Optimal R&D Subsidies in a Two-Sector Quality-Ladder Growth Model*

Dilip Dutta†
University of Sydney

Tapas Mishra‡
University of Southampton

Yibai Yang§
University of Nottingham Ningbo China

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Abstract

This study develops a two-sector quality-ladder growth model to analyze the welfare comparison between a policy regime in which R&D subsidies are differentiated across sectors and another policy regime in which R&D subsidies are uniformly implemented. The findings of this study are as follows. First, sector-specific optimal R&D subsidies are decreasing in the markup of firms and are smaller in the sector that has a larger degree of R&D duplication externality. Second, general optimal R&D subsidies are a weighted average of sector-specific optimal R&D subsidies and also depend on the market sizes of the sectors, which is in contrast to the sector-specific policy design. Finally, sector-specific optimal R&D subsidies can be more welfare-enhancing than general optimal R&D subsidies only if R&D investment is subsidized more (less) heavily in the sector that grows fast (slowly) but possesses a larger (smaller) market size. We calibrate the model to the US economy and the numerical investigation confirms our theoretical results on the welfare difference between the two regimes.

JEL classification: H20; O31; O34

Keywords: Economic growth; General R&D subsidies; R&D duplication externality; Social welfare

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†School of Economics, University of Sydney, Sydney, New South Wales, 2006, Australia. *Email* address: dilip.dutta@sydney.edu.au.

‡Southampton Business School, University of Southampton, Southampton, SO17 1BJ, United Kingdom. *Email* address: t.k.mishra@soton.ac.uk.

§GEP China, IAPS and School of Economics, University of Nottingham Ningbo China, Ningbo, Zhejiang, 315100, China. *Email* address: yibai.yang@hotmail.com.

1 Introduction

It is widely known that research and development (R&D) activities for innovations are considered as one major engine of growth in many industrialized economies. Traditional endogenous growth theory shows that positive externalities from R&D tend to be in a dominate position because it is difficult for inventors to fully appropriate the benefits of innovations (e.g., Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Jones and Williams (2000)). This argument is highly consistent with empirical evidence measuring the social and private returns to R&D (e.g., Griliches and Lichtenberg (1984) and Jones and Williams (1998)). Due to the R&D underinvestment problem, government intervention in terms of subsidizing R&D activities becomes obviously plausible, and this topic has been one key realm for policy design in the recent studies of endogenous growth.[1]

The existing literature has examined various forms of (optimal) R&D subsidies by assuming that the process of innovations is virtually identical across industries.[2] In fact, the organization of R&D can be diverse from sector to sector such that sectoral technological progress and the resulting growth exhibit apparently different patterns. For instance, Ngai and Pissarides (2007) propose a multi-sector growth model where an exogenous difference in technological progress across sectors is assumed to explain substantial variances in the rates of sectoral total factor productivity (TFP) growth. This proposal supports the observation of early empirical studies such as Kravis, Heston, and Summers (1983) with across-sectors data and Baumol, Blackman, and Wolff (1985) with industry-level data, respectively. Therefore, applying the Ngai-Pissarides idea implies that our analysis of R&D subsidies should take into account the important differences of technological progress across sectors. Motivated by the above discussion, a natural question in relation to the optimal design of R&D policy arises, as is written by Aghion and Howitt (1998), "Should R&D subsidies be targeted to particular sectors, industries, or firms, or instead should R&D subsidies be provided on a nondiscriminatory basis?" Moreover, if more R&D subsidies should be targeted to a particular sector or a particular industry, how should they be implemented according to the sector/industry characteristics? Consequently, this study attempts to address these questions by comparing *sector-specific* and *general* optimal R&D subsidies in terms of their implications on the policy instrument design and social welfare.[3]

This paper provides a two-sector quality-ladder growth model in which the rate of investment subsidies on R&D serves as a policy variable that is implementable by the government (i.e., policymakers). We characterize optimal R&D subsidization under two policy regimes such that (a) R&D subsidies are differentiated in each sector and (b) R&D subsidies are devised uniformly across sectors. Furthermore, we compare the welfare implications of these two policy regimes and derive the (necessary) conditions under which the potential welfare improvement is realized from targeting R&D subsidies differently across sectors.

[1]See Takalo, Tanayama, and Toivanen (2013) for a survey showing that since 1990s, R&D subsidies have been one of the largest and most frequently-used form of industrial aid in the US and the European Union.

[2]One exception is Segerstrom (2000), who considers the difference between the process of vertical innovations and that of horizontal innovations in order to derive the conditions under which targeted and general R&D subsidies are growth-enhancing or growth-retarding, respectively.

[3]This study uses *general* optimal R&D subsidies instead of *uniform* optimal R&D subsidies to keep the consistency with the terms in Segerstrom (2000).

We summarize the results from our analysis in the growth-theoretic model as follows. First, sector-specific optimal R&D subsidies are decreasing in the markup of firms and are smaller in the sector that has a larger degree of R&D duplication. The markup of firms measures monopolistic distortions and the degree of R&D duplication measures the *fishing-out effect* in sectoral production of innovations, respectively. A higher level of these factors reinforces the negative R&D externalities and mitigates the R&D underinvestment problem in the sector. Hence, R&D subsidies respond to decline by reducing the equilibrium R&D level. Second, general optimal R&D subsidies are a weighted average of sector-specific optimal R&D subsidies and the optimal weight depends on the market size and the degree of R&D duplication of a sector. The main difference in policy design between these regimes is that the market size of a sector does not affect sector-specific optimal R&D subsidies whereas it affects general optimal R&D subsidies. We find that market size does not lead to a change in the labor ratios within and across sector(s) under sector-specific optimal R&D subsidies relative to the first-best counterparts; in contrast, market size affects the within-sector production-R&D labor ratio under general optimal R&D subsidies relative to the first-best counterpart. Thus, the uniform regime has to take into account the effect of market size in addition to that of the sectoral degree of R&D duplication. Finally, we show that sector-specific optimal R&D subsidies do not necessarily generate a higher level of social welfare than general optimal R&D subsidies, and the magnitude of the welfare difference is determined by the comparisons of market size and R&D duplication externalities between the sectors. Specifically, the sector-specific regime can lead to welfare gains by placing more subsidies on the fast-growing sector only if this sector happens to possess a larger market. In other words, this result indicates an important policy recommendation such that, in most cases, setting a uniform rate of R&D subsidy across sectors ought to be more welfare-improving than differentiating the R&D subsidy rates.

One feature of this study is that we modify the quality-ladder model of Grossman and Helpman (1991) by considering semi-endogenous growth as in Segerstrom (1998) and by incorporating two sectors that differ by market size, technological opportunity, and R&D duplication. It is worthwhile highlighting our intention to choose this modification for the analysis of optimal R&D subsidies. On the one hand, based on the empirical evidence documented by Griliches (1990), Klenow (1996) shows that in a two-sector Romer-type growth model, market size and technological opportunity are two crucial industry characteristics that best account for the across-industries differentials in R&D intensity and productivity growth in the US during 1959 and 1989. On the other hand, Kim (2011) presents more recent US data to reveal the fact that employment, consumption expenditure, and R&D investment have grown faster in the service sector than in the manufacturing sector, which is in line with the findings of other related empirical evidence (e.g., Wölfl (2003), Triplett and Bosworth (2004), and Herrendorf, Rogerson, and Valentinyi (2013)). To explain these sectoral differences, Kim (2011) constructs a two-sector model in the Jones (1995) version of semi-endogenous growth by assuming that the degree of R&D duplication in production of ideas (i.e., the process of innovations) varies across sectors. Therefore, our paper attempts to combine the above aspects (i.e., semi-endogenous growth and three types of across-industries differences) in a standard R&D-based growth framework. In addition, the reason for this study to adopt the Grossman-Helpman fashion of Schumpeterian growth is that we take the advantage of one property in this model such that labor is the only element involved in the (steady-state) equilibrium allocations. Given this nature of the model, it is convenient to derive the socially optimal solution, making the welfare comparison between the two optimal R&D regimes analytically tractable.

This paper firstly relates to some previous interesting studies, such as Ekholm and Torstensson (1997) and Segerstrom (2000), which analyze the impacts of targeted and uniform R&D subsidies on production of innovations and economic growth, respectively. In particular, Ekholm and Torstensson (1997) consider a specific-factor model and examine the conditions under which targeted industry R&D subsidies and the uniform counterparts across sectors are able to increase R&D investment and aggregate production of high-tech goods. Because of the relative impact of different R&D activities on productivity and the difficulty in reallocating resources from the non-high-tech sector to various high-tech industries, they find that as compared to uniform R&D subsidies, R&D subsidies targeted to the high-tech sector may be less effective for expanding production in the targeted sector giving rise to potentially negative welfare effects. Additionally, Segerstrom (2000) presents a generalized version of Howitt (1999) model of scale-free growth involving both horizontal and vertical R&D innovations, which provides a complete characterization about the long-run growth effect of R&D subsidies. He concludes that both general and targeted R&D subsidies stimulate (retard) economic growth if subsidies promote the type of innovations that is the stronger (weaker) engine of growth.

The present paper differs from the above papers as follows. (a) The prior two studies focus on R&D subsidies that are targeted to one particular sector or one particular type of innovations without interventions on the others, whereas our analysis allows a sector-specific investigation for financial aids implementation by targeting the two sectors differently. (b) Ekholm and Torstensson (1997) consider a static general equilibrium model to conduct a comparative-statics analysis for the effects of both targeted and uniform R&D subsidies on increasing high-tech production, so their model is lack of explicit dynamics and the process by which firms undertake R&D expenditures. Our analysis hence complements their study by considering a framework of dynamic general equilibrium models and modeling the optimization of firms' R&D decisions in two different industries. (c) Segerstrom (2000) shows the interesting result such that both targeted and uniform R&D subsidies can either stimulate or hinder long-run economic growth according to the differences in the diminishing returns to innovations (i.e., R&D duplication externalities in our context). Nonetheless, in addition to growth effects, welfare effects are also an important criterion for devising and choosing R&D policy regimes. Therefore, the present study fills this gap in the literature. Specifically, one novel contribution of this study is to provide another aspect of explanation for why general R&D subsidies can be a ubiquitous form of industrial aids in practice, even though the structure of innovations across industries is different and all industries are targeted.

This study also contributes to the literature of dynamic general equilibrium models that explores the policy design and welfare effects of optimal research subsidies in a setup of R&D-based growth. For instance, Şener (2008) investigates an endogenous growth model in which scale effects are removed by the rent-protection approach and the diminishing-technological-opportunities approach. He finds that the steady-state rate of innovations and that of economic growth depend on the R&D subsidy/tax. Also, the simulation exercises suggest that under a wide range of empirical calibrations, the optimal R&D subsidy rate should vary between 5% and 25%. Moreover, Grossmann, Steger, and Trimborn (2013) extend the semi-endogenous growth model of Jones (1995) and show that the first-best optimal growth path can be supported in market equilibrium by a combination of constant intermediate-goods subsidies and time-varying R&D subsidies. By characterizing the optimal transitional dynamics, their results indicate that the welfare losses of implementing the long-run optimal R&D subsidies rather than the dynamically optimal counterparts are quantitatively negligible.[4] Our paper adds to this literature by identifying the across-sectors differences in several industry characteristics in a similar framework of (semi-)endogenous growth eliminating scale effects, given that some of these differences are crucial for policymakers to implement the appropriate form of R&D subsidies.

Finally, this study is related to a small but growing empirical literature that estimates the effects of R&D subsidies on innovative performance and social welfare. For example, using Spanish firm-level data, González, Jaumandreu, and Pazo (2005) find that R&D subsidies play an effective role in stimulating investment for R&D projects, and subsidies that are required to induce firms to engage in R&D are very heterogeneous according to the firm size. The closest empirical study to the current paper is Takalo, Tanayama, and Toivanen (2013), who measure the expected welfare effects of targeted R&D subsidies (i.e., an applicant-specific R&D policy scheme) using project-level data from Finland during 2000 and 2002. Their results show that the estimated benefits of this subsidy policy exceed the opportunity cost of public funds leading to welfare improvements. The present paper complements these interesting empirical studies by focusing on the welfare effect of industry- (sector-)level R&D subsidies in a growth-theoretic framework.

This paper is organized as follows. Section 2 introduces the model setup. Section 3 characterizes the decentralized equilibrium. Section 4 derives the optimal rates of sector-specific R&D subsidies and the counterpart of general R&D subsidies in addition to their welfare comparison. Section 5 calibrates this model to the US economy and provides a numerical analysis to quantify the welfare differences between the two optimal R&D policy regimes. Section 6 concludes this study.

2 The Model

In this section, we construct a multi-industry version of the semi-endogenous Schumpeterian growth model, and the underlying quality-ladder feature is based on the seminar work of Grossman and Helpman (1991). We follow Segerstrom (1998) to remove scale effects in this Schumpeterian model by allowing increasing difficulty in innovations. Furthermore, to consider the effects of R&D subsidies, we modify this Schumpeterian model by introducing a lump-sum tax that is imposed on the representative household to finance R&D subsidization as in the previous studies such as Segerstrom (2000), Şener (2008), and Grossmann, Steger, and Trimborn (2013). We also assume that there are two sectors producing different types of final and intermediate goods. To analyze the comparison between sector-specific R&D subsidies and general R&D subsidies, the two sectors in this model are distinct by three industry-specific characteristics, namely, market size, technological opportunity, and R&D duplication. As stated in Introduction, Klenow (1996) uses the first two characteristics to well explain empirical differences in R&D intensity and productivity growth across industries.[5] The last characteristic generates different rates of creative destruction across

[4]Zeng and Zhang (2007) incorporate elastic labor supply into an expanding-variety growth model to consider the growth and welfare effects of optimal subsidies to intermediate goods and to research, respectively. Their results show that neither a single subsidy nor a mix of subsidies is a socially first-best policy regime.

[5]Chu (2011) shows that these two industry characteristics also help explain welfare differences between sector-specific patent protection and uniform patent protection in a quality-ladder growth model, but his model is subject to scale effects.

sectors in our model, corresponding to the third industry feature studied in Klenow (1996), that is, appropriability.

2.1 Households

Suppose that there is an economy admitting a representative household. The population size of the household is N_t, and it grows at the rate of $n > 0$ such that $\dot{N}_t = nN_t$. The lifetime utility function of the household (based on per capita utility) is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

where c_t denotes the per capita consumption at time t, and $\rho > 0$ is the exogenous discount rate. The law of motion for assets per person that is expressed in real terms (i.e., all prices are denominated in units of final goods) such that

$$\dot{v}_t = (r_t - n)v_t + w_t - c_t - \tau_t, \tag{2}$$

where v_t is the value of per capita assets, w_t denotes the wage rate, r_t is the nominal interest rate, and τ_t is the lump-sum (non-distorting) tax imposed by the government for financing R&D subsidization. The household inelastically supplies one unit of labor, and maximizes (1) subject to (2). Then the standard dynamic optimization yields the familiar Euler equation such that

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - n. \tag{3}$$

Moreover, the household owns a balanced portfolio of all firms in the economy.

2.2 Consumption

Following the previous literature of two-sector R&D-based growth models such as Klenow (1996) and Chu (2011), consumption is derived from the aggregation of two types of final goods according to the following Cobb-Douglas aggregator:

$$C_t = (Y_{1,t})^\alpha (Y_{2,t})^{1-\alpha}, \tag{4}$$

where $Y_{i,t}$ denotes the final goods produced in sector $i \in \{1, 2\}$, and $\alpha \in (0, 1)$ determines the market size of sector 1 in the production of final goods. From profit maximization, the conditional demand functions for $Y_{1,t}$ and $Y_{2,t}$ are given by

$$P_{1,t}Y_{1,t} = \alpha C_t, \tag{5}$$

$$P_{2,t}Y_{2,t} = (1 - \alpha)C_t, \tag{6}$$

where $P_{1,t}$ and $P_{2,t}$ represent the prices of $Y_{1,t}$ and $Y_{2,t}$, respectively.

2.3 Final Goods

Final goods for consumption in sector $i \in \{1, 2\}$ are produced competitively using a unit continuum of fully depreciated intermediate goods indexed by variety $j \in [0, 1]$, which follows the standard Cobb-Douglas production function:

$$Y_{i,t} = \exp\left(\int_0^1 \ln X_{i,t}(j) dj\right), \tag{7}$$

where $X_{i,t}(j)$ is the quantity of intermediate goods in variety j. Denote $P_{i,t}(j)$ as the price of $X_{i,t}(j)$ and assume that there is free entry into the final-goods sectors. This assumption together with (7) yields the demand for variety j such that

$$P_{i,t}Y_{i,t} = P_{i,t}(j)X_{i,t}(j), \tag{8}$$

where the price index of final goods is given by $P_{i,t} = \exp\left(\int_0^1 \ln P_{i,t}(j)dj\right)$ due to cost minimization.

2.4 Intermediate Goods

In each variety $j \in [0, 1]$ of sector $i \in \{1, 2\}$, intermediate goods are produced by a monopolistic leader who holds a patent on the latest innovation and are replaced by the products of an entrant who has a new innovation due to the *Arrow replacement effect.* The current leader has the following production function for the intermediate goods:

$$X_{i,t}(j) = z^{q_{i,t}(j)} L_{i,t}(j), \tag{9}$$

where the parameter $z > 1$ measures the step size of each quality improvement, $q_{i,t}(j)$ is the number of innovations in variety j between time 0 and time t, and $L_{i,t}(j)$ is the employment level of production labor in this variety. Given $z^{q_{i,t}(j)}$, (9) implies that the marginal cost of producing intermediate goods for the current leader in variety j is given by

$$MC_{i,t}(j) = w_t / z^{q_{i,t}(j)}. \tag{10}$$

As commonly assumed in the literature, standard Bertrand competition implies that the current leader charges a markup over the marginal cost to maximize profits. Similar to previous studies such as Li (2001), Goh and Olivier (2002), and Iwaisako and Futagami (2013), because of incomplete patent protection, the markup $\mu > 1$ is a policy instrument that is set by patent authority as patent breadth. Given that fiscal authority has no control over patent policy and takes this policy as given in reality, the patent tool μ is considered as exogenous while designing optimal R&D subsidization. Hence, the monopolistic price is given by

$$P_{i,t}(j) = \mu MC_{i,t}(j) = \mu \left(w_t / z^{q_{i,t}(j)}\right), \tag{11}$$

which is the limit price of the current leader against potential imitations.[6] The case with $\mu = z$

[6]As assumed in Howitt (1999) and Segerstrom (2000), once the incumbent transfers the licensing for production to

corresponds to the markup ratio used in the canonical quality-ladder model of Grossman and Helpman (1991). Consequently, the leader's profit in variety j is given by

$$\Pi_{i,t}^{x}(j) = \left(\frac{\mu - 1}{\mu}\right) P_{i,t}(j) X_{i,t}(j) = \left(\frac{\mu - 1}{\mu}\right) P_{i,t} Y_{i,t}, \tag{12}$$

where we substitute (8) into $\Pi_{i,t}^{x}(j)$ to derive the second equality. Finally, using (9)-(12) yields the relation between the wage costs and the output values in variety j as follows:

$$w_t L_{i,t}(j) = \left(\frac{1}{\mu}\right) P_{i,t}(j) X_{i,t}(j) = \left(\frac{1}{\mu}\right) P_{i,t} Y_{i,t}. \tag{13}$$

2.5 R&D and Innovations

Denote the real value of the most recent innovation in variety j of sector i by $v_{i,t}(j)$. Following the standard literature, we focus on a symmetric equilibrium since $\Pi_{i,t}^{x}(j) = \Pi_{i,t}^{x}$ for $j \in [0,1]$ from (12).[7] Then, $v_{i,t}(j) = v_{i,t}$ in this symmetric equilibrium in which the arrival rate of innovation is equal across varieties within a sector. Denote $\lambda_{i,t}$ as the *aggregate* Poisson arrival rate of innovation in sector i. As a result, the Hamilton-Jacobi-Bellman (HJB) equation for $v_{i,t}$ is given by

$$r_t v_{i,t} = \Pi_{i,t}^{x} + \dot{v}_{i,t} - \lambda_{i,t} v_{i,t}, \tag{14}$$

which is the no-arbitrage condition for the asset value. In equilibrium, the return on this asset $r_t v_{i,t}$ equals the sum of the flow payoffs $\Pi_{i,t}^{x}$, the potential capital gain $\dot{v}_{i,t}$, and the expected capital losses $\lambda_{i,t} v_{i,t}$ because of creative destruction.

New innovations in each variety are invented by a unit continuum of R&D firms indexed by $\nu \in [0,1]$. Each of these firms employs R&D labor $H_{i,t}(\nu)$ for producing inventions with the aid of subsidization. The expected profit of the ν-th R&D firm is

$$\Pi_{i,t}^{r}(\nu) = v_{i,t} \lambda_{i,t}(\nu) - (1 - s_{i,t}) w_t H_{i,t}(\nu), \tag{15}$$

where $s_{i,t} \in (0,1)$ is a subsidy rate to research given the evidence in Impullitti (2010) showing that the subsidy rate to R&D investment in many OECD countries is positive. This R&D subsidy is financed by the household's lump-sum tax. Moreover, the *firm-level* arrival rate of innovation is given by

$$\lambda_{i,t}(\nu) = \bar{\varphi}_{i,t} H_{i,t}(\nu), \tag{16}$$

where $\bar{\varphi}_{i,t}$ is R&D productivity. Then, the free entry into the R&D sector implies the zero-expected-profit condition for R&D such that:

$$v_{i,t} \lambda_{i,t}(\nu) = (1 - s_{i,t}) w_t H_{i,t}(\nu). \tag{17}$$

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the entrant, the incumbent leaves the market and cannot threaten to reenter. Therefore, the constrained monopolistic markup of the current leader, which is reflected by the strength of patent breadth, is subject to potential imitations from other competitive fringes rather than competition from previous innovators.

[7] See Cozzi, Giordani, and Zamparelli (2007) for justifying that the symmetric equilibrium in Schumpeterian growth models features uniqueness and rational expectation.

Denote $Z_{i,t}$ and $H_{i,t}$ as the aggregate level of technology and of R&D labor in sector i, respectively. We follow Chu, Cozzi, Lai, and Liao (2015) to assume that $\bar{\varphi}_{i,t} = \varphi_i / \left[(H_{i,t})^{\delta_i} Z_{i,t} \right]$ in order to combine two sources of R&D externality that are commonly analyzed in the literature. First, the arrival rate of innovation is subject to a fishing-out effect capturing increasing innovation complexity; the scale effects are removed by this formulation as in Segerstrom (1998).[8] Second, $\delta_i \in (0,1)$ captures the usual negative externality of R&D duplication within the industry as in Jones and Williams (2000). We assume that the degree of R&D duplication externality varies across sectors because we want to allow $Z_{1,t}$ and $Z_{2,t}$ to grow at different rates, which will be shown below. Additionally, $\varphi_i > 0$ is the parameter of technological opportunity (i.e., productivity), which is also allowed to vary across sectors as in Klenow (1996) and Chu (2011). In equilibrium, the *aggregate-level* arrival rate of innovation equals the firm-level counterpart for each variety, namely, $\lambda_{i,t}(\nu) = \lambda_{i,t}$, and it can be expressed by

$$\lambda_{i,t} = \int_0^1 \lambda_{i,t}(\nu) d\nu = \frac{\varphi_i H_{i,t}^{1-\delta_i}}{Z_{i,t}}. \tag{18}$$

2.6 Aggregation

Using (7) and (9), we derive the production function for sector $i \in \{1, 2\}$ such that $Y_{i,t} = Z_{i,t} L_{i,t}$, where $Z_{i,t}$ is defined as the sectoral (aggregate) technology given by

$$Z_{i,t} = \exp\left(\ln z \int_0^1 q_{i,t}(j) dj \right) = \exp\left(\ln z \int_0^t \lambda_{i,\iota} d\iota \right), \tag{19}$$

where the second equality of (19) is based on the law of large numbers. Differentiating this equation with respect to time yields the growth rate of technology in sector i given by

$$\frac{\dot{Z}_{i,t}}{Z_{i,t}} = \lambda_{i,t} \ln z = \frac{(H_{i,t})^{1-\delta_i}}{Z_{i,t}} \varphi_i \ln z. \tag{20}$$

3 Decentralized Equilibrium

The equilibrium consists of a sequence of allocations $[C_t, Y_{1,t}, Y_{2,t}, X_{1,t}(j), X_{2,t}(j), L_{1,t}, L_{2,t}, H_{1,t}, H_{2,t}]_{t=0, j\in[0,1]}^{\infty}$ and a sequence of prices $[P_{1,t}, P_{2,t}, P_{1,t}(j), P_{2,t}(j), r_t, w_t, v_{1,t}, v_{2,t}, v_t]_{t=0, j\in[0,1]}^{\infty}$. Moreover, in each instant of time,

• the representative household chooses $[c_t]$ to maximize lifetime utility given $[r_t, w_t]$;

• competitive consumption firms produce $[c_t]$ by combining $[Y_{1,t}, Y_{2,t}]$ to maximize profits given $[P_{1,t}, P_{2,t}]$;

• final-goods producers choose $[Y_{1,t}, Y_{2,t}]$ to maximize profits given $[P_{1,t}, P_{2,t}, P_{1,t}(j), P_{2,t}(j)]$;

• monopolistic leaders for intermediate goods produce $[X_{1,t}(j), X_{2,t}(j)]$ and choose $[P_{1,t}(j), P_{2,t}(j), L_{1,t}, L_{2,t}]$ to maximize profits given $[w_t]$;

• R&D firms choose $[H_{1,t}, H_{2,t}]$ to maximize profits given $[w_t, v_{1,t}, v_{2,t}]$;

[8] Venturini (2012) shows that the assumption of increasing R&D difficulty in R&D-driven growth models is best supported by empirical evidence for the US manufacturing industries during 1975–1996.

- the labor market clears such that $L_{1,t} + L_{2,t} + H_{1,t} + H_{2,t} = N_t$;
- the values of innovations add up to the household's assets value such that $v_{1,t} + v_{2,t} = v_t N_t$.

3.1 Equilibrium Allocations

In this subsection, we first show that the economy is always on a uniquely stable balanced growth path (BGP). Then, we derive the equilibrium labor allocations for a stationary path of R&D subsidies $\{s_{1,t}, s_{2,t}\}_{t=0}^{\infty}$.

Lemma 1. *Holding constant R&D subsidies $\{s_1, s_2\}$, the aggregate economy must jump to a unique and stable balanced growth path.*

Proof. See the Appendix. □

On this BGP, the arrival rate of innovation in each sector $\{\lambda_{i,t}\}$ for $i \in \{1, 2\}$ is stationary in the long run, so according to (20) technologies $\{Z_{1,t}, Z_{2,t}\}$ grow at a constant rate. This analysis implies that the long-run growth rate of the sectoral technology is given by

$$g_i \equiv \frac{\dot{Z}_{i,t}}{Z_{i,t}} = \lambda_{i,t} \ln z = (1 - \delta_i)n, \tag{21}$$

where the steady-state equilibrium value of λ_i is determined by the population growth rate n and the degree of the sectoral R&D duplication externality δ_i. Given the sectoral production function $Y_{i,t} = Z_{i,t} L_{i,t}$, we derive the growth rate of outputs in sector i given by

$$\frac{\dot{Y}_{i,t}}{Y_{i,t}} = \frac{\dot{Z}_{i,t}}{Z_{i,t}} + \frac{\dot{L}_{i,t}}{L_{i,t}} = g_i + n = (2 - \delta_i)n. \tag{22}$$

Differentiating the log of (4) with respect to time yields the growth rate of aggregate consumption such that $\frac{\dot{C}_t}{C_t} = \alpha \frac{\dot{Y}_{1,t}}{Y_{1,t}} + (1-\alpha)\frac{\dot{Y}_{2,t}}{Y_{2,t}}$, and combining (22) implies $\frac{\dot{C}_t}{C_t} \equiv g_C = [2 - (\alpha\delta_1 + (1-\alpha)\delta_2)]\, n$. In addition, the growth rate of per capita consumption is given by $\frac{\dot{c}_t}{c_t} \equiv g_c = [1 - (\alpha\delta_1 + (1-\alpha)\delta_2)]\, n$.

Next, we derive the steady-state equilibrium labor allocations in this economy. Denote $l_{i,t} \equiv L_{i,t}/N_t$ and $h_{i,t} \equiv H_{i,t}/N_t$ for $i \in \{1, 2\}$ as per capita production labor and per capita R&D labor in each sector, respectively. Consequently, we obtain the following result.

Lemma 2. *Given constant R&D subsidies $\{s_1, s_2\}$, the equilibrium labor allocations are stationary and given by*

$$l_1 = \frac{\alpha}{1 + (\mu - 1)\left[\frac{\alpha}{1-s_1}\frac{\lambda_1}{\rho+\lambda_1} + \frac{1-\alpha}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}\right]}, \tag{23}$$

$$l_2 = \frac{1-\alpha}{1 + (\mu - 1)\left[\frac{\alpha}{1-s_1}\frac{\lambda_1}{\rho+\lambda_1} + \frac{1-\alpha}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}\right]}, \tag{24}$$

$$h_1 = \frac{(\mu - 1)\frac{\alpha}{1-s_1}\frac{\lambda_1}{\rho+\lambda_1}}{1 + (\mu - 1)\left[\frac{\alpha}{1-s_1}\frac{\lambda_1}{\rho+\lambda_1} + \frac{1-\alpha}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}\right]}, \tag{25}$$

$$h_2 = \frac{(\mu-1)\frac{1-\alpha}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}}{1+(\mu-1)\left[\frac{\alpha}{1-s_1}\frac{\lambda_1}{\rho+\lambda_1}+\frac{1-\alpha}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}\right]}, \tag{26}$$

where $\lambda_i = (1-\delta_i)n/lnz$ *for* $i \in \{1,2\}$.

Proof. See the Appendix. □

The intuition of the steady-state equilibrium labor allocations is straightforward as follows. (i) A larger α increases both l_1 and h_1, since a larger market size of final good 1 induces the economy to assign more labor to both production and R&D in sector 1. (ii) A larger μ decreases l_1 and l_2 but increases h_1 and h_2; intuitively, a larger patent breadth reallocates labor from production to R&D within each sector. (iii) A larger s_1 increases h_1 but decreases l_1, because a larger R&D subsidy encourages more R&D incentives leading to a reallocation of labor from production to R&D within sector 1. However, both l_2 and h_2 decrease in this case, so it is obvious that the sum of l_1 and h_1 is increasing in s_1.[9] In other words, more labor is devoted to the sector with a higher level of R&D subsidy.[10] (iv) Similar to the effect of s_1 on the labor allocations, a larger λ_1 increases h_1 but decreases l_1, l_2, and h_2. Interestingly, this result implies that a higher arrival rate of innovation (due to a lower degree of R&D duplication given that $\lambda_1 = (1-\delta_1)n/\ln z$) not only reallocates labor from production to R&D within the sector but also from the labor in the other sector.[11] This effect implies that more (less) labors are assigned to the sector with a strong (weak) engine of growth. Finally, since this is a semi-endogenous growth model as in Jones (1995), λ_i of sector i is independent of the sectoral technological opportunity φ_i, which does not enter the steady-state equilibrium labor allocations. Moreover, a permanent increase in R&D subsidies s_1 and s_2 does not change the long-run growth rate of sectoral technology $\dot{Z}_{i,t}/Z_{i,t}$ for $i \in \{1,2\}$ and that of per capita consumption g_c.[12]

4 Optimal R&D Subsidies and Social Welfare

In this section, we first derive the first-best labor allocations. Second, we derive *sector-specific* optimal R&D subsidies that maximize welfare by targeting the R&D subsidy rate to each sector. Third, we derive *general* optimal R&D subsidies that maximize welfare by setting a uniform rate of R&D subsidy across sectors. Then, the first-best allocations are compared to the equilibrium

[9] More precisely, $\frac{\partial(l_1+h_1)}{\partial s_1} = (\mu-1)\frac{\alpha\lambda_1}{\rho+\lambda_1}\left[1-\alpha+\frac{\mu-1}{1-s_2}\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}\right] > 0$.

[10] In a modified Romer (1990) model where innovations and capital accumulation are both engines of long-run economic growth, Chen, Chu, and Lai (2015) show that an R&D subsidy reallocates labor to R&D from production of capital and of final goods. Nevertheless, when the relative productivity between the R&D and the capital-producing sectors is sufficiently large (small), R&D subsidies become growth-enhancing (growth-retarding).

[11] More precisely, $\frac{\partial(l_1+h_1)}{\partial \lambda_1} = (\mu-1)(1-\alpha)\frac{\alpha\rho}{\rho+\lambda_1}\left[\frac{1}{1-s_1}+\frac{1}{1-s_1}\frac{\mu-1}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}\right] > 0$.

[12] Nevertheless, a permanent increase in s_i increases l_i for $i \in \{1,2\}$ as implied by (25) and (26). Given that λ_i is constant in the dynamics analysis, (20) implies that $Z_{i,t}/(N_t)^{1-\delta_i}$ must rise in the short run; $Z_{i,t}$ grows at a higher rate as compared to its long-run rate (i.e., $\dot{Z}_{i,t}/Z_{i,t} > (1-\delta_i)n$). Therefore, a permanent increase in the R&D subsidy rate causes a temporary increase in the growth rate of sectoral technology but a permanent increase in the level of sectoral technology. These effects also hold for the growth rate and the level of per capita consumption. See Kim (2011) for a similar result and discussions on growth-increasing policies through permanently changing exogenous parameters in a semi-endogenous growth model.

allocations under the two R&D subsidization schemes. Finally, we analyze the welfare differences between the optimal R&D policy regimes.

In this economy, the long-run welfare of the representative household is given by its lifetime utility in (1) along the BGP such that

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c}{\rho}\right). \tag{27}$$

Substituting (4), the condition $c_0 = C_0/N_0$, and the growth rate of per capita consumption $g_c = [1-(\alpha\delta_1+(1-\alpha)\delta_2)]\, n$ into (27) yields

$$\rho U = \ln C_0 = \alpha \ln Y_{1,0} + (1-\alpha)\ln Y_{2,0}, \tag{28}$$

where all the exogenous terms have been dropped. Recall the sectoral production function at time 0 such that $Y_{i,0} = Z_{i,0} l_{i,0} N_{i,0}$, where the level of sectoral technology along the BGP is given by $Z_{i,0} = \frac{(N_{i,0})^{1-\delta_i}\varphi_i \ln z}{(1-\delta_i)n}(h_i)^{1-\delta_i}$, which is a function of the R&D labor in sector i. Substituting these two conditions into (28) yields

$$\rho U = \alpha\left[\ln l_1 + (1-\delta_1)\ln h_1\right] + (1-\alpha)\left[\ln l_2 + (1-\delta_2)\ln h_2\right], \tag{29}$$

where all the exogenous terms have again been dropped. In (29), each labor allocation of $\{l_1, l_2, h_1, h_2\}$ depends on both s_1 and s_2.

4.1 First-Best Allocations

To derive the first-best labor allocations along the BGP, the social planner chooses a time path of $\{l_{1,t}, l_{2,t}, h_{1,t}, h_{2,t}\}_{t=0}^{\infty}$ that maximizes the representative household's welfare in (29). Therefore, we obtain the following result.

Lemma 3. *The optimal path of first-best labor allocations $\{\hat{l}_1, \hat{l}_2, \hat{h}_1, \hat{h}_2\}_{t=0}^{\infty}$ in this economy is stationary and given by*

$$\hat{l}_1 = \left[1 + \frac{1-\delta_1}{1+\frac{\rho/n}{1-\delta_1}} + \frac{1-\alpha}{\alpha}\left(1 + \frac{1-\delta_2}{1+\frac{\rho/n}{1-\delta_2}}\right)\right]^{-1}, \tag{30}$$

$$\hat{l}_2 = \frac{1-\alpha}{\alpha}\hat{l}_1, \tag{31}$$

$$\hat{h}_1 = \left[\frac{1-\delta_1}{1+\frac{\rho/n}{1-\delta_1}}\right]\hat{l}_1, \tag{32}$$

$$\hat{h}_2 = \frac{1-\alpha}{\alpha}\left[\frac{1-\delta_2}{1+\frac{\rho/n}{1-\delta_2}}\right]\hat{l}_1. \tag{33}$$

Proof. See the Appendix. □

We will compare (30)-(33) to the steady-state equilibrium labor allocations under sector-specific optimal R&D subsidies in subsection 4.2 and those under general optimal R&D subsidies in subsection 4.3, respectively, so as to analyze the comparative statics of the optimal rates of R&D subsidies under these policy regimes.

4.2 Sector-Specific Optimal R&D Subsidies

To consider the policy regime for sector-specific optimal R&D subsidies denoted by s_1^* and s_2^*, we substitute (23)-(26) into (29) and differentiate ρU with respect to s_1 and s_2, respectively. Combining the two first-order conditions derives the optimal rates of R&D subsidy by which the government targets to different sectors to maximize social welfare. Accordingly, we obtain the following result.

Proposition 1. *The optimal rates of sector-specific R&D subsidies that maximize the welfare of the representative household are given by*

$$s_1^* = 1 - \frac{\mu - 1}{1 - \delta_1} \frac{\lambda_1}{\rho + \lambda_1}, \tag{34}$$

$$s_2^* = 1 - \frac{\mu - 1}{1 - \delta_2} \frac{\lambda_2}{\rho + \lambda_2}. \tag{35}$$

Proof. See the Appendix. □

Substituting (34)-(35) into (23)-(26) yields the equilibrium labor allocations under this policy regime given by

$$l_1^* = \frac{\alpha}{1 + [\alpha(1 - \delta_1) + (1 - \alpha)(1 - \delta_2)]}, \tag{36}$$

$$l_2^* = \frac{1 - \alpha}{1 + [\alpha(1 - \delta_1) + (1 - \alpha)(1 - \delta_2)]}, \tag{37}$$

$$h_1^* = \frac{\alpha(1 - \delta_1)}{1 + [\alpha(1 - \delta_1) + (1 - \alpha)(1 - \delta_2)]}, \tag{38}$$

$$h_2^* = \frac{(1 - \alpha)(1 - \delta_2)}{1 + [\alpha(1 - \delta_1) + (1 - \alpha)(1 - \delta_2)]}. \tag{39}$$

The comparative statics of sector-specific optimal R&D subsidies reveals that the optimal R&D subsidy rate in sector i is increasing in the quality step size z but decreasing in the sectoral degree of R&D duplication δ_i and the size of patent breadth (i.e., markup) μ. In addition, it is independent of the market size (i.e., α for sector 1 and $1 - \alpha$ for sector 2).

The intuition of the results in the first three parameters are straightforward and standard as in the previous literature (e.g., Chu (2011) and Chu, Cozzi, Lai, and Liao (2015)). The first-best ratio of R&D to production labor and the equilibrium counterpart in sector 1 are expressed by $\hat{h}_1/\hat{l}_1 = (1 - \delta_1)g_1/(\rho + g_1)$ and $h_1/l_1 = (\mu - 1)g_1/[(1 - s_1)(\rho \ln z + g_1)]$, respectively, where $g_1 = (1 - \delta_1)n$ and we use the fact that $\lambda_1 = g_1/\ln z$. A larger z decreases h_1/l_1 relative to $\hat{h}_1/\hat{l}_1$, which reflects a worsening of the (positive) surplus-appropriability externality. Thus, s_1^* increases to stimulate the equilibrium level of R&D as response. A larger δ_1 and μ both increase h_1/l_1 relative to $\hat{h}_1/\hat{l}_1$, which captures a strengthening of the (negative) R&D duplication externality and a strengthening of the (negative) business-stealing effect, respectively. Thus, s_1^* decreases to depress the equilibrium level of R&D as response. An analogous reasoning can be applied for explaining the comparative statics of s_2^*.

The above analysis indicates that the market size does not affect the equilibrium ratio of R&D and production labor within a sector, since Lemma 2 reveals that an increase in the market size increases production labor and R&D labor proportionately in the same sector. Furthermore, the market size does not affect labor (re)allocations across sectors either. On the one hand, it is important to notice that comparing (22)-(23) and (31) shows that the ratio of production labor across sectors in equilibrium is socially optimal, i.e., $l_1/l_2 = \hat{l}_1/\hat{l}_2 = \alpha/(1-\alpha)$. On the other hand, combining (25) and (26) yields the across-sector R&D labor ratio in equilibrium such that

$$\frac{h_1}{h_2} = \frac{\alpha}{1-\alpha}\frac{1-s_2}{1-s_1}\frac{g_1/(\rho\ln z + g_1)}{g_2/(\rho\ln z + g_2)}, \tag{40}$$

whereas combining (32) and (33) yields the first-best ratio of R&D labor across sectors given by

$$\frac{\hat{h}_1}{\hat{h}_2} = \frac{\alpha}{1-\alpha}\frac{1-\delta_1}{1-\delta_2}\frac{g_1/(\rho + g_1)}{g_2/(\rho + g_2)}. \tag{41}$$

Comparing (40) and (41) implies that the optimal R&D subsidy rates serve to partially reduce the difference between h_1/h_2 and $\hat{h}_1/\hat{h}_2$; nevertheless, they only operate through the channels of z, δ_1, and δ_2 but not through α. Consequently, this reasoning explains the independence of s_1^* and s_2^* upon the market size. This result also implies that when $\delta_1 = \delta_2 = \delta$, we obtain $s_1^* = s_2^* = s^* = 1 - \frac{\mu-1}{1-\delta}\frac{\lambda}{\rho+\lambda}$, where $\lambda = (1-\delta)n/\ln z$. Intuitively, given that the market size does not play a role in reallocating labors under this policy regime, the between-sector differences in labor allocations no longer exist in the presence of an equal sectoral degree of R&D duplication externality; thus the optimal design for R&D subsidies becomes sector-invariant.

4.3 General Optimal R&D Subsidies

To consider the policy regime for general optimal R&D subsidies denoted by $\bar{s}$, in this subsection the condition $s_1 = s_2 = s$ is set. Then, as before we substitute (23)-(26) into (29) and differentiate ρU with respect to s. This will derive the optimal rate of R&D subsidy by which the government uses uniformly across the two sectors in order to maximize social welfare. Hence, the following result is obtained.

Proposition 2. *The optimal rate of general R&D subsidies that maximizes the welfare of the representative household is given by*

$$\begin{aligned}\bar{s} &= \frac{\alpha(1-\delta_1)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}s_1^* + \frac{(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}s_2^* \\ &= 1 - \frac{(\mu-1)\left(\frac{\alpha\lambda_1}{\rho+\lambda_1} + \frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}\right)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}.\end{aligned} \tag{42}$$

Proof. See the Appendix. □

Substituting (42) into (23)-(26) yields the equilibrium labor allocations under this policy regime given by

$$\bar{l}_1 = \frac{\alpha}{1 + [\alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)]}, \tag{43}$$

$$\bar{l}_2 = \frac{1-\alpha}{1 + [\alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)]}, \tag{44}$$

$$\bar{h}_1 = \frac{\frac{\frac{\alpha\lambda_1}{\rho+\lambda_1}}{\frac{\alpha\lambda_1}{\rho+\lambda_1} + \frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}} [\alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)]}{1 + [\alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)]}, \tag{45}$$

$$\bar{h}_2 = \frac{\frac{\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}}{\frac{\alpha\lambda_1}{\rho+\lambda_1} + \frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}} [\alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)]}{1 + [\alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)]}. \tag{46}$$

Proposition 2 shows that the optimal rate of general R&D subsidies is a weighted average of the optimal rates of sector-specific R&D subsidies, and the optimal weights are $\frac{\alpha(1-\delta_1)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}$ and $\frac{(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}$, respectively. The intuition of this result is simple. In the welfare function (29) that takes into account the equilibrium labor allocations in (23)-(26), the only differences between the sectors stem from the market size and the degree of R&D duplication externality. Therefore, when the rate of R&D subsidy is constrained to be uniform across sectors, a balanced weight must be put based on these differences. Additionally, this proposition confirms the fact that if $\delta_1 = \delta_2 = \delta$, then $\bar{s} = s_1^* = s_2^* = s^*$ implying that there is no difference between sector-specific optimal R&D subsidies and general optimal R&D subsidies in the case of an equal degree of R&D duplication externality across sectors.

The impacts of z, μ, and δ_i for $i \in \{1, 2\}$ on the optimal subsidy rate $\bar{s}$ are equivalent to those on s_1^* and s_2^* and can be explained similarly. However, in contrast to the previous policy regime, the market size now has an effect on $\bar{s}$ depending on the relative magnitude of sectoral R&D duplication externalities such that $\bar{s}$ is increasing (decreasing) in α if $\delta_1 < (>)\delta_2$.[13] In other words, a larger market size in the sector that is more (less) growth-enhancing increases (decreases) the optimal rate of general R&D subsidies.

To gain a better understanding of the comparative statics of α in $\bar{s}$, we rewrite the welfare function in (29) by multiplying $1-s$ on both the denominators and the numerators in (23)-(26) and substituting them into (29), which is given by

$$\rho\tilde{U} = \ln(1-s) - [1 + \alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)] \ln \left[1 - s + (\mu - 1)\left(\frac{\alpha\lambda_1}{\rho+\lambda_1} + \frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}\right)\right], \tag{47}$$

where $\tilde{U}$ is used because we have dropped all exogenous terms. Inspecting (47) shows the following result. On the one hand, an increase in the R&D subsidy rate decreases the production labor in both sectors, which causes a negative welfare effect captured by the term $\ln(1-s)$. On the other hand, an increase in the R&D subsidy rate increases the R&D labor in both sectors, which causes a positive welfare effect captured by the term $-[1 + \alpha(1-\delta_1) + (1-\alpha)(1-\delta_2)] \ln \left[1 - s + (\mu-1)\left(\frac{\alpha\lambda_1}{\rho+\lambda_1} + \frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}\right)\right]$.

[13] Specifically, $\partial\bar{s}/\partial\alpha = \frac{\mu-1}{[\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)]^2}\left[\frac{\lambda_2(1-\delta_1)}{\rho+\lambda_2} - \frac{\lambda_1(1-\delta_2)}{\rho+\lambda_1}\right]$, which is positive (negative) if $\delta_1 < (>)\delta_2$.

In this case, the optimal rate of R&D subsidy $\bar{s}$ simply balances the welfare gains and losses in (47). It is worthwhile noticing that when the long-run growth rate in sector 1 is higher than that in sector 2 due to $\delta_1 < \delta_2$, the positive welfare effect is strengthened by a larger market size in sector 1 since $\partial\rho\tilde{U}/\partial\alpha > 0$. Hence, the optimal subsidy rate $\bar{s}$ is induced to rise for reinforcing the negative welfare effect, which will mitigate the above influence of α. A similar reasoning also explains the comparative statics of α in the case of $\delta_1 > \delta_2$.

4.4 Welfare Difference between the R&D Subsidy Regimes

This section analytically compares the welfare difference between sector-specific optimal R&D subsidies and general R&D optimal subsidies. We substitute (36)-(39) into (29) to compute the discounted level of social welfare under sector-specific R&D subsidies denoted by $\rho U(s_1^*, s_2^*)$, and substitute (43)-(46) into (29) to compute the counterpart under general R&D subsidies denoted by $\rho U(\bar{s}_1, \bar{s}_2)$, respectively. Accordingly, the welfare difference between these two policy regimes is denoted by $\rho\Delta U \equiv \rho U(s_1^*, s_2^*) - \rho U(\bar{s}_1, \bar{s}_2)$.

It is shown that both R&D subsidy regimes achieve the same level of consumption production because their allocations of production labor are identical and the sectoral ratio is efficient (i.e., $l_1^*/l_2^* = \bar{l}_1/\bar{l}_2 = \hat{l}_1/\hat{l}_2 = \alpha/(1-\alpha)$). Moreover, given that general optimal R&D subsidies depend on the market sizes of the sectors but sector-specific R&D subsidies do not, this disparity in the optimal policy design, along with the relative magnitude of R&D duplication, creates a wedge on the R&D labor allocations between the regimes. Therefore, the welfare difference only stems from the discrepancy of the R&D labor allocations, which affect the underlying levels of sectoral technology under the regimes. Then, we obtain the following result.

Proposition 3. *Sector-specific optimal R&D subsidies can generate a higher level of welfare than general optimal R&D subsidies only if the degree of R&D duplication externality is smaller in the sector that has a larger market size, namely,* $\delta_1 < (>)\delta_2$ *when* $\alpha > (<)1/2$.

Proof. See the Appendix. □

Using the equilibrium labor allocations under the optimal regimes, the welfare difference can be expressed as $\rho\Delta U = \alpha(1-\delta_1)\ln(h_1^*/\bar{h}_1) + (1-\alpha)(1-\delta_2)\ln(h_2^*/\bar{h}_2)$. Intuitively, given that the levels of production labor are the same under both regimes (i.e., $l_i^* = \bar{l}_i$ for $i \in \{1, 2\}$), more (less) labor that is allocated to R&D in sector 1 under sector-specific R&D subsidies than under general R&D subsidies enlarges (shrinks) the welfare difference because the technology level in sector 1 under the former regime rises (declines). However, this case also implies that less (more) labor is assigned to R&D in sector 2 under sector-specific R&D subsidies, which lowers (raises) the technology level in sector 2 under this regime and shrinks (enlarges) the welfare difference. Whether sector-specific or general R&D subsidies are more welfare-improving depends on the relative impact of the sectoral ratios of R&D labor on the welfare difference in addition to their weights of significance, namely $\alpha(1-\delta_1)$ and $(1-\alpha)(1-\delta_2)$.

When sector 1 exhibits a lower degree of R&D duplication externality than sector 2 (i.e., $\delta_1 < \delta_2$), the R&D labor in sector 1 relative to sector 2 under sector-specific R&D subsidies is higher than the counterpart under general R&D subsidies, namely $(h_1^*/h_2^*)/(\bar{h}_1/\bar{h}_2) = (\rho+\lambda_1)/(\rho+\lambda_2) > 1$. In other words, the negative effect of R&D duplication externality in sector 1 is less severe under sector-specific R&D subsidies as compared to under general R&D subsidies. Given a larger market size in sector 1 ($\alpha > 1/2$), the former regime internalizes this across-sectors difference in the R&D externality effect and responds by setting a larger R&D subsidy rate in sector 1 than in sector 2 (i.e., $s_1^* > s_2^*$) so that $h_1^*/h_2^* = (\alpha/(1-\alpha))((1-\delta_1)/(1-\delta_2)) > 1$. Nevertheless, the latter regime neglects the effect of the across-sectors difference in δ, so a uniform subsidy rate leads to a lower sectoral ratio of R&D labors (i.e., $\bar{h}_1/\bar{h}_2 = (\alpha/(1-\alpha))((1-\delta_1)/(1-\delta_2))((\rho+\lambda_2)/(\rho+\lambda_1)) < h_1^*/h_2^*$). Furthermore, due to equal allocations of production labor between the regimes, it must be the case that $h_1^*/\bar{h}_1 > 1 > h_2^*/\bar{h}_2$ to ensure $\rho\Delta U > 0$. The above analysis implies that relative to general R&D subsidies, the welfare gain brought by more R&D labor allocated in sector 1 under sector-specific R&D subsidies dominates the welfare loss brought by less R&D labor allocated in sector 2, so that sector-specific R&D subsidies are optimal. A similar reasoning can be applied to explain the positive welfare difference for the situation when $\delta_1 > \delta_2$ and $\alpha < 1/2$. This result also implies an interesting insight: if the welfare effect of reallocating R&D labors is sufficiently strong, then R&D investment in a sector that grows relatively fast and possesses a larger market size at the same time could be subsidized more heavily to generate welfare benefits.

Contrarily, for $\delta_1 < \delta_2$ and $\alpha < 1/2$, the sector-specific R&D regime may still increase the subsidy rate in sector 1 more than in sector 2 leading to $h_1^*/h_2^* = (\alpha/(1-\alpha))((1-\delta_1)/(1-\delta_2)) > 1$. Nevertheless, the fact that $(h_1^*/h_2^*)/(\bar{h}_1/\bar{h}_2) > 1$ continues to hold. Suppose that a uniform subsidy rate raises the sectoral ratio of R&D labor to a level that is not significantly lower than h_1^*/h_2^*, then the welfare loss due to less labor assigned in sector 2 under sector-specific R&D subsidies outweighs the welfare gain due to more labor assigned in sector 1. Accordingly, general R&D subsidies becomes optimal in this case.

Finally, it is worthwhile noting that Proposition 3 provides only the necessary condition for sector-specific R&D subsidies to be optimal. If the welfare effect of labor allocations is not very strong, then general optimal R&D subsidies could be more welfare-enhancing under the same condition (i.e., $\delta_1 < (>)\delta_2$ when $\alpha > (<)1/2$). Consequently, given that the welfare difference between the above policy regimes is ambiguous depending on the parameter values, in the next section where this model is calibrated to the US economy, we start off with the decentralized equilibrium to quantify the respective welfare gains by implementing these R&D subsidy regimes.

5 Conclusion

In this paper, we present a two-sector quality-ladder growth model to compare the welfare effect of two different R&D policy regimes, namely, sector-specific optimal R&D subsidies and general optimal R&D subsidies. The two sectors in our study are differentiated in terms of three industry characteristics: market size, technological opportunity, and R&D duplication externality. Under the former regime R&D subsidies are implemented differently across sectors, whereas under the latter regime R&D subsidies are implemented uniformly. As for the optimal design of R&D subsidies, it is found that sector-specific optimal R&D subsidies are decreasing in the markup of firms and are smaller (larger) in the sector with more (less) R&D duplication externalities, namely in the slow- (fast-)growing sector. However, general optimal R&D subsidies are a weighted average of sector-specific optimal R&D subsidies. In contrast to the independence of the sector-specific policy design on the market sizes of the sectors, general optimal R&D subsidies decrease (increase) when the market size of the slow- (fast-)growing sector becomes larger. Finally, sector-specific R&D subsidies can be more welfare-enhancing than general R&D subsidies only if the R&D subsidies are set to be larger (smaller) in the fast- (slow-) growing sector that possesses a larger (smaller) market size. In other words, this result implies an important policy implication such that, in most cases, setting a uniform rate of R&D subsidy across sectors tends to yield a higher level of social welfare than differentiating the R&D subsidy rates.

Appendix

Proof for Lemma 1

Define a transformed variable $\Phi_{1,t} \equiv P_{1,t}Y_{1,t}/v_{1,t}$. Differentiating $\Phi_{1,t}$ with respect to time yields

$$\frac{\dot{\Phi}_{1,t}}{\Phi_{1,t}} \equiv \frac{\dot{P}_{1,t}}{P_{1,t}} + \frac{\dot{Y}_{1,t}}{Y_{1,t}} - \frac{\dot{v}_{1,t}}{v_{1,t}} = \frac{\dot{c}_t}{c_t} + n - \frac{\dot{v}_{1,t}}{v_{1,t}}, \tag{A.1}$$

where the second equality is obtained by using (5). Substituting (12) into the no-arbitrage condition for $v_{1,t}$ (14) yields

$$\frac{\dot{v}_{1,t}}{v_{1,t}} = r_t - \left(\frac{\mu-1}{\mu}\right)\Phi_{1,t} + \lambda_{1,t}. \tag{A.2}$$

Combining (A.2) and the Euler equation for c_t in (3) yields a differential equation for $\Phi_{1,t}$ given by

$$\frac{\dot{\Phi}_{1,t}}{\Phi_{1,t}} = \left(\frac{\mu-1}{\mu}\right)\Phi_{1,t} - \lambda_{1,t} - \rho. \tag{A.3}$$

Inspecting the labor-market-clearing condition $L_{1,t}/N_t + L_{2,t}/N_t + H_{1,t}/N_t + H_{2,t}/N_t = 1$ implies that the production labor and R&D labor $L_{i,t}$ and $H_{i,t}$ for $i \in \{1, 2\}$ grow equiproportionately at the rate of population growth n; otherwise the labor-market-clearing condition will be violated such that the per capita employment level of labor will converge to infinity or zero. Then, differentiating (18) with respect to time yields a differential equation for $\lambda_{1,t}$ given by

$$\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} = (1-\delta_1)n - \lambda_{1,t}\ln z, \tag{A.4}$$

where we use the fact that $H_{1,t}$ grows at the rate of n. Therefore, we obtain a dynamic system that consists of (A.3) and (A.4) in terms of $\Phi_{1,t}$ and λ_t.

Linearizing (A.3) and (A.4) around the steady-state equilibrium yields

$$\begin{bmatrix} \dot{\Phi}_{1,t} \\ \dot{\lambda}_{1,t} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Phi_{1,t} - \Phi_1 \\ \lambda_{1,t} - \lambda_1 \end{bmatrix}, \tag{A.5}$$

where $a_{11} = \left(\frac{\mu-1}{\mu}\right)\Phi_1$, $a_{12} = -\Phi_1$, $a_{21} = 0$, and $a_{22} = -\lambda_1 \ln z$. Denote the Jacobian matrix in (A.5) as $\mathcal{J}$. Then the determinant of the Jacobian matrix equals the multiplication of the two characteristic roots of the differential equation system (A.5), such that $\text{Det}\mathcal{J} = a_{11}a_{22} - a_{12}a_{21} = -\left(\frac{\mu-1}{\mu}\right)\Phi_1\lambda_1\ln z < 0$. This result implies that the two characteristic roots have opposite signs. Given that $\Phi_{1,t}$ and $\lambda_{1,t}$ are both jump variables, as depicted in Figure 1, this dynamic system is characterized by global instability such that $\Phi_{1,t}$ and $\lambda_{1,t}$ must jump to their steady-state equilibrium values given by Φ_1 and λ_1. An analogous proof would show that $\Phi_{2,t}$ and $\lambda_{2,t}$ must also jump to their steady-state equilibrium values. Hence, the economy is always on a unique and stable balanced growth path along which each variable grows at a constant (possibly zero) rate. The above findings continue to hold given that R&D subsidies s_1 and s_2 are constant.

Proof for Lemma 2

Setting $\dot{\Phi}_{1,t} = 0$ and $\dot{\lambda}_{1,t} = 0$ in (A.3) and (A.4) yields their steady-state values such that $\Phi_1 = \frac{\mu}{\mu-1}(\lambda_1 + \rho)$ and $\lambda_1 = (1-\delta_1)n/\ln z$, both of which are stationary. Hence, we make use of (3) and (A.1) to derive that $\dot{v}_{1,t}/v_{1,t} \equiv g_v = g_c + n = r_t - \rho$. Combining g_v and the no-arbitrage condition for R&D in (14) implies $v_{1,t} = \Pi^x_{1,t}/(\rho + \lambda_1)$. Given a stationary path of R&D subsidies $\{s_1, s_2\}_{t=0}^{\infty}$, substituting this equation into the R&D free-entry condition in (17) yields the ratio of R&D and production labors in sector 1 given by

$$\frac{h_{1,t}}{l_{1,t}} = \frac{\mu-1}{1-s_1}\frac{\lambda_1}{\rho+\lambda_1}, \tag{A.6}$$

which is stationary. The ratio of R&D and production labors in sector 2 is analogously derived and given by

$$\frac{h_{2,t}}{l_{2,t}} = \frac{\mu-1}{1-s_2}\frac{\lambda_2}{\rho+\lambda_2}. \tag{A.7}$$

Furthermore, using (5), (6), and (13) yields the ratio of production labor across sectors such that

$$\frac{l_{1,t}}{l_{2,t}} = \frac{\alpha}{1-\alpha}. \tag{A.8}$$

Finally, solving (A.6)-(A.8) with the labor-market-clearing condition $l_{1,t}+l_{2,t}+h_{1,t}+h_{2,t} = 1$ yields the steady-state equilibrium labor allocations in (23)-(26).

Proof for Lemma 3

The social planner maximizes (1) subject to the constraints of consumption production (i.e., (4) and $Y_{i,t} = Z_{i,t}L_{i,t}$ for $i \in \{1,2\}$), the law of motion for sectoral technologies in (20) (i.e., $\dot{Z}_{i,t} = \varphi_i \ln z \,(H_{i,t})^{1-\delta_i}$ for $i \in \{1,2\}$), and the labor-market-clearing condition (i.e., $L_{1,t} + L_{2,t} + H_{1,t} + H_{2,t} = N_t$), which yields the following current-value Hamiltonian

$$\begin{aligned}\Theta_t =& \ln\frac{(Z_{1,t}L_{1,t})^{\alpha}(Z_{2,t}L_{2,t})^{1-\alpha}}{N_t} + \theta_{1,t}\left(\varphi_1 \ln z H_{1,t}^{1-\delta_1}\right) + \theta_{2,t}\left(\varphi_2 \ln z H_{2,t}^{1-\delta_2}\right) \\ &+ \theta_{3,t}\left(N_t - L_{1,t} - L_{2,t} - H_{1,t} - H_{2,t}\right),\end{aligned} \tag{A.9}$$

where $\theta_{1,t}$, $\theta_{2,t}$, and $\theta_{3,t}$ are the co-state variables for the constraints. Therefore, the FOCs for $L_{i,t}$, $H_{i,t}$, and $Z_{i,t}$ are given by

$$\frac{\partial\Theta_t}{\partial L_{1,t}} = \frac{\alpha}{L_{1,t}} - \theta_{3,t} = 0, \tag{A.10}$$

$$\frac{\partial \Theta_t}{\partial L_{2,t}} = \frac{1-\alpha}{L_{2,t}} - \theta_{3,t} = 0, \tag{A.11}$$

$$\frac{\partial \Theta_t}{\partial H_{1,t}} = \theta_{1,t}\varphi_1 \ln z (1-\delta_1) H_{1,t}^{-\delta_1} - \theta_{3,t} = 0, \tag{A.12}$$

$$\frac{\partial \Theta_t}{\partial H_{2,t}} = \theta_{2,t}\varphi_2 \ln z (1-\delta_2) H_{2,t}^{-\delta_2} - \theta_{3,t} = 0, \tag{A.13}$$

$$\frac{\partial \Theta_t}{\partial Z_{1,t}} = \frac{\alpha}{Z_{1,t}} = \rho\theta_{1,t} - \dot{\theta}_{1,t}, \tag{A.14}$$

$$\frac{\partial \Theta_t}{\partial Z_{2,t}} = \frac{1-\alpha}{Z_{2,t}} = \rho\theta_{2,t} - \dot{\theta}_{2,t}. \tag{A.15}$$

Using (A.14) and the fact that $\dot{Z}_{1,t} = (1-\delta_1)nZ_{1,t}$ along the BGP, we obtain a differential equation such that $\dot{\theta}_{1,t}Z_{1,t} + \theta_{1,t}\dot{Z}_{1,t} = [\rho + (1-\delta_1)n]\theta_{1,t}Z_{1,t} - \alpha$. Setting this equation to zero yields $\theta_{1,t}Z_{1,t} = \alpha/[\rho + (1-\delta_1)n]$. Similarly, we derive $\theta_{2,t}Z_{2,t} = (1-\alpha)/[\rho + (1-\delta_2)n]$ from (A.15). Thus, combining these results along with (A.10)-(A.13) yields

$$\frac{H_{1,t}}{L_{1,t}} = \frac{1-\delta_1}{1 + \frac{\rho/n}{1-\delta_1}}, \tag{A.16}$$

$$\frac{H_{2,t}}{L_{2,t}} = \frac{1-\delta_2}{1 + \frac{\rho/n}{1-\delta_2}}, \tag{A.17}$$

where we use the definition that $\lambda_{i,t} = \varphi_i \left(H_{i,t}\right)^{1-\delta_i}/Z_{i,t}$ and the balanced-growth level of λ_i such that $\lambda_i = (1-\delta_i)n/\ln z$ for $i \in \{1,2\}$. Finally, solving (A.10)-(A.11), (A.16)-(A.17), and the labor-market-clearing condition yields the equilibrium allocations given by (30)-(33).

Proof for Proposition 1

Under the policy regime of sector-specific optimal R&D subsidies, the government chooses s_1 and s_2 separately to maximize the welfare of the representative household. We then substitute (23)-(26) into ρU in (29) and differentiate it with respect to s_1 and s_2, respectively, to obtain the optimal rates of R&D subsidy. This yields the following two first-order conditions such that

$$s_1 = 1 - \frac{[1 + (1-\alpha)(1-\delta_2)]\,(\mu - 1)\frac{\alpha\lambda_1}{\rho+\lambda_1}}{\alpha(1-\delta_1)\left[1 + (\mu-1)\frac{1}{1-s_2}\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}\right]}, \tag{A.18}$$

$$s_2 = 1 - \frac{[1 + \alpha(1-\delta_1)]\,(\mu - 1)\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}}{(1-\alpha)(1-\delta_2)\left[1 + (\mu-1)\frac{1}{1-s_1}\frac{\alpha\lambda_1}{\rho+\lambda_1}\right]}. \tag{A.19}$$

Combining (A.18) and (A.19) yields a set of roots given by $\{s_1 = 1, s_2 = 1\}$ and another set of roots given by (34) and (35).[14] However, the former set of roots $\{s_1 = 1, s_2 = 1\}$ violates the definition for the range of R&D subsidies, which is abandoned. Therefore, the latter set of roots is selected

[14]See the complementary *Mathematica* files for the derivation in this proof, which are available upon request.

as the solution for sector-specific optimal R&D subsidies.

Proof for Proposition 2

Under the policy regime of general optimal R&D subsidies, the government chooses the same level of s in both sectors to maximize the welfare of the representative household. We apply the condition $s_1 = s_2 = s$ in (23)-(26) and substitute them into ρU in (29). Differentiating this equation with respect to s, we obtain the optimal subsidy rate $\bar{s}$ given by (42). Moreover, substituting s_1^* and s_2^* given by (34)-(35) into (42) verifies that the optimal rate of general R&D subsidies is a weighted average of the optimal rates of sector-specific R&D subsidies, where the optimal weights are $\frac{\alpha(1-\delta_1)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}$ and $\frac{(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}$, respectively.

Proof for Proposition 3

Substituting (36)-(39) and (43)-(46) into (29) computes $\rho U(s_1^*, s_2^*)$ and $\rho U(\bar{s}_1, \bar{s}_2)$, respectively. Comparing these welfare levels yields the difference given by

$$\begin{aligned}\rho\Delta U = {} & \alpha(1-\delta_1)\underbrace{\left[\ln\frac{\frac{\alpha\lambda_1}{\rho+\lambda_1}+\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}}{\frac{\alpha\lambda_1}{\rho+\lambda_1}} - \ln\frac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)}\right]}_{M_1} \\ & + (1-\alpha)(1-\delta_2)\underbrace{\left[\ln\frac{\frac{\alpha\lambda_1}{\rho+\lambda_1}+\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}}{\frac{(1-\alpha)\lambda_2}{\rho+\lambda_2}} - \ln\frac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{(1-\alpha)(1-\delta_2)}\right]}_{M_2},\end{aligned} \tag{A.20}$$

where we use the fact that $l_i^* = \bar{l}_i$ for $i \in \{1, 2\}$. Denote the term in the first bracket by M_1 and the one in the second bracket by M_2. It is clear that when $\delta_1 = \delta_2$, we obtain $M_1 = M_2 = 0$ and $\rho\Delta U = 0$, implying that there is no welfare difference between $\rho U(s_1^*, s_2^*)$ and $\rho U(\bar{s}_1, \bar{s}_2)$ since optimal R&D subsidies under the two regimes are identical, namely $s_1^* = s_2^* = \bar{s}$. Furthermore, it can be shown that when $\delta_1 < \delta_2$, $M_1 > 0 > M_2$, whereas when $\delta_1 > \delta_2$, $M_1 < 0 < M_2$, which makes the sign of $\rho\Delta U$ ambiguous. Inspecting (A.20), it is known that the sign of $\rho\Delta U$ depends on the relationship between $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)}$ and $\frac{\rho+\lambda_1}{\rho+\lambda_2}$ in addition to the relative magnitude of δ_1 and δ_2. Consequently, for $\delta_1 \neq \delta_2$, we will have the following six cases for the welfare comparison, and each case is proved by contradiction.

Case A.1. Suppose $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \geq \frac{\rho+\lambda_1}{\rho+\lambda_2} > 1$. This case corresponds to $\delta_1 < \delta_2$. Then, we can derive (A.20) as follows

$$\begin{aligned}\rho\Delta U > {} & (1-\alpha)(1-\delta_2)\left[\begin{array}{l}\ln\left(1+\dfrac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\dfrac{\rho+\lambda_1}{\rho+\lambda_2}\right)+\ln\left(1+\dfrac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\dfrac{\rho+\lambda_2}{\rho+\lambda_1}\right) \\ -2\ln\dfrac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{(1-\alpha)(1-\delta_2)}\end{array}\right] \\ \geq {} & 2(1-\alpha)(1-\delta_2)\left[\ln\left(1+\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\frac{\rho+\lambda_1}{\rho+\lambda_2}\right)-\ln\left(1+\frac{\alpha}{1-\alpha}\frac{1-\delta_1}{1-\delta_2}\right)\right],\end{aligned} \tag{A.21}$$

where the first inequality is obtained by using $\delta_1 < \delta_2$ and $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} > 1$ whereas the second inequality is obtained by using $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \geq \frac{\rho+\lambda_1}{\rho+\lambda_2}$. To ensure that the expression in the second line of (A.21) is nonnegative, it is easy to show that the condition $\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1} \geq 1$ must be satisfied, but it violates the implication of the assumption that is imposed. Therefore, it is only possible that $\rho U(s_1^*, s_2^*) \leq \rho U(\bar{s}_1, \bar{s}_2)$ in this case.

Case A.2. Suppose $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \geq 1 > \frac{\rho+\lambda_1}{\rho+\lambda_2}$. This case corresponds to $\delta_1 > \delta_2$. Then, we can derive (A.20) as follows

$$\begin{aligned}\rho\Delta U &\leq \alpha(1-\delta_1)\left[\begin{aligned}&\ln\left(1+\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\frac{\rho+\lambda_1}{\rho+\lambda_2}\right)+\ln\left(1+\frac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\frac{\rho+\lambda_2}{\rho+\lambda_1}\right)\\&-2\ln\frac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)}\end{aligned}\right]\\&< 2\alpha(1-\delta_1)\left[\ln\left(1+\frac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\frac{\rho+\lambda_2}{\rho+\lambda_1}\right)-\ln\left(1+\frac{1-\alpha}{\alpha}\frac{1-\delta_2}{1-\delta_1}\right)\right],\end{aligned} \tag{A.22}$$

where the first inequality is obtained by using $\delta_1 > \delta_2$ and $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \geq 1$ whereas the second inequality is obtained by using $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} > \frac{\rho+\lambda_1}{\rho+\lambda_2}$. To ensure that the expression in the second line of (A.22) is nonpositive, it is easy to show that the condition $\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1} > 1$ must be satisfied, but it violates the implication of the assumption that is imposed. Therefore, it is only possible that $\rho U(s_1^*, s_2^*) \geq \rho U(\bar{s}_1, \bar{s}_2)$ in this case.

Case A.3. Suppose $1 > \frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \geq \frac{\rho+\lambda_1}{\rho+\lambda_2}$. This case corresponds to $\delta_1 > \delta_2$. Then, we can derive (A.20) as follows

$$\begin{aligned}\rho\Delta U &> \alpha(1-\delta_1)\left[\begin{aligned}&\ln\left(1+\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\frac{\rho+\lambda_1}{\rho+\lambda_2}\right)+\ln\left(1+\frac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\frac{\rho+\lambda_2}{\rho+\lambda_1}\right)\\&-2\ln\frac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)}\end{aligned}\right]\\&\geq 2\alpha(1-\delta_1)\left[\ln\left(1+\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\frac{\rho+\lambda_1}{\rho+\lambda_2}\right)-\ln\left(1+\frac{1-\alpha}{\alpha}\frac{1-\delta_2}{1-\delta_1}\right)\right],\end{aligned} \tag{A.23}$$

where the first inequality is obtained by using $\delta_1 > \delta_2$ and $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} < 1$ whereas the second inequality is obtained by using $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \geq \frac{\rho+\lambda_1}{\rho+\lambda_2}$. To ensure that the expression in the second line of (A.23) is nonnegative, it is easy to show that the condition $\delta_1 \leq \delta_2$ must be satisfied, but it violates the implication of the assumption that is imposed. Therefore, it is only possible that $\rho U(s_1^*, s_2^*) \leq \rho U(\bar{s}_1, \bar{s}_2)$ in this case. For the case such that $1 \geq \frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} > \frac{\rho+\lambda_1}{\rho+\lambda_2}$, it also corresponds to $\delta_1 > \delta_2$. The proof for this case is similar to that for Case A.3, which still leads to the result $\rho U(s_1^*, s_2^*) \leq \rho U(\bar{s}_1, \bar{s}_2)$.

Case A.4. Suppose $1 < \frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \leq \frac{\rho+\lambda_1}{\rho+\lambda_2}$. This case corresponds to $\delta_1 < \delta_2$. Then, we can derive

(A.20) as follows

$$
\begin{aligned}
\rho\Delta U > (1-\alpha)(1-\delta_2) & \begin{bmatrix} \ln\left(1+\dfrac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\dfrac{\rho+\lambda_1}{\rho+\lambda_2}\right)+\ln\left(1+\dfrac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\dfrac{\rho+\lambda_2}{\rho+\lambda_1}\right) \\ -2\ln\dfrac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{(1-\alpha)(1-\delta_2)} \end{bmatrix} \\
\geq 2(1-\alpha)(1-\delta_2) & \left[\ln\left(1+\frac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\frac{\rho+\lambda_2}{\rho+\lambda_1}\right)-\ln\left(1+\frac{\alpha}{1-\alpha}\frac{1-\delta_1}{1-\delta_2}\right)\right],
\end{aligned}
\tag{A.24}
$$

where the first inequality is obtained by using $\delta_1 < \delta_2$ and $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} > 1$ whereas the second inequality is obtained by using $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \leq \frac{\rho+\lambda_1}{\rho+\lambda_2}$. To ensure that the expression in the second line of (A.24) is nonnegative, it is easy to show that the condition $\delta_1 \geq \delta_2$ must be satisfied, but it violates the implication of the assumption that is imposed. Therefore, it is only possible that $\rho U(s_1^*, s_2^*) \leq \rho U(\bar{s}_1, \bar{s}_2)$ in this case. For the case such that $1 \leq \frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} < \frac{\rho+\lambda_1}{\rho+\lambda_2}$, it also corresponds to $\delta_1 < \delta_2$. The proof for this case is similar to that for Case A.4, which still leads to the result $\rho U(s_1^*, s_2^*) \leq \rho U(\bar{s}_1, \bar{s}_2)$.

Case A.5. Suppose $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \leq 1 < \frac{\rho+\lambda_1}{\rho+\lambda_2}$. This case corresponds to $\delta_1 < \delta_2$. Then, we can derive (A.20) as follows

$$
\begin{aligned}
\rho\Delta U \leq (1-\alpha)(1-\delta_2) & \begin{bmatrix} \ln\left(1+\dfrac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\dfrac{\rho+\lambda_1}{\rho+\lambda_2}\right)+\ln\left(1+\dfrac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\dfrac{\rho+\lambda_2}{\rho+\lambda_1}\right) \\ -2\ln\dfrac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{(1-\alpha)(1-\delta_2)} \end{bmatrix} \\
< 2(1-\alpha)(1-\delta_2) & \left[\ln\left(1+\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\frac{\rho+\lambda_1}{\rho+\lambda_2}\right)-\ln\left(1+\frac{\alpha}{1-\alpha}\frac{1-\delta_1}{1-\delta_2}\right)\right],
\end{aligned}
\tag{A.25}
$$

where the first inequality is obtained by using $\delta_1 < \delta_2$ and $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \leq 1$ whereas the second inequality is obtained by using $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} < \frac{\rho+\lambda_1}{\rho+\lambda_2}$. To ensure that the expression in the second line of (A.25) is nonpositive, it is easy to show that the condition $\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1} < 1$ must be satisfied, but it violates the implication of the assumption that is imposed. Therefore, it is only possible that $\rho U(s_1^*, s_2^*) \geq \rho U(\bar{s}_1, \bar{s}_2)$ in this case.

Case A.6. Suppose $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \leq \frac{\rho+\lambda_1}{\rho+\lambda_2} < 1$. This case corresponds to $\delta_1 > \delta_2$. Then, we can derive (A.20) as follows

$$
\begin{aligned}
\rho\Delta U > \alpha(1-\delta_1) & \begin{bmatrix} \ln\left(1+\dfrac{(1-\alpha)\lambda_2}{\alpha\lambda_1}\dfrac{\rho+\lambda_1}{\rho+\lambda_2}\right)+\ln\left(1+\dfrac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\dfrac{\rho+\lambda_2}{\rho+\lambda_1}\right) \\ -2\ln\dfrac{\alpha(1-\delta_1)+(1-\alpha)(1-\delta_2)}{\alpha(1-\delta_1)} \end{bmatrix} \\
\geq 2\alpha(1-\delta_1) & \left[\ln\left(1+\frac{\alpha\lambda_1}{(1-\alpha)\lambda_2}\frac{\rho+\lambda_2}{\rho+\lambda_1}\right)-\ln\left(1+\frac{1-\alpha}{\alpha}\frac{1-\delta_2}{1-\delta_1}\right)\right],
\end{aligned}
\tag{A.26}
$$

where the first inequality is obtained by using $\delta_1 > \delta_2$ and $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} < 1$ whereas the second inequality is obtained by using $\frac{\alpha(1-\delta_1)}{(1-\alpha)(1-\delta_2)} \leq \frac{\rho+\lambda_1}{\rho+\lambda_2}$. To ensure that the expression in the second line of (A.26) is nonnegative, it is easy to show that the condition $\frac{(1-\alpha)\lambda_2}{\alpha\lambda_1} \leq 1$ must be satisfied, but it violates the implication of the assumption that is imposed. Therefore, it is only possible that $\rho U(s_1^*, s_2^*) \leq \rho U(\bar{s}_1, \bar{s}_2)$ in this case.

Accordingly, summing up the cases where $\rho U(s_1^*, s_2^*) \geq \rho U(\bar{s}_1, \bar{s}_2)$ is straightforward to show that the condition $\frac{\alpha}{1-\alpha} \geq \frac{1-\delta_2}{1-\delta_1} > 1$ must hold in Case A.2 and the condition $\frac{\alpha}{1-\alpha} \leq \frac{1-\delta_2}{1-\delta_1} < 1$ must hold in Case A.5, respectively. In other words, $\rho U(s_1^*, s_2^*)$ can be greater than $\rho U(\bar{s}_1, \bar{s}_2)$ only if $\delta_1 > (<)\delta_2$ in the presence of $\alpha/(1-\alpha) > (<)1$. This completes the proof.

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