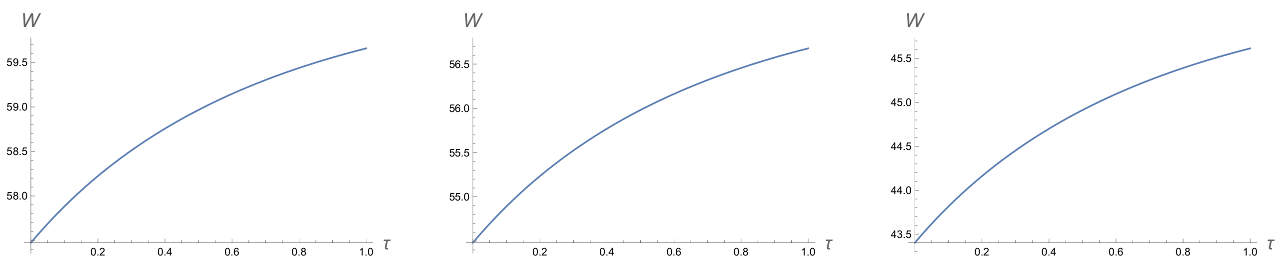
(a) $\psi = 0.1$ (b) $\psi = 0.05$ (c) $\psi = 0$

Figure 2: Welfare effects of τ with various weights of pollution.



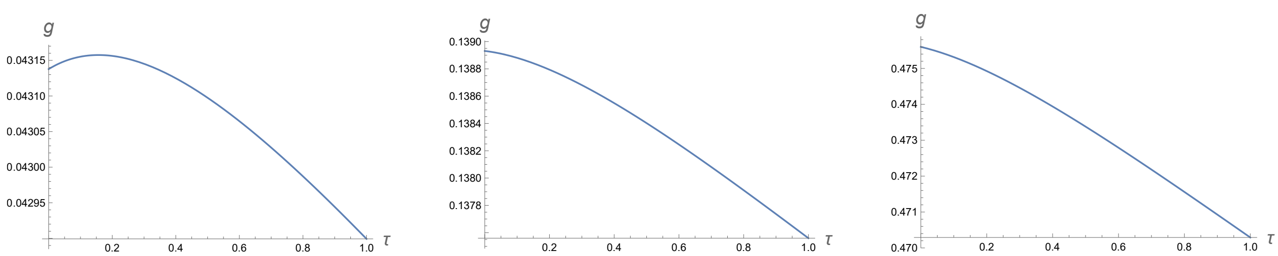
(a) $b = 0.008$ (b) $b = 0.005$ (c) $b = 0.001$

Figure 3-1: Pollution effects of τ with various abatement spending ratios.



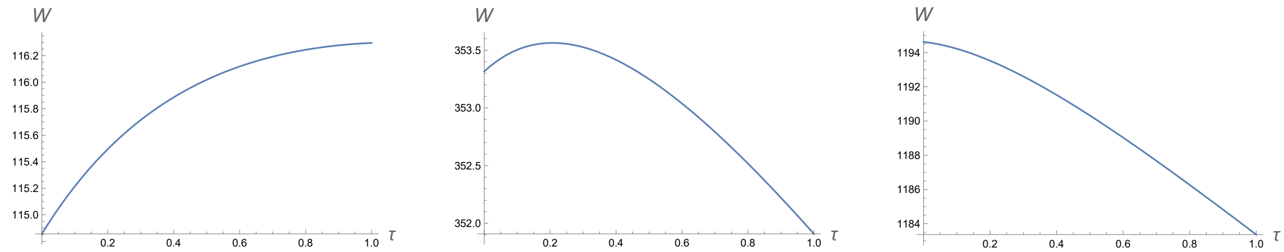
(a) $b = 0.008$ (b) $b = 0.005$ (c) $b = 0.001$

Figure 3-2: Welfare effects of τ with various abatement spending ratios.



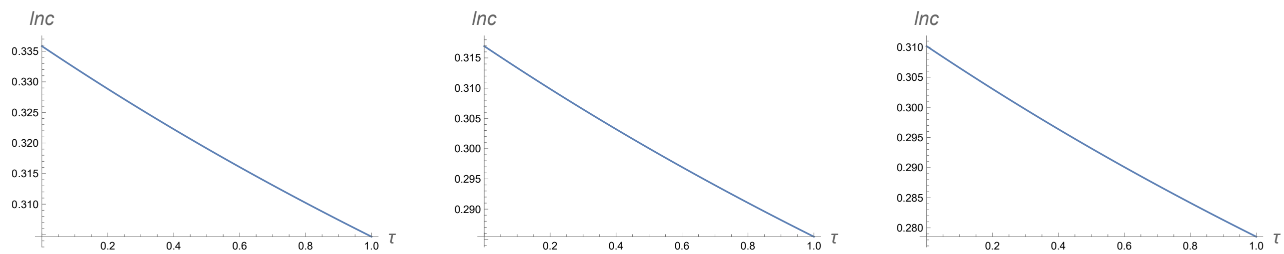
(a) $\varphi/\phi = 0.3$ (b) $\varphi/\phi = 0.1$ (c) $\varphi/\phi = 0.03$

Figure 4-1: Growth effects of τ with various relative productivities of R&D to capital.



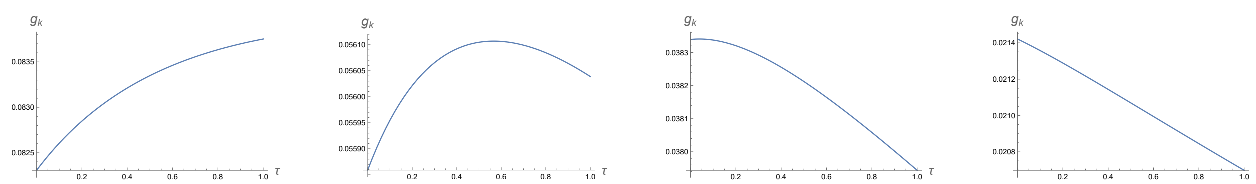
(a) $\varphi/\phi = 0.3$ (b) $\varphi/\phi = 0.1$ (c) $\varphi/\phi = 0.03$

Figure 4-2: Welfare effects of τ with various relative productivities of R&D to capital.



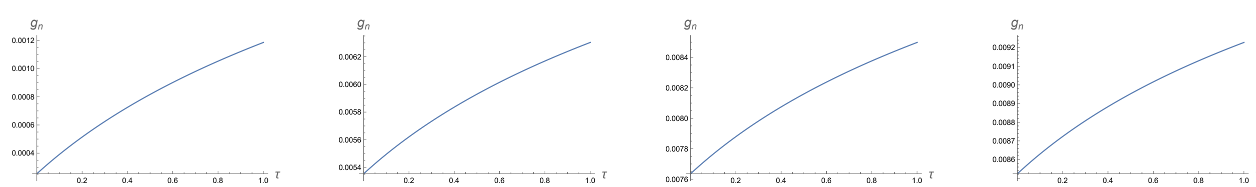
(a) $\varphi/\phi = 0.3$ (b) $\varphi/\phi = 0.1$ (c) $\varphi/\phi = 0.03$

Figure 4-3: Consumption effects of τ with various relative productivities of R&D to capital.



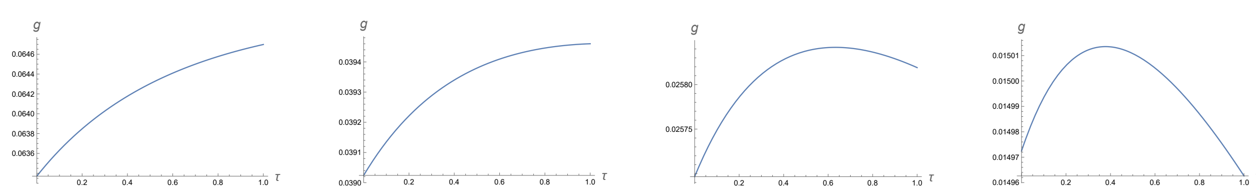
(a) $1/\alpha = 1.3$ (b) $1/\alpha = 1.5$ (c) $1/\alpha = 1.7$ (d) $1/\alpha = 2$

Figure 5-1: Capital growth effects of τ with various price markups.



(a) $1/\alpha = 1.3$ (b) $1/\alpha = 1.5$ (c) $1/\alpha = 1.7$ (d) $1/\alpha = 2$

Figure 5-2: Innovation growth effects of τ with various price markups.



(a) $1/\alpha = 1.3$ (b) $1/\alpha = 1.5$ (c) $1/\alpha = 1.7$ (d) $1/\alpha = 2$

Figure 5-3: Overall GDP growth effects of τ with various price markups.

Online Appendix for "Environmental Regulation Stringency and Allocation between R&D and Physical Capital: A Two-Engine Growth Model"

Not Intended for Publication unless Requested

Appendix A: Proof of Lemma 1

We first divide both sides of the household budget constraint (2) by w_t, which yields

$$\frac{\dot{a}_t}{w_t} = r_t \frac{a_t}{w_t} + 1 - \frac{c_t}{w_t} - \frac{\mu_t}{w\,t}. \tag{A.1}$$

Substituting (16) and (20) into the expression for the household's total assets $a_t = v_{n_t} n_t + v_{k_t} K_t$, we obtain: $a_t = \left(\frac{\phi+\varphi}{\phi\varphi}\right) w_t$. This implies that $\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t}$ and $\left(\frac{\phi+\varphi}{\phi\varphi}\right)\frac{\dot{a}_t}{a_t} = \frac{\dot{a}_t}{w_t}$. Substituting these relations into (A.1), we then have

$$\left(\frac{\phi+\varphi}{\phi\varphi}\right)\frac{\dot{w}_t}{w_t} = \left(\frac{\phi+\varphi}{\phi\varphi}\right) r_t + 1 - \frac{c_t}{w_t} - \frac{\mu_t}{w\,t}.$$

By using the Euler equation (3), the labor demand function (5), and the good-market clearing condition $Y_t = C_t$, some manipulation allows us to further rewrite the equation as

$$\frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\phi\varphi}{\phi+\varphi}\left(\frac{1+t_t}{1-\alpha}\right) l_{y,t} - \left(\frac{\phi\varphi}{\phi+\varphi} + \rho\right),$$

where $t_t \equiv \mu_t/Y_t$. Next, using the government balanced budget constraint (25) and (10) yields $t_t \equiv \frac{\mu_t}{Y_t} = b_t - \alpha^2 \frac{(1-\gamma)\left(\frac{e_t}{k_t}\right)^\sigma}{1+(1-\gamma)\left(\frac{e_t}{k_t}\right)^\sigma}$. Substituting it into the above equation with a stationary path of b_t yields a single non-linear differential equation with respect to $l_{y,t}$

$$\frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\phi\varphi}{\phi+\varphi}\left(\frac{1}{1-\alpha}\right)\left[1 + b - \alpha^2 \frac{(1-\gamma)\left(\frac{e_t}{k_t}\right)^\sigma}{1+(1-\gamma)\left(\frac{e_t}{k_t}\right)^\sigma}\right] l_{y,t} - \left(\frac{\phi\varphi}{\phi+\varphi} + \rho\right), \tag{A.2}$$

where $(e_t/k_t)^\sigma$ is an implicit function of $l_{y,t}$.

To find out the relation between $l_{y,t}$ and e_t/k_t, using (20) and (9), equation (11) can be written as

$$\frac{\gamma k_t^{\sigma-1}}{(1-\gamma)e_t^{\sigma-1}} = \frac{\alpha p_t x_t n_t \left[\frac{\gamma k_t^\sigma}{\gamma k_t^\sigma + (1-\gamma)e_t^\sigma}\right]}{(1+\tau)w_t}.$$

Rearranging it and using (4), (6), and (5) yield

$$\frac{\gamma k_t^\sigma + (1-\gamma)e_t^\sigma}{(1-\gamma)e_t^\sigma} = \frac{\delta\alpha^2}{(1+\tau)}\frac{l_{y,t}}{(1-\alpha)}\frac{k_t}{e_t}. \tag{A.3}$$

Plugging it into (A.2) yields

$$\frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\phi\varphi}{\phi+\varphi}\left(\frac{1+b}{1-\alpha}\right)l_{y,t} - \left[\left(\frac{1+\tau}{\delta}\right)\frac{e_t}{k_t}+1\right]\frac{\phi\varphi}{\phi+\varphi} - \rho.$$

Finally, we take the derivative of e_t/k_t with respect to $l_{y,t}$, apply the implicit function theorem to (A.3), and obtain

$$\frac{\partial(e_t/k_t)}{\partial l_{y,t}} = \frac{\delta\alpha^2}{(1+\tau)(1-\alpha)}\left[\frac{\gamma}{1-\gamma}(1-\sigma)\left(\frac{e_t}{k_t}\right)^{-\sigma}+1\right]^{-1} > 0.$$

Accordingly, we thus have

$$\frac{\partial(\dot{l}_{y,t}/l_{y,t})}{\partial l_{y,t}} = \frac{\phi\varphi}{\phi+\varphi}\left[\frac{1+b}{1-\alpha} - \left(\frac{1+\tau}{\delta}\right)\frac{\partial(e_t/k_t)}{\partial l_{y,t}}\right] = \frac{\phi\varphi}{\phi+\varphi}\left(\frac{1}{1-\alpha}\right)\left[1+b-\frac{\alpha^2}{\frac{\gamma}{1-\gamma}(1-\sigma)\left(\frac{e_t}{k_t}\right)^{-\sigma}+1}\right] > 0.$$

Given that $l_{y,t}$ is a jump variable, a positive eigenvalue implies that the steady-state labor allocation is unique and locally determinate. It also implies that $l_{y,t}$ has to jump to its stationary path immediately after any shock. Accordingly, e_t/k_t jumps to its stationary path according to (29), and $l_{r,t}$ jumps to its stationary path according to (26). Finally, provided that both $l_{y,t}$ and e_t/k_t jump to the steady state, $l_{k,t}$ and $l_{e,t}$ jump to their stationary path accordingly from (27) and (28), respectively. With steady-state labor allocation, the growth rates of innovation g_n and capital g_K are determined by (15) and (19), and accordingly the GDP growth rate g is obtained by (31).

Appendix B: Proof of Lemma 2

Combining (5) with (16) yields

$$\varphi n_t v_{n,t} = (1-\alpha)\frac{y_t}{l_{y,t}}.$$

Taking the derivative with respect to time of the log of the above expression with the fact that $l_{y,t}$ grows at a zero rate along the BGP, we thus obtain

$$\frac{\dot{n}_t}{n_t} + \frac{\dot{v}_{n,t}}{v_{n,t}} = \frac{\dot{y}_t}{y_t} - \frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\dot{c}_t}{c_t}, \tag{B.1}$$

Note that the final-good clearing condition refers to $y_t = c_t$. Next, substituting (15), (13), and (3) into $\dot{n}_t/n_t$, $\dot{v}_{n,t}/v_{n,t}$ and $\dot{c}_t/c_t$, respectively, yields

$$\varphi l_{r,t} + r_t - \frac{\pi_{x,t}}{v_{n,t}} = r_t - \rho$$

Under symmetry across varieties in equilibrium, by using (5), (9), and (16), the above equation can be further rewritten as

$$l_r = \alpha l_y - \frac{\rho}{\varphi},$$

as shown in (26).

Substituting (5) into (20) similarly yields

$$\phi K_t v_{k,t} = (1-\alpha)\frac{y_t}{l_{y,t}}. \tag{B.2}$$

By differentiating the log of the above equation with respect to t, we have

$$\frac{\dot{K}_t}{K_t} + \frac{\dot{v}_{k,t}}{v_{k,t}} = \frac{\dot{y}_t}{y_t} - \frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\dot{c}_t}{c_t}. \tag{B.3}$$

By plugging (19), (17), and (3) into $\dot{K}_t/K_t$, $\dot{v}_{k,t}/v_{k,t}$, and $\dot{c}_t/c_t$, respectively, the relation becomes

$$\phi l_{k,t} = \frac{q_{k,t}}{v_{k,t}} - \rho.$$

Moreover, we combine this equation with (9), (12), and (20) to derive the equilibrium labor devoted to the capital-good production $l_{k,t}$, which is a function of $l_{y,t}$ and e_t/k_t

$$l_k = \frac{1}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}\frac{\alpha^2}{1-\alpha}l_y - \frac{\rho}{\phi},$$

as shown in (27).

With the equilibrium capital production labor $l_{k,t}$ derived above, by combining $\phi l_{k,t} = \frac{q_{k,t}}{v_{k,t}} - \rho$ with (9), (11), and (21), the equilibrium labor devoted to the production of polluting inputs $l_{e,t}$ is

$$l_e = \frac{\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}\frac{\alpha^2}{1-\alpha}\frac{l_y}{1+\tau},$$

which is a function of $l_{y,t}$ and e_t/k_t. Notice that from (5), a constant abatement share in GDP $b = \frac{w_t l_{b,t}}{Y_t}$ implies that $l_b = bl_y/(1-\alpha)$. Accordingly, by substituting the equilibrium conditions for l_r, l_k, l_e, and l_b into the labor-market clearing condition, we can obtain the following equilibrium relation between $l_{y,t}$ and e_t/k_t:

$$l_y = \frac{1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}}{1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\left[\frac{1}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}+\frac{\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}\frac{1}{1+\tau}\right]},$$

as shown in (29).

Finally, with the relation $e/k = \delta l_e$, by substituting the above expression of l_e into (28), we obtain

$$\frac{e}{k} = \delta\frac{\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}{1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}}\frac{\alpha^2}{1-\alpha}\frac{l_y}{1+\tau}, \tag{B.4}$$

which can be used to solve e/k implicitly.

Appendix C: Proof of Lemma 3

In the appendix we prove that e/k is decreasing in τ. From (28), (29), and (B.4), it is easy to derive

$$1 = \frac{\delta\alpha^2(1-\gamma)\left(\frac{e}{k}\right)^{\sigma-1}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)}{(1+b)(1+\tau)\gamma+[(1-\alpha^2+b)(1+\tau)+\alpha^2](1-\gamma)\left(\frac{e}{k}\right)^{\sigma}}. \tag{C.1}$$

By differentiating $\eta \equiv e/k$ with respect to τ from (C.1), we obtain

$$\frac{\partial\eta}{\partial\tau} = \frac{-(1+b)\gamma\left(\frac{e}{k}\right)-(1-\alpha^2+b)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma+1}}{(1-\sigma)(1+b)(1+\tau)\gamma+[(1-\alpha^2+b)(1+\tau)+\alpha^2](1-\gamma)\left(\frac{e}{k}\right)^{\sigma}} < 0. \tag{C.2}$$

The negative relation is true for all values of σ that lie in the range $(-\infty, 1)$.

Appendix D: Proof of Proposition 1

First, we rewrite (21) as $l_e = E/(\delta K) = (e/k)/\delta$. Given Lemma 3, it is straightforward to show that l_e is decreasing in τ.

Second, we reexpress (29) as $l_y = (1+\rho/\varphi+\rho/\phi)/\left[1+\alpha+b/(1-\alpha)+\alpha^2\Phi/(1-\alpha)\right]$, where $\Phi \equiv 1/\left[1+(1-\gamma)(e/k)^{\sigma}/\gamma\right]+[1/(1+\tau)]/\left[1+\gamma(e/k)^{-\sigma}/(1-\gamma)\right]$. Accordingly, $\partial l_y/\partial\tau = -l_y[\alpha^2/(1-\alpha)](\partial\Phi/\partial\tau)/[1+\alpha+b/(1-\alpha)+\alpha^2\Phi/(1-\alpha)]$, where

$$\frac{\partial\Phi}{\partial\tau} = \underbrace{\frac{-\sigma\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{e}{k}\right)^{\sigma-1}\frac{\partial(e/k)}{\partial\tau}}{\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\right]^2}}_{<0} + \underbrace{\frac{-1}{(1+\tau)^2}\left[\frac{1}{1+\frac{\gamma}{1-\gamma}\left(\frac{e}{k}\right)^{-\sigma}}\right]}_{<0} + \underbrace{\frac{-\sigma\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{e}{k}\right)^{-\sigma-1}\frac{\partial(e/k)}{\partial\tau}}{(1+\tau)\left[1+\frac{\gamma}{1-\gamma}\left(\frac{e}{k}\right)^{-\sigma}\right]^2}}_{<0} < 0. \tag{D.1}$$

By applying Lemma 3, we then have that $\partial l_y/\partial\tau > 0$ for $\sigma < 0$.

Third, by using $\partial l_y/\partial\tau > 0$, (26) immediately yields $\partial l_r/\partial\tau > 0$ given $\sigma < 0$.

Finally, we substitute (29) into (27) to obtain

$$l_k = \frac{\frac{\alpha^2}{1-\alpha}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)}{\left(1+\alpha+\frac{b}{1-\alpha}\right)\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\right]+\frac{\alpha^2}{1-\alpha}\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\frac{1}{1+\tau}\right]} - \frac{\rho}{\phi}. \tag{D.2}$$

Taking the derivative of (D.2) with respect to τ yields

$$\frac{\partial l_k}{\partial\tau} = -\frac{\frac{\alpha^2}{1-\alpha}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{e}{k}\right)^{\sigma}\Omega}{\left\{\left(1+\alpha+\frac{b}{1-\alpha}\right)\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\right]+\frac{\alpha^2}{1-\alpha}\left[1+\frac{1-\gamma}{\gamma}\left(\frac{e}{k}\right)^{\sigma}\frac{1}{1+\tau}\right]\right\}^2}, \tag{D.3}$$

where $\Omega \equiv \sigma\left\{1+\alpha+b/(1-\alpha)+\alpha^2/[(1-\alpha)(1+\tau)]\right\}(e/k)^{-1}[\partial(e/k)/\partial\tau]-\alpha^2/[(1-\alpha)(1+\tau)^2]$. It is easy to see that the sign of $\partial l_k/\partial\tau$ depends on the sign of Ω. Given that $\partial(e/k)/\partial\tau$ is negative as shown in Lemma 3, the sign of Ω therefore depends on the value of σ. The following two cases are possible.

(i) For $\sigma \geq 0$, Ω is negative. In this case, it is straightforward to obtain $\partial l_k/\partial\tau > 0$.

(ii) For $\sigma < 0$, we can prove that there exists a sequence of $\sigma < 0$ for which $\Omega > 0$. Specifically, by plugging $\partial(e/k)/\partial\tau$ into Ω, we have

$$\Omega = \sigma\left(1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\frac{1}{1+\tau}\right)\left[\frac{-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma}}{(1-\sigma)(1+b)(1+\tau)\gamma+[(1-\alpha^2+b)(1+\tau)+\alpha^2](1-\gamma)\left(\frac{e}{k}\right)^{\sigma}}\right]$$
$$-\frac{\alpha^2}{1-\alpha}\frac{1}{(1+\tau)^2},$$

implying that Ω is a continuous function of σ. We separate the proof into the following two cases.

(a) $e/k > 1$: As $\sigma \to -\infty$, $\lim\Omega = \frac{1+\alpha+b/(1-\alpha)}{1+\tau}$. Given Ω, we define $A_1 \equiv \lim\Omega = \frac{1+\alpha+b/(1-\alpha)}{1+\tau}$. Thus, for any $\epsilon > 0$, there exists $\sigma^\star < 0$ such that for all $\sigma < \sigma^\star$, $\mid \Omega(\sigma) - A_1 \mid < \epsilon$. By construction, setting $\epsilon = A_1$ yields $\mid \Omega(\sigma) - A_1 \mid < A_1$, which implies that $0 < \Omega(\sigma) < 2A_1$. We thus find a sequence of $\sigma < \sigma^\star$ such that $\Omega(\sigma) > 0$ in this case.

(b) $e/k \leq 1$: We first find that as $\sigma \to -\infty$, $\lim\Omega = \infty$. Thus, we can claim that for any positive constant, say, $A_2 > 0$, there exists $\sigma^{\star\star} < 0$ such that for all $\sigma < \sigma^{\star\star}$, $\Omega(\sigma) > A_2 > 0$. We thus find a sequence of $\sigma < \sigma^{\star\star}$ such that $\Omega(\sigma) > 0$ in this case.

The two cases guarantee that there always exists a sequence of σ that guarantees $\partial l_k/\partial\tau < 0$.

Finally, we identify the threshold value of σ that makes $\Omega = 0$. Setting Ω equal to zero and rearranging it yield a threshold value $\bar{\sigma}$ (which is the smallest real root) solved implicitly by:

$$\sigma = -\frac{\frac{1}{1+\tau}\left(\frac{\alpha^2}{1-\alpha}\right)\left[(1-\sigma)(1+b)\gamma+\left(1-\alpha^2+b+\frac{\alpha^2}{1+\tau}\right)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma}\right]}{\left(1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\frac{1}{1+\tau}\right)\left[(1+b)\gamma+(1-\alpha^2+b)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma}\right]}.$$

It is straightforward to see that $\Omega > 0$ and thus $\partial l_k/\partial\tau < 0$, when $\sigma < \bar{\sigma}$. Thus, by Bolzano's theorem, we can ensure the existence of the threshold $\bar{\sigma}$. See Appendix F for more details.

Appendix E: Proof of Proposition 2

Given Proposition 1, this proposition immediately follows from (15) and (19).

Appendix F: Proof of Proposition 3

To prove the inverted-U relation between the economic growth g and the environmental tax τ, we need to prove that $\eta \equiv e/k$ is a monotonically decreasing function of τ in which $\eta(\tau)$ converges to a positive constant when τ converges to 0.

We first prove that when τ converges to 0, $\eta(\tau)$ converges to a positive constant. The proof proceeds with the following two steps.

Step (a): rearranging (30) yields

$$F(\eta) \equiv (1+b)(1+\tau)\gamma + [(1-\alpha^2+b)(1+\tau)+\alpha^2](1-\gamma)\eta^{\sigma} - \delta\alpha^2(1-\gamma)\eta^{\sigma-1}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right) = 0.$$

By setting $\tau = 0$, the above function can be reduced to

$$F(\eta)|_{\tau=0} = (1+b)\gamma + (1+b)(1-\gamma)\eta^{\sigma} - \delta\alpha^2(1-\gamma)\eta^{\sigma-1}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)^{\sigma} = 0. \tag{F.1}$$

Given $\sigma < 0$, by letting $\eta \to 0$, we can show that $\lim_{\eta\to 0} F(\eta)|_{\tau=0} = -\infty < 0$ by L'Hospital's rule. Moreover, if $\eta \to \infty$, then we find $\lim_{\eta\to\infty} F(\eta)|_{\tau=0} = (1+b)\gamma > 0$. According to Bolzano's theorem, the above two results imply that there exists at least one root of η that solves $F(\eta) = 0$ when $\tau = 0$.

Step (b): rearranging (F.1) yields

$$F(\eta) = \frac{\eta\left[(1+b)\gamma + (1+b)(1-\gamma)\eta^{\sigma}\right]}{\eta^{\sigma}} - \delta\alpha^2(1-\gamma)\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)^{\sigma} = 0.$$

By taking its derivative with respect to η, we further obtain

$$\begin{aligned}F'(\eta) &= \left[(1+b)\gamma\eta^{-\sigma} + (1+b)(1-\gamma)\right] - \sigma\eta^{-\sigma-1}(1+b)\gamma\eta = 0\\ &\Rightarrow (1+b)\gamma\left[1-\sigma\right]\eta^{-\sigma} = -(1+b)(1-\gamma) \Rightarrow \eta^{*} = -\left[\frac{\gamma(1-\sigma)}{1-\gamma}\right]^{\frac{1}{\sigma}} < 0,\end{aligned}$$

given that $\sigma < 0$. In addition, it is easy to verify that $F''(\eta) > 0$. Thus, this negative solution $\eta^{*} < 0$ for $F'(\eta) = 0$ implies that $F(\eta)$ is monotonically increasing in η for all $\eta > 0$.

Steps (a) and (b) guarantee that there is only one positive root η (denoted as $\eta^{\star}$) that solves $F(\eta) = 0$ at $\tau = 0$. This further implies that $\eta^{\star}$ is also the limit of η as $\tau \to 0$.

We are ready to prove the relation between g and τ. Recall from (33) that $\partial g/\partial(1+\tau) = f(\tau)\Gamma$, with

$$f(\tau) \equiv \frac{-\frac{\alpha^3}{1-\alpha}\frac{1-\gamma}{\gamma}\eta^{\sigma}\left(1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}\right)}{\left(\left(1+\alpha+\frac{b}{1-\alpha}\right)\left(1+\frac{1-\gamma}{\gamma}\eta^{\sigma}\right)+\frac{\alpha^2}{1-\alpha}\left[1+\frac{1-\gamma}{\gamma}\eta^{\sigma}\frac{1}{1+\tau}\right]\right)^2(1+\tau)^2},$$

$$\Gamma \equiv (1+\tau)^2\sigma\frac{\eta_{\tau}}{\eta}\Pi - \left[\phi\frac{\alpha^2}{1-\alpha} + (1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}\eta^{\sigma}\right)\right],$$

where $\Pi \equiv \phi\left[1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\frac{1}{(1+\tau)}\right] - \varphi\frac{(1-\alpha)\tau}{1+\tau}$ and $\eta_{\tau} = \frac{-(1+b)\gamma\eta-(1-\alpha^2+b)(1-\gamma)\eta^{\sigma+1}}{(1-\sigma)(1+b)(1+\tau)\gamma+[(1-\alpha^2+b)(1+\tau)+\alpha^2](1-\gamma)\eta^{\sigma}} < 0$. To ensure an inverted U-relation between g and τ, some parameter conditions have to be specified.

To show that an inverted-U relation between g and τ does exist (i.e., $\Gamma < 0$ when τ is small, but $\Gamma > 0$ when τ is large), we first explore the condition for $\Gamma < 0$. Using Lemma 4 (i.e., $\tau \to 0$ $\eta \to \eta^{\star}$) and substituting (C.2) into Γ, we obtain the limit of Γ when $\tau \to 0$, and imposing the condition for it to be less than zero yields

$$\begin{aligned}\lim_{\tau\to 0}\Gamma &\equiv \sigma\frac{\eta_{\tau}\,|_{\eta=\eta^{\star}}}{\eta^{\star}}\Pi - \left[\phi\frac{\alpha^2}{1-\alpha} + (1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}\eta^{\star\sigma}\right)\right] < 0\\ &\Leftrightarrow \sigma\frac{-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\eta^{\star\sigma}}{(1-\sigma)(1+b)\gamma+(1+b)(1-\gamma)\eta^{\star\sigma}}\Pi - \left[\phi\frac{\alpha^2}{1-\alpha} + (1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}\eta^{\star\sigma}\right)\right] < 0.\end{aligned}$$

Rearranging it yields

$$\frac{\varphi}{\phi} > \frac{-\sigma\left[(1+b)\gamma+(1-\alpha^2+b)(1-\gamma)\eta^{\star\sigma}\right]\left[1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\right]}{\left[(1-\sigma)(1+b)\gamma+(1+b)(1-\gamma)\eta^{\star\sigma}\right]\left(1+\frac{1-\gamma}{\gamma}\eta^{\star\sigma}\right)} - \frac{\alpha^2}{1+\frac{1-\gamma}{\gamma}\eta^{\star\sigma}}.$$

This inequality guarantees that $B_1 \equiv \lim_{\tau\to 0}\frac{\partial g}{\partial(1+\tau)} = f(0)\lim_{\tau\to 0}\Gamma > 0$ when $\tau \to 0$.

Second, we explore the condition for $\Gamma > 0$. To prove that there is a negative term $\frac{\partial g}{\partial(1+\tau)}$ as τ goes to a sufficiently large value, we cannot take the limit of $\frac{\partial g}{\partial(1+\tau)}$ given that it will converge to zero. This is because a dominant force from the common factor $\frac{-\frac{\alpha^3}{1-\alpha}\frac{1-\gamma}{\gamma}\eta^{\sigma}\left(1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}\right)}{\left\{\left(1+\alpha+\frac{b}{1-\alpha}\right)\left(1+\frac{1-\gamma}{\gamma}\eta^{\sigma}\right)+\frac{\alpha^2}{1-\alpha}\left[1+\frac{1-\gamma}{\gamma}\eta^{\sigma}\frac{1}{1+\tau}\right]\right\}^2(1+\tau)^2}$ drives the entire $\frac{\partial g}{\partial(1+\tau)}$ to converge faster than Γ, making $\frac{\partial g}{\partial(1+\tau)}$ go to zero directly. Therefore, we first find the limit of Γ and use the sign-preserving property of limits to argue that there is a finite negative value for $\frac{\partial g}{\partial(1+\tau)}$ as τ goes to a sufficiently large value. Hence, taking the limit of Γ when $\tau \to \infty$ yields

$$\lim_{\tau\to\infty}\Gamma = \sigma\frac{(1+\tau)^2\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\eta^{\sigma}\right]\Pi}{(1-\sigma)(1+b)(1+\tau)\gamma+\left[(1-\alpha^2+b)(1+\tau)+\alpha^2\right](1-\gamma)\eta^{\sigma}} - \left[\frac{\phi\alpha^2}{1-\alpha}+(1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}\eta^{\sigma}\right)\right].$$

To address the above problem of finding the limit of Γ, we first consider the case of $z_1\sigma \geq 0$ ($\sigma \leq 0$) where $z\sigma$ is a multiple of σ that denotes the highest exponent of τ in η^{σ}. The problem can then be equivalently transformed to

$$\begin{aligned}
\lim_{\tau\to\infty}\Gamma =& \sigma\frac{\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma}\right](1+\tau)^2}{(1-\sigma)(1+b)\tau\gamma+\left[(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma+1}+\alpha^2(1-\gamma)(\tau)^{z\sigma}\right.}\Pi \\
&-\left[\phi\frac{\alpha^2}{1-\alpha}+(1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}(\tau)^{z\sigma}\right)\right]. \\
=& \left\{\sigma\frac{\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma}\right](1+\tau)^2}{(1-\sigma)(1+b)\tau\gamma+\left[(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma+1}+\alpha^2(1-\gamma)(\tau)^{z\sigma}\right.}\right. \\
&\left.\frac{\phi(1+b)+\left[\phi(1-\alpha^2+b)-\varphi(1-\alpha)^2\right]\tau}{(1-\alpha)(1+\tau)}\right\}-\left[\phi\frac{\alpha^2}{1-\alpha}+(1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}(\tau)^{z\sigma}\right)\right] \\
=& \left\{\sigma\frac{\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma}\right](1+\tau)^2}{(1-\sigma)(1+b)\tau\gamma+\left[(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma+1}+\alpha^2(1-\gamma)(\tau)^{z\sigma}\right.}\right. \\
&\left.\frac{\phi(1+b)+\left[\phi(1-\alpha^2+b)-\varphi(1-\alpha)^2\right]\tau}{(1-\alpha)(1+\tau)}\right\}- \\
&\frac{\left[(1-\sigma)(1+b)\tau\gamma+\left[(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma+1}+\alpha^2(1-\gamma)(\tau)^{z\sigma}\right](1-\alpha)(1+\tau)\right.}{\left[(1-\sigma)(1+b)\tau\gamma+\left[(1-\alpha^2+b)(1-\gamma)(\tau)^{z\sigma+1}+\alpha^2(1-\gamma)(\tau)^{z\sigma}\right](1-\alpha)(1+\tau)\right.} \\
&\left[\phi\frac{\alpha^2}{1-\alpha}+(1-\alpha)\varphi\left(1+\frac{1-\gamma}{\gamma}(\tau)^{z\sigma}\right)\right].
\end{aligned}$$

In this case, the problem can be further reduced to the following by considering only the term with the highest exponent, and the result is given by

$$\lim_{\tau\to\infty}\Gamma=\sigma\frac{\left[\phi(1-\alpha^2+b)-\varphi\,(1-\alpha)^2\right]\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\right](\tau)^{z\sigma+3}}{(1-\alpha)[(1-\alpha^2+b)(1-\gamma)\,(\tau)^{z\sigma+2}}$$
$$-\frac{(1-\alpha)[(1-\alpha^2+b)(1-\gamma)\,(1-\alpha)\,\varphi\frac{1-\gamma}{\gamma}\,(\tau)^{2z\sigma+2}}{(1-\alpha)[(1-\alpha^2+b)(1-\gamma)\,(\tau)^{z\sigma+2}}$$
$$\lim_{\tau\to\infty}\Gamma=\sigma\frac{\left[\phi(1-\alpha^2+b)-\varphi\,(1-\alpha)^2\right]\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\right](\tau)^{z\sigma+3}}{(1-\alpha)[(1-\alpha^2+b)(1-\gamma)\,(\tau)^{z\sigma+2}}=\infty,$$

given that $z\sigma+3>2z\sigma+2\Rightarrow\sigma>1/z$. Notice that the case where $z_1\sigma+3<2z_1\sigma+2$ implies that $\lim_{\tau\to\infty}\Gamma=-\infty$, which fails the condition $\Gamma>0$ for which we are looking.

Second, we consider the case $z\sigma<0$. Since $z\sigma$ is the highest exponent, $-z\sigma$ becomes the lowest exponent in this case. We set $d\equiv-z\sigma>0$ and $\tau_v\equiv\frac{1}{\tau}$. The problem of finding the limit of Γ can be equivalently transformed to the following with the τ_v term only featuring the lowest exponent d in η^σ,

$$\lim_{\tau_v\to0}\Gamma=\sigma\frac{\left[-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\,(\tau_v)^d\right]\left(1+\frac{1}{\tau_v}\right)^2}{(1-\sigma)(1+b)\left(1+\frac{1}{\tau_v}\right)\gamma+[(1-\alpha^2+b)\left(1+\frac{1}{\tau_v}\right)+\alpha^2](1-\gamma)\,(\tau_v)^d}\Pi(\tau_v)$$
$$-\left[\phi\frac{\alpha^2}{1-\alpha}+(1-\alpha)\,\varphi\left(1+\frac{1-\gamma}{\gamma}\,(\tau_v)^d\right)\right].$$
$$=\sigma\frac{\left(1+\frac{1}{\tau_v}\right)^2\left(-(1+b)\gamma-(1-\alpha^2+b)(1-\gamma)\,(\tau_v)^d\right)}{(1-\sigma)(1+b)\left(1+\frac{1}{\tau_v}\right)\gamma+[(1-\alpha^2+b)\left(1+\frac{1}{\tau_v}\right)+\alpha^2](1-\gamma)\,(\tau_v)^d}$$
$$\frac{\phi(1+b)+\left[\phi(1-\alpha^2+b)-\varphi\,(1-\alpha)^2\right]\tau_v^{-1}}{(1-\alpha)(1+\frac{1}{\tau_v})}-$$
$$\frac{\left[(1-\sigma)(1+b)\left(1+\frac{1}{\tau_v}\right)\gamma+[(1-\alpha^2+b)\left(1+\frac{1}{\tau_v}\right)+\alpha^2](1-\gamma)\,(\tau_v)^d\right](1-\alpha)(1+\frac{1}{\tau_v})}{\left[(1-\sigma)(1+b)\left(1+\frac{1}{\tau_v}\right)\gamma+[(1-\alpha^2+b)\left(1+\frac{1}{\tau_v}\right)+\alpha^2](1-\gamma)\,(\tau_v)^d\right](1-\alpha)(1+\frac{1}{\tau_v})}$$
$$\left[\phi\frac{\alpha^2}{1-\alpha}+(1-\alpha)\,\varphi\left(1+\frac{1-\gamma}{\gamma}\,(\tau_v)^d\right)\right].$$

The problem can be further reduced to the following by considering only the term with the lowest exponent, and the result is given by

$$\lim_{\tau_v\to0}\Gamma=\sigma\frac{-(1+b)\gamma\left[\phi(1-\alpha^2+b)-\varphi\,(1-\alpha)^2\right]\tau_v^{-3}}{(1-\sigma)(1+b)\gamma(1-\alpha)\tau_v^{-2}}-\frac{(1-\sigma)(1+b)\gamma(1-\alpha)\tau_v^{-2}\left(\phi\frac{\alpha^2}{1-\alpha}+(1-\alpha)\,\varphi\right)}{(1-\sigma)(1+b)\gamma(1-\alpha)\tau_v^{-2}}$$
$$\Rightarrow\lim_{\tau_v\to0}\Gamma=\sigma\frac{-(1+b)\gamma\left[\phi(1-\alpha^2+b)-\varphi\,(1-\alpha)^2\right]\tau_v^{-3}}{(1-\sigma)(1+b)\gamma(1-\alpha)\tau_v^{-2}}=\infty.$$

Therefore, we conclude that for $\sigma > 1/z$, $\lim_{\tau_v \to 0} \Gamma = \infty$.

By using the sign-preserving property of limits, we can always claim that $\lim_{\tau_v \to 0^+} \Gamma = \infty$ implies that there exists $\epsilon > 0$ such that when $0 < \tau_v - 0 < \epsilon$, $\Gamma(\tau_v) > 0$ (i.e., $\Gamma(1/\tau) > 0$). By plugging τ into the common factor $f(\tau)$, we find $f(\tau) = \frac{-\frac{\alpha^3}{1-\alpha}\frac{1-\gamma}{\gamma}\eta(\tau)^\sigma\left(1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}\right)}{\left\{\left(1+\alpha+\frac{b}{1-\alpha}\right)\left(1+\frac{1-\gamma}{\gamma}\eta(\tau)^\sigma\right)+\frac{\alpha^2}{1-\alpha}\left[1+\frac{1-\gamma}{\gamma}\eta(\tau)^\sigma\frac{1}{1+\tau}\right]\right\}^2(1+\tau)^2} > 0$. We then infer that $B_2 \equiv \frac{\partial g}{\partial(1+\tau)} = f(\tau)\Gamma(1/\tau) < 0$.

Finally, by applying Bolzano's theorem again, $B_1 B_2 < 0$ implies that there is at least one positive root $\tau^\star$ that solves the equation $\frac{\partial g}{\partial(1+\tau)} = 0$, which can be rewritten as

$$\frac{\varphi}{\phi} = \overline{\left(\frac{\varphi}{\phi}\right)} \equiv \frac{(1+\tau^\star)^2\,\sigma\frac{\eta_\tau}{\eta}\left[1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\frac{1}{(1+\tau^\star)}\right]-\frac{\alpha^2}{1-\alpha}}{(1+\tau^\star)^2\,\sigma\frac{\eta_\tau}{\eta}\frac{(1-\alpha)\tau^\star}{1+\tau^\star}+(1-\alpha)\left(1+\frac{1-\gamma}{\gamma}\eta^\sigma\right)},$$

where η implicitly solves

$$\eta = \frac{\delta\alpha^2(1-\gamma)\eta^\sigma\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)}{(1+b)(1+\tau^\star)\gamma+[(1-\alpha^2+b)(1+\tau^\star)+\alpha^2](1-\gamma)\eta^\sigma}.$$

In a neighborhood of the root $\tau^\star$ (i.e., there is $\epsilon > 0$ such that $\mid \tau - \tau^\star \mid < \epsilon$) forms an inverted-U relation in $\frac{\partial g}{\partial(1+\tau)}$. In other words, for $\tau^\star > \tau_l > \tau^\star - \epsilon$, $\frac{\partial g}{\partial(1+\tau)} > 0$ if and only if $\frac{\varphi}{\phi} > \overline{\left(\frac{\varphi}{\phi}\right)}(\tau_l)$ and for $\tau^\star < \tau_u < \tau^\star + \epsilon$, $\frac{\partial g}{\partial(1+\tau)} < 0$ if and only if $\frac{\varphi}{\phi} < \overline{\left(\frac{\varphi}{\phi}\right)}(\tau_u)$.

Appendix G: Flexible Labor Supply

In this appendix we consider an extension of the baseline model in which labor is elastically supplied and verify the robustness of the double-dividend. To the end, we modify the instantaneous utility function at time t as follows

$$u_t = \ln c_t + \theta \ln l_t - \psi \ln s_t,$$

where l_t is the level of leisure, and θ is the intensity of the leisure preference relative to consumption. Accordingly, the household's budget constraint is given by: $\dot{a}_t = r_t a_t + w_t(1 - l_t) - c_t + \mu_t$. Thus, we can easily obtain the leisure-consumption tradeoff, which is satisfied with

$$l_t = \frac{\theta c_t}{w_t} = \frac{\theta y_t}{w_t} = \left(\frac{\theta}{1-\alpha}\right) l_{y,t}, \tag{G.1}$$

where the second equality comes from the resource constraint, and the third one comes from (5). Along BGP, using (3), (11), (17), (20), and the labor-market clearing condition yields

$$1 = l_y + \frac{\theta}{1-\alpha}l_y + \frac{b}{1-\alpha}l_y + \alpha l_y - \frac{\rho}{\varphi} + \frac{\gamma k_j^\sigma}{\gamma k_j^\sigma + (1-\gamma)e_j^\sigma}\frac{\alpha^2}{1-\alpha}l_y - \frac{\rho}{\phi} + \frac{(1-\gamma)e_j^\sigma}{\gamma k_j^\sigma + (1-\gamma)e_j^\sigma}\frac{\alpha^2}{1-\alpha}\frac{l_y}{1+\tau}.$$

Rearranging it yields the equilibrium production labor l_y such that

$$l_y = \frac{1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}}{1+\alpha+\frac{b+\theta}{1-\alpha}+\frac{\alpha^2}{1-\alpha}\left[\frac{\gamma k_j^\sigma}{\gamma k_j^\sigma+(1-\gamma)e_j^\sigma}+\frac{(1-\gamma)e_j^\sigma}{\gamma k_j^\sigma+(1-\gamma)e_j^\sigma}\frac{1}{1+\tau}\right]}.$$

Its form is analogous to (29) with an additional constant term $(b+\theta)/(1-\alpha)$, containing the leisure parameter θ showing up in the denominator.

Following the same logic in the main text, we can solve e/k implicitly by plugging it into (30), and taking the derivative of $\eta = e/k$ with respect to τ yields

$$\frac{\partial\eta}{\partial\tau} = \frac{-(1+b+\theta)\gamma\eta-(1-\alpha^2+b+\theta)(1-\gamma)\eta^{\sigma+1}}{(1-\sigma)(1+b+\theta)(1+\tau)\gamma+[(1-\alpha^2+b+\theta)(1+\tau)+\alpha^2](1-\gamma)\eta^\sigma} < 0.$$

Lemma 3 thus holds. Furthermore, the relation between (29) and other equilibrium labor allocations holds, similarly as in the main text. Thus, it is straightforward to show that the effects of τ on all the other equilibrium labor allocations follow those in the main text (i.e., Proposition 1); namely, an increase in τ raises l_y (i.e., $\partial l_y/\partial(1+\tau)>0$), l_r (i.e., $\partial l_r/\partial(1+\tau)>0$), and l_b (i.e., $\partial l_y/\partial(1+\tau)>0$) and reduces l_e (i.e., $\partial l_e/\partial(1+\tau)<0$). In addition, l_k could be either increasing or decreasing in τ. The new feature here is that a higher τ increases leisure l (i.e., $\partial l/\partial(1+\tau)>0$) given (G.1). Qualitatively, the growth effect of τ thus continues to follow the implication of Proposition 3.

Finally, imposing the BGP equilibrium yields the steady-state welfare function

$$W = \frac{1}{\rho}\left\{\underbrace{(1-\alpha)\ln l_y+\frac{\alpha}{\sigma}\ln\left[\gamma+(1-\gamma)\delta l_e^\sigma\right]}_{\text{consumption effect}}+\underbrace{\frac{1}{\rho}\left[(1-\alpha)\varphi l_r+\alpha\phi l_k\right]}_{\text{growth effect}}+\underbrace{\psi\left[\ln(l_e)^{-1}+\ln l_b\right]}_{\text{pollution effect}}+\underbrace{\theta\ln l_y}_{\text{leisure effect}}\right\},$$

where the exogenous terms have been dropped. This equation shows that the welfare effects of τ on consumption, growth, and pollution follow the same sign as in the main text. There is an additional positive effect on leisure that improves welfare, which implies that the double dividends of the pollution tax can be robust in the generalized model with flexible labor supply.

Appendix H: Unit Tax for Environmental Policy

In this appendix we show that an equivalence between the ad valorem tax and the unit tax for environmental policy exists in terms of their effects on the equilibrium labor allocation under an appropriate choice of numeraire. Accordingly, the growth and welfare implications are also similar.

Denote the price of final goods as p_y, which is normalized to unity in the baseline model. Thus, for the final-good sector, the first-order conditions of the profit maximization problem are modified as

$$w_t = (1-\alpha)\frac{p_{y,t}y_t}{l_{y,t}} \quad \text{and} \quad x_t(j) = l_{y,t}\left[\frac{\alpha p_{y,t}}{p_t(j)}\right]^{\frac{1}{1-\alpha}}. \tag{H.1}$$

As for the intermediate-good sector, the intermediate-good firm j maximizes the profit $\pi_{x,t}(j) = p_t(j)x_t(j) - q_{k,t}k_t(j) - (q_{e,t} + \tau_{e,t})e_t(j)$ subject to (7) and (49). Notice that the environmental tax $\tau_{e,t}$ now enters the profit-maximization problem when environmental taxation is in the form of the unit tax, instead of the ad valorem tax. The first-order conditions of the maximization allow us to obtain the relation: $\frac{\gamma k_t(j)^{\sigma-1}}{(1-\gamma)e_t(j)^{\sigma-1}} = \frac{q_{k,t}}{q_{e,t}+\tau_{e,t}}$.

As for the R&D sector, the settings of the capital-producing sector, pollutant-producing sector, labor market, asset market, final-good market, and the abatement technology are all identical to those of the baseline model. Moreover, for the households, the standard usual Euler equation $\dot{E}_{c,t}/E_{c,t} = r_t - \rho$ also holds.

As for the government, the budget constraint balances such that $T_{y,t} + \tau_{e,t}E_t = w_t l_{b,t}$, where abatement spending remains as a constant share b of GDP such that $w_t l_{b,t}/(p_{y,t}Y_t) = b$, which, with (H.1), implies the relation between $l_{b,t}$ and $l_{y,t}$ such that

$$l_b = \left(\frac{b}{1-\alpha}\right) l_y. \tag{H.2}$$

The next step is to determine the equilibrium labor allocation. By using the same logic as in the baseline model, we derive the relation between $l_{y,t}$ and $l_{r,t}$ such that

$$\varphi l_r = \alpha\varphi l_y - \rho, \tag{H.3}$$

the relation between $l_{y,t}$ and $l_{k,t}$ such that

$$l_k = \frac{\alpha^2}{1-\alpha}\left[\frac{\gamma k^\sigma}{\gamma k^\sigma + (1-\gamma)e^\sigma}\right] l_y - \frac{\rho}{\phi}, \tag{H.4}$$

and the relation between $l_{y,t}$ and $l_{e,t}$ such that

$$l_e = \frac{(1-\gamma)e^\sigma}{\gamma k^\sigma + (1-\gamma)e^\sigma}\left(\frac{\alpha^2}{1-\alpha}\right)\left(\frac{q_e}{q_e+\tau_e}\right) l_y. \tag{H.5}$$

With these relations, we solve for l_y as a function of e/k. Plugging (H.2), (H.3), (H.4), and (H.5) into the labor-market clearing condition yields

$$l_y = \frac{1 + \frac{\rho}{\phi} + \frac{\rho}{\varphi}}{(1+\alpha) + \frac{b}{1-\alpha} + \frac{\alpha^2}{1-\alpha}\left[\frac{\gamma k^\sigma}{\gamma k^\sigma+(1-\gamma)e^\sigma} + \frac{(1-\gamma)e^\sigma}{\gamma k^\sigma+(1-\gamma)e^\sigma}\left(\frac{q_e}{q_e+\tau_e}\right)\right]}. \tag{H.6}$$

As in the baseline model, we derive an implicit function for the equilibrium level of $\eta \equiv e/k$. A few steps of manipulation yield

$$1 = \frac{\delta\alpha^2(1-\gamma)\eta^{\sigma-1}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right)}{(1+b)\left(\frac{q_e+\tau_e}{q_e}\right)\gamma + \left[(1-\alpha^2+b)\left(\frac{q_e+\tau_e}{q_e}\right) + \alpha^2\right](1-\gamma)\eta^\sigma}.$$

It can be seen that this equation with a unit tax is equivalent to the equation with an ad valorem tax shown in (C.1) in the proof of Lemma 3, once q_e is normalized to unity (namely, the price of polluting inputs is chosen as the numeraire).

In this case, it is straightforward to prove that e/k is decreasing in τ_e, because if q_e is set to be unity, then

$$\frac{\partial(e/k)}{\partial\tau_e} = \frac{-(1+b)\gamma\left(\frac{e}{k}\right) - (1-\alpha^2+b)(1-\gamma)\left(\frac{e}{k}\right)^{\sigma+1}}{(1-\sigma)(1+b)(1+\tau_e)\gamma + [(1-\alpha^2+b)(1+\tau_e)+\alpha^2](1-\gamma)\left(\frac{e}{k}\right)^{\sigma}}. \tag{H.7}$$

This is an identical form of (C.2), such that $\partial(e/k)/\partial\tau_e$ is negative for all values of σ that lie in the range $(-\infty, 1)$.

We now turn to the effects of τ_e on the equilibrium labor allocations and show that they are (qualitatively) equivalent to those in the baseline model.

We focus on $\partial l_y/\partial\tau_e$ first. Normalizing $q_e = 1$ and taking the derivative of (H.6) with respect to $1+\tau_e$ yield

$$\frac{\partial l_y}{\partial(1+\tau_e)} = \frac{-\frac{\alpha^2}{1-\alpha}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)}{\left(1+\alpha+\frac{b}{1-\alpha}+\frac{\alpha^2}{1-\alpha}D_1\right)^2}\left[\frac{\partial D_1}{\partial(1+\tau_e)}\right] > 0,$$

where $D_1 \equiv \left(\frac{1}{1+\frac{1-\gamma}{\gamma}\eta^\sigma} + \frac{\frac{1-\gamma}{\gamma}\eta^\sigma}{1+\frac{1-\gamma}{\gamma}\eta^\sigma}\frac{q_e}{q_e+\tau_e}\right)$, which implies that $\partial D_1/\partial(1+\tau_e) < 0$.

Given the relation between l_y and τ_e, we can analyze $\partial l_k/\partial(1+\tau_e)$. Substituting (H.6) into (H.4) yields $l_k = \frac{\alpha^2}{1-\alpha}\left(\frac{1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}}{D_2}\right) - \frac{\rho}{\phi}$, where $D_2 = \left(1+\alpha+\frac{b}{1-\alpha}\right)\left(1+\frac{1-\gamma}{\gamma}\eta\right) + \frac{\alpha^2}{1-\alpha}\left(1+\frac{1-\gamma}{\gamma}\eta^\sigma\frac{q_e}{q_e+\tau_e}\right)$. Normalizing $q_e = 1$ and taking the derivative with respect to $(1+\tau_e)$ yields

$$\frac{\partial l_k}{\partial(1+\tau_e)} = \frac{\alpha^2}{1-\alpha}\left(1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}\right)\frac{-\eta^\sigma}{D_2^2}\left[\left(1+\alpha+\frac{b}{1-\alpha}\right)\frac{1-\gamma}{\gamma}(\sigma\eta^{-1}\eta_{\tau e}) + \frac{\alpha^2}{1-\alpha}\frac{1-\gamma}{\gamma}\left(\frac{\sigma\eta^{-1}\eta_{\tau e}}{(1+\tau_e)} + \frac{-1}{(1+\tau_e)^2}\right)\right].$$

It follows from the above equation that (a) as $\sigma \to 0$, $\frac{\partial l_k}{\partial(1+\tau_e)} = \frac{\alpha^2}{1-\alpha}\left(1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}\right)\frac{\frac{\alpha^2}{1-\alpha}\frac{1-\gamma}{\gamma}}{D_2^2}\left[\frac{1}{(1+\tau_e)^2}\right] > 0$ and (b) as σ becomes sufficiently negative, then the term

$$-\left[\left(1+\alpha+\frac{b}{1-\alpha}\right)\frac{1-\gamma}{\gamma}\eta^{-1}\underbrace{(\sigma\eta_{\tau e})}_{>0} + \frac{\alpha^2}{1-\alpha}\frac{1-\gamma}{\gamma}\left(\frac{1}{(1+\tau_e)}\eta^{-1}\underbrace{(\sigma\eta_{\tau e})}_{>0} + \frac{-1}{(1+\tau_e)^2}\right)\right]$$

is negative, which implies that $\partial l_k/\partial(1+\tau_e) < 0$. This result is in line with Proposition 2 (ii).

Recalling (21) yields $l_e = \eta/\delta$. Thus, it is obvious that an increase in τ_e decreases l_e through decreasing η; i.e., $\partial l_e/\partial(1+\tau_e) < 0$. In addition, recalling (H.3) yields $l_r = \alpha l_y - \rho/\varphi$, implying that an increase in τ_e raises l_r through raising l_y; i.e., $\partial l_r/\partial(1+\tau_e) > 0$. Finally, recalling (H.2) yields $l_b = bl_y/(1-\alpha)$, implying that an increase in τ_e raises l_b through raising l_y; i.e., $\partial l_b/\partial(1+\tau_e) > 0$.

To sum up, the effects of the unit tax τ_e are equivalent to the effects of the ad valorem tax on the equilibrium labor allocations. Given that the growth and welfare effects of τ_e depend only on the equilibrium labor allocations $\{l_y, l_r, l_k, l_e, l_b\}$, we thus infer that the growth and welfare implications of a tightening of environmental policy are analogous to those in the baseline model.

Appendix I: Empirical Analysis

I.1: Data Source and Description

The main source of the data is the Penn World Table (PWT), version 10.0 (https://www.rug.nl/ggdc/productivity/pwt/pwt-releases/pwt100). PWT 10.0 includes annual data on output, input, productivity, as well as the demographic characteristics and capital stock of 183 countries between 1950 and 2019. The data of carbon tax are obtained from the Carbon Pricing Dashboard of the World Bank (https://carbonpricingdashboard.worldbank.org). The Carbon Pricing Dashboard is a platform that provides data of carbon pricing initiatives worldwide between 1990 and 2022. The carbon tax is measured as the price per ton of carbon dioxide equivalent (tCO2e) in US dollar on April 1, 2022.

We construct the consolidated dataset by merging the PWT data with the data of the values of carbon tax and the R&D expenditure as a percentage of GDP. To avoid any abnormal violability due to the outbreak of the COVID pandemic, the data for 2019 are excluded. Furthermore, among the countries that adopted the carbon tax, Canada and Mexico had implemented the carbon tax in some of their regions at different time periods. Since the carbon tax was not implemented nationwide, these two countries are excluded from the dataset. The final sample includes 178 countries covering annual data for 24 years (and 8 periods for the 3-year averages of the variables for the robustness check) from 1995 to 2018 (see Appendix I.2 for the list of countries). Among those countries, 19 of them had executed a carbon tax between 1995 and 2018. A list of the countries with carbon taxes, together with the year the tax went into effect, is presented in Appendix I.3. The two outcome variables, the growth rates of TFP and capital stock, are constructed using the PWT data in which TFP and capital stock are the values of the country at 2017 constant national prices. The growth rates are obtained by taking the difference between the natural logs of the values in years t and $t-1$ and converting them into percentages (i.e., $[\ln(x_t) - \ln(x_{t-1})] \cdot 100$).

Since the effects of carbon tax on the growth rates of TFP and capital stock often occur with a lag, we apply one-year lags of the values of carbon tax and R&D as the independent variables in our estimations to capture their lagged effects.

We conduct three robustness tests for this analysis. First, according to the Carbon Pricing Dashboard, among the countries that implemented a carbon tax, some of them imposed two different tax rates on different types of fossil fuels. For those countries, the variable *carbon tax* (*avg*) is the average value of the unit carbon taxes among different types of fossil fuels. As a robustness check, we create another two variables *carbon tax* (*high*) and *carbon tax* (*low*), which take the highest and lowest tax rates of the countries that imposed the carbon tax.

Second, we re-run the regressions after transforming all the variables into three-year averages in order to exclude any potential short-run noise from additional random occurrences that might have happened. After taking the three-year average of all variables of interests, the annual data collapse into eight three-year periods (i.e., the first period covers the three-year averages of all variables between 1995 and 1997, and the same transformation is applied to the remaining years). This data transformation reduces the sample to 928 and 1,424 observations for the regressions on TFP and capital stock, respectively.

Third, we re-estimate the main regression models using the Environmental Policy Stringency Index (EPSI) provided by OECD, instead of the carbon tax, as our variable of interest. The data of EPSI are obtained from https://stats.oecd.org/Index.aspx?DataSetCode=EPS. The OECD Environmental Policy Stringency Index is a country-specific indicator of environmental policy stringency. Stringency is measured as the degree to which environmental policies impose an explicit or implicit price on polluting or ecologically detrimental behavior. The index is based on the degree of stringency of 13 environmental policy instruments, primarily related to climate and air pollution, and covers 40 countries from 1990 to 2020. To align with our main regressions, we confine the data to be between 1995 and 2018. Since the EPSI data cover only 40 countries, we also exclude the countries that are indicated with the implementation of carbon pricing initiatives in the Carbon Pricing Dashboard of the World Bank, but are not included in the EPSI data. The regression results for the robustness tests are shown in Appendix I.3.

I.2: List of Countries Involved in the Sample

Table 7: List of all the countries

Albania	Cyprus	Kyrgyz Republic	Sao Tome and Principe
Algeria	Czechia	Lao PDR	Saudi Arabia
Angola	Denmark	Latvia	Senegal
Anguilla	Djibouti	Lebanon	Serbia
Antigua and Barbuda	Dominica	Lesotho	Seychelles
Argentina	Dominican Republic	Liberia	Sierra Leone
Armenia	Ecuador	Lithuania	Singapore
Aruba	Egypt, Arab Rep.	Luxembourg	Slovak Republic
Australia	El Salvador	Macao SAR, China	Slovenia
Austria	Equatorial Guinea	Madagascar	South Africa
Azerbaijan	Estonia	Malawi	Spain
Bahamas, The	Eswatini	Malaysia	Sri Lanka
Bahrain	Ethiopia	Maldives	St. Kitts and Nevis
Bangladesh	Fiji	Mali	St. Lucia
Barbados	Finland	Malta	St. Vincent and the Grenadines
Belarus	France	Mauritania	Sudan
Belgium	Gabon	Mauritius	Suriname
Belize	Gambia, The	Moldova	Sweden
Benin	Georgia	Mongolia	Switzerland
Bermuda	Germany	Montenegro	Syrian Arab Republic
Bhutan	Ghana	Montserrat	Taiwan
Bolivia	Greece	Morocco	Tajikistan
Bosnia and Herzegovina	Grenada	Mozambique	Tanzania
Botswana	Guatemala	Myanmar	Thailand
Brazil	Guinea	Namibia	Togo
British Virgin Islands	Guinea-Bissau	Nepal	Trinidad and Tobago
Brunei Darussalam	Haiti	Netherlands	Tunisia
Bulgaria	Honduras	New Zealand	Turkiye
Burkina Faso	Hong Kong SAR, China	Nicaragua	Turkmenistan
Burundi	Hungary	Niger	Turks and Caicos Islands
Cabo Verde	Iceland	Nigeria	Uganda
Cambodia	India	North Macedonia	Ukraine
Cameroon	Indonesia	Norway	United Arab Emirates
Cayman Islands	Iran, Islamic Rep.	Oman	United Kingdom
Central African Republic	Iraq	Pakistan	United States
Chad	Ireland	Panama	Uruguay
Chile	Israel	Paraguay	Uzbekistan
China	Italy	Peru	Venezuela, RB
Colombia	Jamaica	Philippines	Vietnam
Comoros	Japan	Poland	West Bank and Gaza
Congo, Dem. Rep.	Jordan	Portugal	Yemen, Rep.
Congo, Rep.	Kazakhstan	Qatar	Zambia
Costa Rica	Kenya	Romania	Zimbabwe
Cote d'Ivoire	Korea, Rep.	Russian Federation	
Croatia	Kuwait	Rwanda	

Table 8: List of countries with carbon tax and the years of implementation

Country	Year of implementation of carbon tax
Chile	2017
Colombia	2017
Denmark	1992
Estonia	2000
Finland	1990
France	2014
Iceland	2010
Ireland	2010
Japan	2012
Latvia	2004
Norway	1991
Poland	1990
Portugal	2015
Slovenia	1996
Spain	2014
Sweden	1991
Switzerland	2008
Ukraine	2011
United Kingdom	2013

I.3: Robustness Checks for the Empirical Analysis

As discussed in Appendix I.1, we conduct three robustness tests for our empirical analysis. The first two tests are (a) using different measures of carbon tax and (b) taking the three-year averages of all the variables. The estimation results are presented in Table 9. In the table, columns (1) and (2) show the estimates obtained from using the variable carbon tax (high), columns (3) and (4) show the estimates obtained from using the variable carbon tax (low), and columns (5) and (6) show the estimates obtained from using the three-year averages of all the variables.

For the estimates shown in Table 9, columns (1) to (4) are in general comparable to those in Table 6. Regardless of which measure of carbon tax we use, the carbon tax seems to have a positive impact on the growth of TFP and a negative impact on the growth of capital stock. Though the magnitude of the effects of the carbon tax tends to be slightly larger when we use the highest carbon tax rates, the coefficient estimates are very close to one another.

Table 9: Estimation results from using different measures of variables

	(1)	(2)	(3)	(4)	(5)	(6)
	TFP	*Capital Stock*	*TFP*	*Capital Stock*	*TFP*	*Capital Stock*
L.Carbon Tax (high)	0.347**	-0.323***				
	(0.133)	(0.121)				
L.Carbon Tax (low)			0.328***	-0.296**		
			(0.117)	(0.116)		
L.Carbon Tax (avg)					0.283	-0.211*
					(0.219)	(0.124)
Constant	1.122**	3.504***	1.131**	3.498***	0.834**	3.449***
	(0.544)	(0.260)	(0.544)	(0.261)	(0.322)	(0.208)
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Year/Period FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2668	4094	2668	4094	928	1424
Within R2	0.061	0.046	0.061	0.046	0.075	0.039

Notes: Robust standard errors clustered at the country level are in parentheses. ***, **, and * denote significance at 1%, 5%, and 10%, respectively.

The results in columns (5) and (6), which are obtained from using the variables of the three-year averages, also have the same signs and comparable magnitudes of the estimates in Table 6, columns (3-a) and (3-b). Though the coefficient for *L.Carbon Tax (avg)* is statistically insignificant, it may be due to a smaller sample size after taking the three-year averages of the variables.

The third robustness test is using the Environmental Policy Stringency Index (EPSI) provided by OECD, instead of the carbon tax, as our variable of interest. Particularly, we estimate the following models:

$$TFPGR_{i,t} = \beta_0 + \beta_1 ESPI_{i,t-1} + \zeta_t + \chi_i + u_{i,t},$$
$$CSGR_{i,t} = \gamma_0 + \gamma_1 ESPI_{i,t-1} + \zeta_t + \chi_i + v_{i,t}.$$

The summary statistics of the variables applied in this robustness check can be found below.

Table 10: Summary statistics of variables in the regressions using EPSI

Variable	Obs.	Mean	Std. dev.	Min.	Max.
TFP	2,484	0.664	4.227	-44.954	45.532
Capital Stock	2,484	3.233	2.863	-3.440	24.465
L.EPSI	2,484	0.599	1.098	0.000	4.220

Notes: *TFP* and *Capital Stock* denote the annual growth rate of TFP and capital stock in percent, respectively. *EPSI* is the Environmental Policy Stringency Index (EPSI) provided by OECD.

The estimation results are presented in Table 11. In the table, columns (1) and (2) show the results from using the variable EPSI, columns (3) and (4) show the results from using the three-year averages of all the variables, and columns (5) and (6) show the results from adding the linear time trends for each country using the three-year averages of all the variables.

Table 11: Estimation results from using EPSI

	(1)	(2)	(3)	(4)	(5)	(6)
	TFP	*Capital Stock*	*TFP*	*Capital Stock*	*TFP*	*Capital Stock*
L.EPSI	0.084	-0.755***	0.117	-0.767***	0.22	-1.037***
	(0.188)	(0.185)	(0.183)	(0.195)	(0.514)	(0.259)
Constant	1.221**	3.733***	0.878***	3.683***	1.911***	3.901***
	(0.553)	(0.257)	(0.308)	(0.203)	(0.307)	(0.123)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Linear Time Trend	No	No	No	No	Yes	Yes
Observations	2484	3887	864	1352	864	1352
Within R2	0.055	0.052	0.07	0.049	0.275	0.58

Notes: Robust standard errors clustered at the country level are in parentheses. ***, **, and * denote significance at 1%, 5%, and 10%, respectively.

The estimates shown in Table 11 are in general consistent with the results in Table 6. Though we cannot compare the magnitudes of the coefficients with those in Table 6 since EPSI is a unit-less index, similar to the carbon tax, EPSI imposes a positive effect on the growth of TFP and a negative effect on the growth of capital stock. These results are consistent under different specifications including using annual data in columns (1) and (2), three-year averages in columns (3) and (4), and adding the country-specific linear time trends in (5) and (6). Notice that the coefficient estimate of EPSI is insignificant for TFP. It could be because EPSI involves multiple environmental policy instruments, which may have a mixed impact on TFP. Moreover, the number of observations is smaller when we estimate the models using EPSI since, as discussed in Section I.1, the countries that had implemented the carbon pricing initiatives, but not covered by EPSI, were excluded from the regressions. This may induce a higher error variance in our estimation, and hence the estimates could be less precise.