Welfare Effects of Patent Protection in a Growth Model with R&D and Capital Accumulation

Yibai Yang*
University of Macau†
University of Nottingham Ningbo China‡

May 27, 2017

Abstract

This study explores the welfare effects of patent protection in a Romer-type expanding-variety model in which R&D and capital accumulation are both engines of growth. It shows that the comparison between the productivity of R&D and that of capital plays an important role in the welfare analysis. When the relative productivity of R&D compared to capital is high (low), social welfare takes an inverted-U shape for (is decreasing in) the strength of patent protection, and the welfare-maximizing degree of patent protection is no greater than (identical to) the growth-maximizing degree. Moreover, the model is calibrated to the US economy and the numerical results support these welfare implications.

JEL classification: O31; O34; O40
Keywords: Economic growth; Patent protection; R&D; Capital accumulation

*The author thanks Chien-Yu Huang for the useful comments and discussion. The author also gratefully acknowledges the hospitality and support of Academia Sinica, where part of the research was completed. *Email* address: yibai.yang@hotmail.com.

†Department of Economics, University of Macau, Taipa, Macau, China.

‡GEP, IAPS, and School of Economics, University of Nottingham Ningbo China, Ningbo, Zhejiang, 315100, China.

1 Introduction

In this study, we explore the effects of patent protection on economic growth and social welfare in a Romer-type expanding variety model. We consider research and development (R&D) and capital accumulation as non-complementary growth engines and allow the growth rates of these factors to be determined independently, given that both innovation growth and capital growth significantly contribute to output growth, as shown in growth accounting studies.[1]. We find that the relative productivity of R&D compared to capital is particularly crucial for the effect of tightening intellectual property rights (IPR) protection on not only economic growth but also on social welfare. According to Park (2008), although patent protection in many countries has strengthened since the agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), patent protection indexes remain considerably lower than the upper bound; even high-income countries, such as the US, Japan, and Germany, do not pursue the strongest patent rights index.[2] Therefore, this study attempts to provide a new rationale from the viewpoint of welfare analysis to explain why countries may not prefer a high degree of patent protection.[3]

R&D and capital accumulation are both considered key driving forces of economic growth, but they are affected very differently by a tightening of IPR that better protects innovations. On the one hand, strengthening patent protection enables firms with market power to charge higher prices through enlarging markups. This increases the amount of profits generated by the invention and positively affects R&D activities, which stimulates economic growth. On the other hand, strengthening patent protection leads firms with market power to decrease the production volume because of larger markups. This reduces the demand for capital inputs used in manufacturing and negatively affects capital accumulation, which impedes economic growth. Summing up the two opposing effects, tightening patent rights lowers (raises) the rate of economic growth if the relative productivity of R&D compared to capital is low (high), namely if the engine of R&D growth is less (more) efficient. In other words, the lowest (highest) degree of patent protection tends to maximize economic growth in this framework.

In addition, strengthening patent protection has important implications for social welfare in this two-engine growth model. Stronger patent protection leads to a negative welfare effect stemming from a lower steady-state level of consumption, since the demand of capital inputs and therefore, the amount of production reduces. When the relative productivity of R&D is low, tightening IPR protection depresses economic growth, yielding a negative welfare effect that reinforces the negative welfare effect by decreased consumption. Thus, social welfare monotonically decreases in the strength of patent rights and is maximized by the same degree of growth-maximizing patent protection. In contrast, when the relative productivity of R&D is high, tightening IPR protection enhances economic growth, yielding a positive welfare effect. For low levels of IPR protection, this positive effect tends to overwhelm the negative welfare effect of lower consumption, which makes the

[1] Barro and Sala-I-Martin (2004) (Table 10.1) illustrate that in OECD countries, the contribution of total factor productivity growth and capital growth as a fraction of GDP growth are similarly large.

[2] The measure of patent protection in Park (2008), known as the Ginarte-Park index, includes 122 countries and sets a scale from 0 to 5. The average scale of the Ginarte-Park index is 3.34 in 2005, showing that a number of countries, especially developing ones, do not have a considerably high degree of IPR protection.

[3] Iwaisako (2013) indicates that the degree of patent protection that maximizes social welfare is relatively weak in a country where public services are limited.

welfare level positively correlated with the strength of patent rights. Nevertheless, for high levels of IPR protection, the former positive effect tends to be dominated by the latter negative welfare effect, which instead makes the welfare level negatively correlated with the strength of patent rights. Overall, social welfare exhibits an inverted-U shape in the degree of patent protection, and thus, the welfare-maximizing degree cannot exceed the growth-maximizing degree in this case. Consequently, it seems that patent policy that is growth-enhancing could deteriorate social welfare, and our analysis demonstrates that this may be true only for an economy with a more productive R&D-growth engine relative to the capital-growth engine, provided that the initial enforcement of IPR is sufficiently strict. In summary, from the perspective of social welfare, the argument that "stronger is always better" for patent protection does not apply in either case of the relative R&D productivity.[4]

This study is closely related to the literature on dynamic general equilibrium (DGE) models that examine the effects of patent protection on economic growth and social welfare. The seminal work in this literature is Judd (1985), who reveals that infinite patent length maximizes social welfare. Subsequent studies apply variants of the endogenous growth model to show that strengthening patent protection in the form of different instruments could generate a negative or non-monotonic effect on economic growth and social welfare (see Goh and Olivier (2002) and Pan, Zhang, and Zou (2017) on patent breadth; Futagami and Iwaisako (2007) on patent length; Furukawa (2007) on patent protection against imitation; O'Donoghue and Zweimüller (2004), Chu (2009), Chu, Cozzi, and Galli (2012), and Chu and Pan (2013) on blocking patents). However, these studies rely on R&D-based growth models in which either capital accumulation is absent or capital accumulation and R&D complement one another (namely, the growth rate of capital is determined such that it is equal to the growth rate of innovations), and these settings differ from the empirical findings of the growth accounting studies. Therefore, the present analysis complements the above papers by analyzing the growth and welfare effects of IPR protection in an expanding-variety model with two engines of growth, where capital growth and innovation growth are disproportionally determined, that is, they are negatively correlated.

Furthermore, this study contributes to a small but growing literature that explores the growth and welfare implications for various policy tools in a two-engine growth model. Chu, Lai, and Liao (2012) examine the growth and welfare effects of the interaction between monetary and patent policies in a growth model with R&D and capital accumulation, where consumption is subject to a cash-in-advance constraint. They show that strengthening patent protection has a monotonic effect on economic growth and social welfare, implying that given a monetary policy, the strongest patent rights would maximize both growth and welfare. In contrary to these findings, the current study reveals that stricter patent rights may lead to a non-monotonic effect on social welfare and render the welfare-maximizing solution for patent protection different from (below) the growth-maximizing one, which better conforms to existing empirical evidence. Iwaisako and Futagami (2013) present a two-engine growth model to show that the relationship between patent protection and economic growth is mixed (i.e., positive, negative, or non-monotonic), depending on the relative productivity of R&D compared to capital. In addition to the growth effects, it is important to consider the welfare

[4]This argument for patent protection does not apply from the economic-growth perspective either. See Thompson and Rushing (1996), Park (2005), Falvey, Foster, and Greenaway (2006), and Horii and Iwaisako (2007) for theoretical analysis and empirical evidence that present a mixed relationship between patent protection and economic growth.

effects of patent protection in such a framework. Thus, our study fills this gap by analytically and quantitatively investigating the welfare implications of patent protection according to the impacts on input allocations (in this study, labor allocations). Recently, Chen, Chu, and Lai (2015) analyze the growth effects of subsidy policy in a similar two-engine growth model and show how subsidizing the R&D sector may be growth-retarding since it has opposing impacts on research and capital. This study complements theirs by focusing on patent policy and including a welfare analysis.

The remainder of this study is organized as follows. Section 2 presents the expanding-variety model with two growth engines. Moreover, this section investigates the welfare implications of patent protection by deriving and comparing the growth- and welfare-maximizing level of patent rights. Section 3 calibrates the model and conducts the numerical analysis. Section 4 concludes.

2 The Model

To analyze the growth and welfare effects of patent protection in a DGE framework in which innovations and capital accumulation are both engines of growth, we follow Iwaisako and Futagami (2013) to adopt Romer (1990) expanding-variety model by incorporating a capital-producing sector in addition to an innovation-producing one. In addition, the level of patent breadth that affects the degree of firms' market power is influenced by patent authority policy, which reflects the strength of IPR protection.

2.1 Households

Suppose that the economy admits a unit continuum of identical households, and their utility function is given by

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt, \tag{1}$$

where $\rho > 0$ represents the discount rate, and C_t is the households' consumption of final goods at time t, whose price is normalized to unity. Assume that there is no population growth in the economy. Each household is endowed with one unit of time for labor.[5] Therefore, the value of households' total assets evolves according to

$$\dot{V}_t = R_t V_t + W_t - C_t, \tag{2}$$

where V_t is the real value of households' assets, W_t denotes the real wage rate, and R_t is the real interest rate. Then, the standard dynamic optimization implies the usual Euler equation

$$\frac{\dot{C}_t}{C_t} = R_t - \rho. \tag{3}$$

Moreover, the households own a balanced portfolio of all firms in the economy.

[5] One can introduce the labor supply of households L_t by assuming a separable utility function $u(C_t, L_t) = \ln C_t + \theta \ln(1 - L_t)$, where the intertemporal elasticity of substitution for consumption equals that for leisure (i.e., $1 - L_t$). Nevertheless, the equilibrium labor supply L will be unaffected by patent breadth μ in this setting, and therefore, the qualitative result in our study is robust to the one in this alternative setting. Derivations are available upon request.

2.2 Final Goods

Final goods y_t are competitively produced by using a continuum of intermediate goods $X_t(j)$ for $j \in [0, N_t]$ and a fixed input factor S_t (e.g., land) according to

$$Y_t = (S_t)^{1-\alpha} \int_0^{N_t} [X_t(j)]^\alpha \, dj, \tag{4}$$

where N_t is the number of varieties for intermediate goods. Notice that since S_t is assumed to be fixed, S_t equals its initial level S_0 and is constant over time. Because of free entry into the final-goods sector, the conditional demand function for intermediate goods is given by

$$P_t(j) = \alpha \left[S_0/X_t(j)\right]^{1-\alpha}, \tag{5}$$

where $P_t(j)$ is the price of $X_t(j)$ relative to the final goods.

2.3 Intermediate Goods

In each variety $j \in [0, N_t]$, intermediate goods are manufactured by a monopolist who rents capital and hires labor, according to a standard Cobb-Douglas production function given by

$$X_t(j) = A \left[K_t(j)\right]^\gamma \left[L_{x,t}(j)\right]^{1-\gamma}, \tag{6}$$

where $K_t(j)$ is the amount of capital inputs, $L_{x,t}(j)$ is the employment level of production labor, and γ is the share parameter of capital. Then, applying cost minimization to (6), the marginal cost of producing intermediate goods for the monopolist in variety j is

$$MC_t(j) = \frac{1}{A} \left(\frac{Q_t}{\gamma}\right)^\gamma \left(\frac{W_t}{1-\gamma}\right)^{1-\gamma}, \tag{7}$$

where Q_t denotes the rental price of capital.

To consider the degree of patent protection, the monopolist is allowed to charge a markup over the marginal production cost for profit maximization. Following previous studies such as Li (2001), Goh and Olivier (2002), Iwaisako and Futagami (2013), and Pan, Zhang, and Zou (2017), we assume that markup $\mu_t \in (1, 1/\alpha]$ is represented by the strength of patent breadth, which is a policy instrument that can be set by patent authority. The upper bound of μ_t is the unconstrained markup value in the existing literature (e.g., Barro and Sala-I-Martin (2004)). Therefore, the profit-maximizing price is given by

$$P_t(j) = \mu_t MC_t(j). \tag{8}$$

Given $MC_t(j)$ and the pricing strategy, the monopolist chooses $K_t(j)$ and $L_{x,t}(j)$ to maximize her profit $\Pi_{x,t}(j) = P_t(j)X_t(j) - Q_t K_t(j) - W_t L_{x,t}(j)$ subject to (5) and (6). Hence, the monopolist's profit is

$$\Pi_{x,t}(j) = \left(\frac{\mu_t - 1}{\mu_t}\right) P_t(j) X_t(j). \tag{9}$$

The factor payments for labor and capital inputs employed in the intermediate-goods production are given by

$$W_t L_{x,t}(j) = \left(\frac{1-\gamma}{\mu_t}\right) P_t(j) X_t(j), \tag{10}$$

$$Q_t K_t(j) = \left(\frac{\gamma}{\mu_t}\right) P_t(j) X_t(j). \tag{11}$$

These equations are conditions that determine the input allocations in this sector.

2.4 Inventions and R&D

The value of invented variety j is denoted as $V_{n,t}(j)$. Following the standard literature, we focus on the symmetric equilibrium such that $\Pi_{x,t}(j) = \Pi_{x,t}$ and $V_{n,t}(j) = V_{n,t}$. In fact, this symmetric equilibrium is ensured by (5), (7), and (8), in which the monopolists face the same marginal production cost and charge an identical price across varieties. Then, the familiar no-arbitrage condition for the asset value is

$$R_t V_{n,t} = \Pi_{x,t} + \dot{V}_{n,t}, \tag{12}$$

which implies that the return on this asset $R_t V_{n,t}$ equals the sum of flow profits as a monopolist $\Pi_{x,t}$ and potential capital gain $\dot{V}_{n,t}$.

New innovations for each variety are invented by a unit continuum of R&D firms indexed by $\iota \in [0,1]$. Each of these firms employs R&D labor $L_{r,t}(\iota)$ to produce inventions. The expected profit of the ι-th R&D firm is

$$\Pi_{r,t}(\iota) = V_{n,t}\dot{N}_t(\iota) - W_t L_{r,t}(\iota), \tag{13}$$

where $\dot{N}_t(\iota) = \varphi N_t L_{r,t}(\iota)$ is the number of inventions created by firm ι, depending on the existing number of varieties. φ is R&D productivity at time t. In equilibrium, the number of inventions occurring at the aggregate level equals the counterpart at the firm level for each variety, namely, $\dot{N}_t = \dot{N}_t(\iota)$. Then, free entry into the R&D sector implies the following zero-expected-profit condition:

$$\varphi N_t V_{n,t} = W_t. \tag{14}$$

This equation is the condition that determines labor allocation for R&D.

2.5 Capital Production

The value of one unit of capital in variety j is denoted as $V_{k,t}(j)$. Similarly, symmetry across varieties implies $V_{k,t}(j) = V_{k,t}$. Then, the no-arbitrage condition for the capital asset is

$$R_t V_{k,t} = Q_{k,t} + \dot{V}_{k,t}. \tag{15}$$

Here as well, this equation implies that the return on asset $R_t V_{k,t}$ equals the sum of the rental price of capital $Q_{k,t}$ and capital gain $\dot{V}_{k,t}$.

Capital goods for each variety are produced by a unit continuum of capital-producing firms indexed $\nu \in [0,1]$. Each of these firms employs capital-producing labor $L_{k,t}(\nu)$ for the production.

The expected profit of the ν-th capital-producing firm is

$$\Pi_{k,t}(\nu) = V_{k,t}\dot{K}_t(\nu) - W_t L_{k,t}(\nu), \tag{16}$$

where $\dot{K}_t(\nu) = \phi A_{k,t} L_{k,t}(\nu)$ is the amount of capital goods produced by firm ν. $\phi A_{k,t}$ denotes the effectiveness of capital production at time t. Following Romer (1986), Chu, Lai, and Liao (2012), and Iwaisako and Futagami (2013), it is assumed that $A_{k,t} = K_t$, implying that this effectiveness increases in the accumulated capital stock. In this setting, knowledge spillovers from past production experiences are present to capture the usual capital externality as in the AK model, enabling sustainable growth for physical capital.[6] In equilibrium, the amount of capital goods created at the aggregate level equals the counterpart at the firm level for each variety, namely, $\dot{K}_t = \dot{K}_t(\nu)$. Then, free entry into the capital-producing sector implies the following zero-expected-profit condition:

$$\phi K_t V_{k,t} = W_t. \tag{17}$$

This equation is the condition that determines labor allocation for capital accumulation.

2.6 Decentralized Equilibrium

An equilibrium consists of a sequence of allocations $[C_t, Y_t, X_t(j), K_t(j), L_{x,t}, L_{r,t}, L_{k,t}]_{t=0}^{\infty}$ and a sequence of prices $[P_t(j), R_t, W_t, Q_t, V_{n,t}, V_{k,t}]_{t=0}^{\infty}$, and a sequence of policies $[\mu_t]_{t=0}^{\infty}$. Moreover, in each instant of time,

- households choose $[C_t]$ to maximize their utility given $[R_t, W_t]$;
- final-goods firms produce $[Y_t]$ and choose $[X_t(j)]$ to maximize profits given $[W_t, P_t(j)]$;
- intermediate-goods monopolist in industry $j \in [0, N_t]$ produces $[X_t(j)]$ and chooses $[K_t(j), L_{x,t}(j)]$ to maximize profits given $[Q_t, W_t]$;
- R&D firms choose $[L_{r,t}]$ to maximize profits given $[W_t, V_{n,t}]$;
- capital-producing firms choose $[L_{k,t}]$ to maximize profits given $[W_t, V_{k,t}]$;
- the final-goods market clears such that $C_t = Y_t$;
- the labor market clears such that $L_{x,t} + L_{r,t} + L_{k,t} = 1$, where $L_{x,t} = \int_0^{N_t} L_{x,t}(j)dj$;
- the capital-goods market clears such that $K_t = \int_0^{N_t} K_t(j)dj$; and
- the values of intangible and tangible assets add up to households' assets value such that $V_{n,t}N_t + V_{k,t}K_t = V_t$.

In this subsection, we derive the equilibrium labor allocations and examine the effect of patent protection (i.e., μ_t) on these allocations. First, we obtain the following result that characterizes the dynamics of the model.

Lemma 1. *Holding μ constant, the economy jumps to a unique and stable balanced growth path.*

Proof. See the Appendix. □

[6] As will be shown, capital accumulation and variety expansion serve as non-complementary engines of growth in this model, because the growth of physical capital is pinned down independently of the growth of the number of varieties.

On the balanced growth path, as shown in the proof of Lemma 1, the equilibrium labor allocations are stationary given a stationary path of μ_t. Imposing balanced growth on (A.3) and (A.5) in Appendix A yields

$$L_{r,t} = \left(\frac{\mu - 1}{1-\gamma}\right) L_{x,t} - \frac{\rho}{\varphi}, \tag{18}$$

$$L_{k,t} = \left(\frac{\gamma}{1-\gamma}\right) L_{x,t} - \frac{\rho}{\phi}, \tag{19}$$

which are the two equations that solve for $\{L_x, L_r, L_k\}$. The last equation is simply the labor-market-clearing condition such that

$$L_{x,t} + L_{r,t} + L_{k,t} = 1. \tag{20}$$

Solving (18)-(20), the equilibrium labor allocations are given by

$$L_x = \frac{1-\gamma}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right), \tag{21}$$

$$L_r = \frac{\mu - 1}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) - \frac{\rho}{\varphi}, \tag{22}$$

$$L_k = \frac{\gamma}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) - \frac{\rho}{\phi}. \tag{23}$$

The allocations in (22) and (23) show that R&D labor L_r is increasing in patent breadth μ, whereas capital-producing labor L_k is decreasing in it.[7] In other words, a strengthening of patent protection stimulates R&D investment but depresses investment in capital accumulation. The positive effect of μ on L_r and negative effect on L_k are based on our setting that follows Iwaisako and Futagami (2013), in which increasing patent breadth allows intermediate-goods monopolists to charge higher prices and decrease the production amount. The former effect attracts investment for innovations, whereas the latter reduces the demand for capital goods.

2.7 Growth-Maximizing Patent Breadth

This subsection considers the growth effects of patent breadth. Using symmetry across varieties in (4) yields $Y_t = (S_t)^\alpha (N_t)^{1-\alpha} (X_t)^{1-\alpha}$. Substituting (6) into this equation and taking differentiation with respect to t yields

$$g_y \equiv \frac{\dot{Y}_t}{Y_t} = (1-\alpha)\frac{\dot{N}_t}{N_t} + \alpha\gamma\frac{\dot{K}_t}{K_t} = (1-\alpha)\varphi L_r + \alpha\gamma\phi L_k, \tag{24}$$

where we use the fact that S_t is constant over time. In (24), the growth rate of the number of varieties $\dot{N}_t/N_t = \varphi L_r$ and that of physical capital $\dot{K}_t/K_t = \phi L_k$ are differently determined, and thus, R&D and capital accumulation are non-complementary engines of growth for final outputs.

[7] The parameter space is restricted to ensure that L_r and L_k are bounded between 0 and 1, which will be further discussed in the numerical analysis in Section 3.

Given the opposing effects of μ on L_r and L_k, μ has an ambiguous impact on the growth rate of final goods g_y. In particular, substituting (22) and (23) into (24) yields $g_y = (1+\rho/\varphi+\rho/\phi)[(1-\alpha)\varphi(1-1/\mu)+\alpha\gamma^2\phi/\mu]$. Taking the derivative of g_y with respect to μ implies that if the relative productivity of R&D compared to capital is high (low), namely $\varphi/\phi > (<)\alpha\gamma^2/(1-\alpha)$, and thus, g_y is positively (negatively) correlated with μ. This result is consistent with the existing empirical evidence that documents a mixed relationship between patent protection and economic growth.[8]

Lemma 2. *A larger patent breadth increases R&D but decreases capital production. Strengthening patent protection is growth-enhancing (-retarding) for a high (low) relative R&D productivity, i.e.,* $\varphi/\phi > (<)\alpha\gamma^2/(1-\alpha)$.

Proof. Proven in the text. □

Given that the range of patent breadth is $\mu \in (1, 1/\alpha]$, it is straightforward to derive the level of patent protection that maximizes the growth rate of outputs, denoted by μ_g, as follows.

Proposition 1. *The growth-maximizing degree of patent protection is given by (i)* $\mu_g = (1+\rho/\varphi+\rho/\phi)/(1+\rho/\phi)$ *if* $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$ *and (ii)* $\mu_g = 1/\alpha$ *if* $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$.

Proof. When $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, g_y is monotonically decreasing in μ. Moreover, an increase in μ increases L_r but decreases L_x and L_k. Thus, the lowest level of μ that is feasible to attain is to make $L_r \to 0$. Then, setting (22) to zero yields the result in (i).

When $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$, g_y is monotonically increasing in μ. Thus, the highest level of μ that is feasible to attain is to set μ at its upper bound $1/\alpha$, yielding the result in (ii). □

In other words, when the relative productivity of R&D is high (low), the economy allocates more labor to the R&D- (capital-)producing sector to maximize economic growth by strengthening (weakening) the level of patent breadth as much as possible.

2.8 Welfare-Maximizing Patent Breadth

This section considers the welfare effects of patent breadth. In addition, we compare growth- and welfare-maximizing patent breadth and show the condition under which the former is greater than the latter.

Imposing balanced growth on (1) yields the households' lifetime utility:

$$U = \frac{1}{\rho}\left(\ln C_0 + \frac{g_c}{\rho}\right), \tag{25}$$

where $g_c = g_y$ (the growth rate of consumption equals that of outputs) and $C_0 = Y_0 = S_0^{1-\alpha}N_0^{1-\alpha}X_0^{\alpha} = S_0^{1-\alpha}N_0^{1-\alpha}\left(AK_0^{\gamma}L_x^{1-\gamma}\right)^{\alpha}$ by using (4) and (6). Dropping the exogenous terms and substituting the growth rate of outputs in (24) yield

$$U = \frac{\alpha(1-\gamma)}{\rho}\ln L_x + \frac{1}{\rho^2}\left[(1-\alpha)\varphi L_r + \alpha\gamma\phi L_k\right], \tag{26}$$

[8]It is obvious that when $\varphi/\phi = \alpha\gamma^2/(1-\alpha)$, adjusting patent breadth has no impact on the growth of outputs.

where the labor allocations follow (21)-(23). To show the impact of patent protection on the steady-state level of welfare, taking the derivative of (32) with respect to μ yields

$$\frac{\partial U}{\partial \mu} = -\frac{\alpha(1-\gamma)}{\rho\mu} + \frac{1}{\rho^2\mu^2}\left(1+\frac{\rho}{\varphi}+\frac{\rho}{\phi}\right)\left[(1-\alpha)\varphi - \alpha\gamma^2\phi\right]. \tag{27}$$

Therefore, it can be seen that the sign of $\partial U/\partial\mu$ is determined by that of $(1-\alpha)\varphi - \alpha\gamma^2\phi$. There are two cases to be considered on the basis of the relative productivity of R&D compared to that of capital φ/ϕ. If $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, then we obtain $\partial U/\partial\mu < 0$, implying that social welfare is monotonically decreasing in the level of patent breadth. In contrast, if $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$, then whether a strengthening of patent protection increases or decreases social welfare depends on the level of patent breadth. In particular, the cutoff value

$$\tilde{\mu} = \frac{1}{\rho(1-\gamma)}\left(1+\frac{\rho}{\phi}+\frac{\rho}{\varphi}\right)\left[\varphi\left(\frac{1-\alpha}{\alpha}\right) - \gamma^2\phi\right] \tag{28}$$

exists such that the welfare level rises (declines) as patent protection becomes stricter if $\mu < (>)\tilde{\mu}$. Therefore, it is straightforward to obtain the following result.

Lemma 3. *Social welfare (i) monotonically decreases in the degree of patent protection for $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$ and (ii) has an inverted-U shape for the degree of patent protection for $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$.*

Proof. Proven in the text. □

Notice that the critical value of relative R&D productivity that pins down the welfare effect of patent protection is identical to the one that pins down the growth effect as shown in Lemma 1 (namely $\alpha\gamma^2/(1-\alpha)$). Consequently, Lemma 3 reveals that when the relative R&D productivity is low, strengthening patent protection impedes not only economic growth but also social welfare. A larger patent breadth always decreases production labor L_x, leading to a negative welfare effect (i.e., the first term on the RHS of (27)) because the initial (steady-state) level of consumption C_0 decreases. When $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, the welfare effect of broadening patent breadth through economic growth (i.e., the second term on the RHS of (27)) also becomes negative since the negative impact on depressing capital accumulation (i.e., capital labor L_k) dominates the positive counterpart on enhancing innovations (i.e., R&D labor L_r). The above two welfare effects reinforce each other to make U decrease in μ. This result may be able to partially explain why certain developing countries in which capital accumulation is a more effective engine of growth do not pursue a high degree of patent protection.

On the other hand, when the relative productivity of R&D is high (recall $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$), the positive impact of broadening patent breadth on L_r overwhelms the negative counterpart on L_k, which yields a positive welfare effect through economic growth. In such a case, if patent breadth is initially narrow (i.e., $\mu < \tilde{\mu}$), then this positive effect by economic growth strictly dominates the negative welfare effect of smaller consumption by the decrease in L_x, and therefore, strengthening patent protection simultaneously stimulates economic growth and increases social welfare. Alternatively, if patent breadth is initially board (i.e., $\mu > \tilde{\mu}$), then the positive effect by economic growth is dominated by the negative welfare effect by the decrease in L_x, and thus, strengthening patent protection continues to promote economic growth but tends to diminish social welfare. This result is consistent with the argument that strengthening patent protection to the highest level may not be optimal, which is the current situation in many developed economies.

Hence, the above analysis implies that extending patent breadth is not always beneficial in terms of promoting economic growth and raising social welfare, because the relative productivity of R&D compared to capital plays a crucial role in the growth and welfare implications of implementing patent policy. Notice that the inverted-U shape of social welfare on patent protection only applies when $\mu_w < 1/\alpha$. If $\mu_w \geq 1/\alpha$, there does not exist a level of patent breadth that satisfies $\mu > \mu_w$ since μ cannot exceed $1/\alpha$. In this case, only the increasing part of the hump shape is valid in broadening patent breadth, implying that the positive effect by economic growth strictly dominates the negative welfare effect by the decline in L_x for all possible ranges of μ. Consequently, social welfare becomes monotonically increasing in patent protection as in the case of economic growth.

In addition, the previous positive analysis implies that the welfare-maximizing degree of patent protection, denoted by μ_w, depends on the welfare effect of patent breadth, which can be summarized by the following result.

Proposition 2. *The welfare-maximizing degree of patent protection is given by (i)* $\mu_w = (1+\rho/\varphi+\rho/\phi)/(1+\rho/\phi) = \mu_g$ *if* $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$*, and (ii)* $\mu_w = \tilde{\mu} \leq \mu_g$ *if* $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$*.*

Proof. When $\varphi/\phi \leq \alpha\gamma^2/(1-\alpha)$, U is monotonically decreasing in μ, so the derivation of μ_w is the same as that of μ_g; setting (22) to zero yields the result in (i).

When $\varphi/\phi > \alpha\gamma^2/(1-\alpha)$, U has a hump shape on μ. Thus, U is maximized at $\mu = \tilde{\mu}$. Also, denote $\mu_m \equiv 1/\alpha = \mu_g$. Then, using $\tilde{\mu}$ in (28) and μ_m, (27) can be rewritten as

$$\frac{\partial U}{\partial \mu} = -\frac{1-\gamma}{\rho\mu^2}\left(\alpha\mu - \frac{\tilde{\mu}}{\mu_m}\right). \tag{29}$$

Given that obtaining $\tilde{\mu}$ in (28) (namely μ_w) requires $\partial U/\partial\mu = 0$, it is easy to show that $\mu_w/\mu_g = \tilde{\mu}/\mu_m = \alpha\mu \leq 1$ since $\mu \leq 1/\alpha$. This yields the result in (ii). □

3 Numerical Analysis

In this section, we calibrate the model to the US economy to numerically evaluate the growth and welfare effects of tightening patent protection in this two-engine growth model. This analysis also enables us to compare the growth-maximizing level of patent breadth and the welfare-maximizing level, which verifies the analytical results.

To perform this numerical exercise, we assign steady-state values to the following structural parameters $\{\mu, \rho, \varphi, \phi, \alpha, \gamma\}$. As for patent breadth μ, we focus on values ranging from 1.1 to 1.4 by taking into account the empirical estimates of the markup reported in Jones and Williams (2000) (i.e., 1.05–1.4) and Laitner and Stolyarov (2004) (i.e., roughly 1.1). We use $\mu = 1.1$ as the market level and gradually increase the magnitude of μ to strengthen the degree of patent protection. To calibrate other parameters, we follow Chu, Lai, and Liao (2012) to use certain empirical moments based on the US data from 1999 to 2010. We set the growth rates of outputs g_y and capital g_k

Table 1: Calibrated parameter values.

μ	ρ	φ	ϕ	α	γ
1.1	0.0047	0.0744	0.0364	0.7143	0.8960

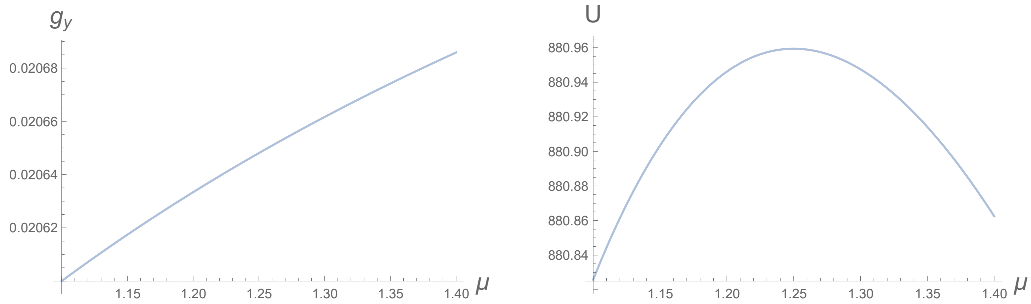
to 2.06% and 3.07%, and R&D investment as a percentage of GDP (namely R&D intensity) to 0.0268. Furthermore, α is set to the value that allows the whole range of μ to satisfy the restriction such that $\alpha \leq 1/\mu$. Finally, we set the discount rate ρ to be a conservative value, which is the largest value in the range consistent with the above empirical moments and ensures that equilibrium R&D labor L_r and equilibrium capital labor L_k are bounded between 0 and 1. These estimates, therefore, yield the calibrated values of R&D productivity φ, capital productivity ϕ, factor share of intermediate goods α, the factor share of capital γ, as shown in Table 1.

3.1 The Results

In this calibrated economy, as the degree of patent protection μ rises from 1.1 to 1.4, R&D labor L_r increases from 0.0448 to 0.2774, while production labor L_x and capital labor L_k decrease from 0.1129 to 0.0887 and from 0.8423 to 0.6339. Consequently, R&D growth rate g_n increases from 0.3335% to 2.0648%, whereas the capital growth rate g_k decreases from 3.07% to 2.3105%. These changes are the implication of labor allocations as shown in (21)-(23).

More importantly, the combination of the calibrated values in Table 1 implies that the relative productivity of R&D compared to capital (i.e., φ/ϕ) in the steady state is above the threshold value that pins down the choice of growth- and welfare-maximizing patent breadth as specified in Propositions 1 and 2 (i.e., $\alpha\gamma^2/(1-\alpha) = 2.0069$). On the one hand, economic growth g_y is monotonically increasing in μ from 2.06% at $\mu = 1.1$ to 2.0686% at $\mu = 1.4$, implying that the positive R&D channel for economic growth overwhelms the negative capital channel. Hence, growth-maximizing patent breadth is obtained by the corner solution such that $\mu_g = 1/\alpha = 1.4$. On the other hand, social welfare U has an inverted-U shape for μ, which first increases from 880.826 at $\mu = 1.1$ to 880.959 at $\mu = 1.25$ and then decreases to 880.863 at $\mu = 1.4$. This result implies that given a high level of efficiency in R&D relative to capital accumulation, recalling (26) shows that at the low levels of μ, the positive welfare effect through growth is sufficient to dominate the negative welfare effect through smaller consumption (due to the decline in manufacturing labor). Nevertheless, as μ increases, this domination increasingly weakens and finally, the negative welfare effect dominates the positive one. Hence, welfare-maximizing patent breadth is given by an interior solution such that $\mu_w = \tilde{\mu} = 1.25$. Furthermore, it is obvious to see that $u_w < u_g$, which is in line with the analytical result in Proposition 2. The above results are illustrated in Figure 1 and results for particular values of μ are summarized in Table 2.

In addition, it can be shown that the welfare-maximizing level of patent protection μ_w leads to a socially suboptimal outcome that results in a welfare loss. This is because given the steady-state level of relative productivity of R&D as calibrated, the social optimum requires to shut down the weaker growth engine (namely R&D) and only use the stronger growth engine (namely capital



(a) Economic Growth (b) Social Welfare

Figure 1: The growth and welfare effects of a strengthening of patent protection: benchmark

Table 2: Growth and welfare effects of patent protection: benchmark.

$\varphi/\phi = 2.0423$							
μ	1.1	1.15	1.2	1.25	1.3	1.35	1.4
L_x	0.1129	0.1080	0.1035	0.0994	0.0955	0.0920	0.0887
L_r	0.0448	0.0920	0.1352	0.1750	0.2118	0.2458	0.2774
L_k	0.8423	0.8000	0.7613	0.7256	0.6927	0.6622	0.6339
g_n	0.3335%	0.6847%	1.0068%	1.3030%	1.5765%	1.8297%	2.0648%
g_k	3.0700%	2.9159%	2.7746%	2.6447%	2.5247%	2.4136%	2.3105%
g_y	2.0600%	2.0617%	2.0633%	2.0648%	2.0662%	2.0674%	2.0686%
U	880.826	880.903	880.846	880.959	880.948	880.914	880.863

accumulation) in the economy.[9] [10] However, the two growth engines are simultaneously used under μ_w as displayed in Table 2. Hence, μ_w does not achieve the first-best optimal allocations.

3.2 Sensitivity Checks

In this subsection, we conduct two sensitivity checks on our numerical exercise to examine the extent to which the quantitative results would change under a different level of relative productivity of R&D compared to capital.

First, the steady-state value φ/ϕ in the previous subsection is calibrated to be higher than the threshold value $\alpha\gamma^2/(1-\alpha)$, which implies that strengthening patent protection is always growth-enhancing but has a non-monotonic effect on social welfare. Here, we consider an alternative value of $\varphi/\phi = 2$ by lowering the value of φ while keeping the value of ϕ unchanged, such that this value is below the aforementioned threshold value. Figure 2 and Table 3 summarize the variations in the impacts of patent protection on economic growth and social welfare in this case. Under this value of

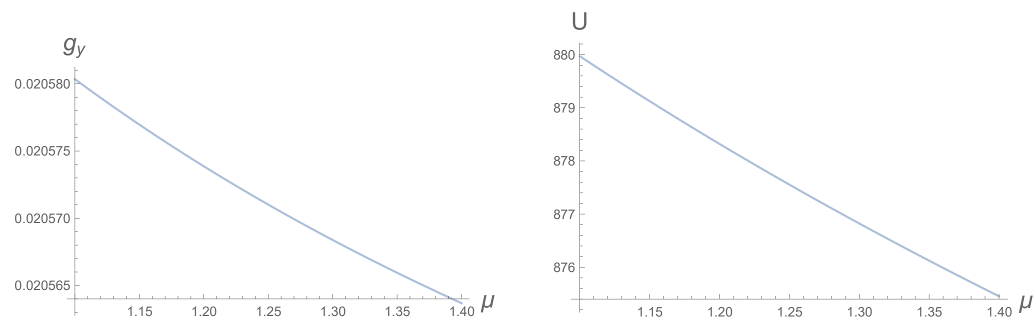
[9]Under this calibration, the first-best optimal labor allocations denoted with superscript $*$ are given by $L_x^* = 0.0151$, $L_r^* = 0$, and $L_k^* = 0.9849$, yielding R&D growth rate $g_n^* = 0$, capital growth rate $g_k^* = 3.5897\%$, output growth rate $g_y^* = 2.2297\%$, and welfare level 954.752.

[10]See Appendix B for the derivation of the first-best allocations. The computation for the numerical result can be seen in the complementary *Mathematica* file and it is available upon request.

Table 3: Growth and welfare effects of patent protection: low relative productivity of R&D.

$\varphi/\phi = 2$							
μ	1.1	1.15	1.2	1.25	1.3	1.35	1.4
L_x	0.1130	0.1081	0.1036	0.0995	0.0956	0.0921	0.0888
L_r	0.0436	0.0908	0.1341	0.1740	0.2107	0.2448	0.2764
L_k	0.8434	0.8011	0.7623	0.7266	0.6936	0.6631	0.6348
g_n	0.3176%	0.6620%	0.9777%	1.2681%	1.5362%	1.7845%	2.0150%
g_k	3.0740%	2.9197%	2.7783%	2.6482%	2.5281%	2.4169%	2.3136%
g_y	2.0580%	2.0577%	2.0574%	2.0571%	2.0568%	2.0566%	2.0564%
U	879.971	879.124	878.320	877.554	876.823	876.124	875.454

φ/ϕ, both the growth rate of outputs g_y and the level of welfare U become monotonically decreasing in patent protection μ. Accordingly, growth- and welfare-maximizing patent breadth coincide with one another; both are represented by the narrowest patent breadth such that $\mu_g = \mu_w = 1.1$, which again supports the implication of Proposition 2 (i.e., $\mu_w = \mu_g$).[11] This result shows that a low φ/ϕ implies a low efficiency level in R&D relative to capital accumulation, and thus, this induces the positive growth effect of a larger μ through R&D to be strictly dominated by the negative growth effect through capital accumulation within the calibrated range of μ; therefore, extending patent breadth is growth-retarding. In addition, this overall negative growth effect of a tightening of patent protection reinforces the negative welfare effect through smaller consumption, which also makes welfare decrease in the level of μ.



(a) Economic Growth (b) Social Welfare

Figure 2: Growth and welfare effects of strengthening patent protection: low φ/ϕ

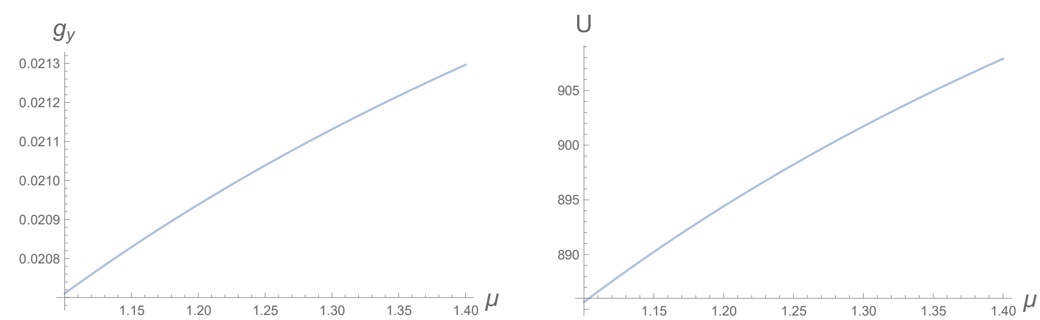
Furthermore, we consider an alternative value of $\varphi/\phi = 2.25$ by raising the value of φ while maintaining that of ϕ as in the previous exercises, such that it is greater than the threshold value $\alpha\gamma^2/(1-\alpha)$. Figure 3 and Table 4 summarize the changes in the growth and welfare implications of tightening patent protection in this case. Under this value of φ/ϕ, both the growth rate of outputs g_y and the level of welfare U monotonically increase in patent protection μ. Accordingly, growth-

[11] In fact, under $\varphi/\phi = 2$ and $\phi = 0.0364$, the growth- and welfare-maximizing patent breadth implied by Proportions 1 and 2 are given by $(1 + \rho/\varphi + \rho/\phi)/(1 + \rho) = 1.0576$, which is smaller than the lower bound of the calibrated value of μ. Thus, a corner solution for μ_g and μ_w is used in this case.

Table 4: Growth and welfare effects of patent protection: high relative productivity of R&D.

$\varphi/\phi = 2.25$							
μ	1.1	1.15	1.2	1.25	1.3	1.35	1.4
L_x	0.1124	0.1075	0.1030	0.0989	0.0951	0.0915	0.0883
L_r	0.0501	0.0971	0.1401	0.1797	0.2163	0.2502	0.2816
L_k	0.8375	0.7954	0.7569	0.7214	0.6886	0.6583	0.6301
g_n	0.4112%	0.7963%	1.1493%	1.4741%	1.7739%	2.0515%	2.3092%
g_k	3.0525%	2.8992%	2.7586%	2.6293%	2.5099%	2.3994%	2.2967%
g_y	2.0710%	2.0829%	2.0938%	2.1039%	2.1131%	2.1217%	2.1296%
U	885.653	890.238	894.412	898.227	901.723	904.939	907.904

and welfare-maximizing patent breadth coincide with each other again, which are represented by the the broadest patent breadth $\mu_g = \mu_w = 1.4$. This result is still consistent with the implication of Proposition 2 (i.e., $\mu_w \leq \mu_g$).[12] The explanation is analogous to the previous discussion; this result reveals that a high φ/ϕ implies a high efficiency level in R&D relative to capital accumulation, such that the positive growth effect of a larger μ through R&D governs the negative growth effect through capital accumulation within the calibrated range of μ. Therefore, extending patent breadth is growth-stimulating. In addition, the overall positive growth effect of tightening patent protection is sufficient to dominate the negative welfare effect through smaller consumption, which also makes welfare increase in the level of μ.



(a) Economic Growth (b) Social Welfare

Figure 3: Growth and welfare effects of strengthening patent protection: high φ/ϕ

Finally, when the relative productivity of R&D φ/ϕ is low (i.e., 2), the first-best allocations that correspond to this case are identical to those for the benchmark case. Therefore, the welfare-maximizing level of patent protection μ_w does not achieve the first-best allocations either as shown in Table 3, given that the growth engine of R&D should be abandoned in the social optimum. Even when φ/ϕ is high (i.e., 2.25), welfare-maximizing patent breadth μ_w cannot attain the first-best outcome (See Footnote 10 and Table 4), since the social optimum now tends to shut down the

[12] Similarly, under $\varphi/\phi = 2.25$ and $\phi = 0.0364$, the growth- and welfare-maximizing patent breadth implied by Proportions 1 and 2 are given by $\tilde{\mu}$ in (28), which exceeds the upper bound of the calibrated value of μ. Thus, a corner solution continues to be used for μ_g and μ_w.

weaker growth engine of capital accumulation and instead relies on only the stronger growth engine of R&D, which is obviously not the case with μ_w under this high value of φ/ϕ.[13]

4 Conclusion

This study explores the impacts of patent protection on economic growth and social welfare in a Romer-type expanding-variety model in which R&D and capital accumulation are both engines of growth. The current setup allows the growth rates of innovation and capital to be determined independently. Previous studies on this type of two-engine growth models reveal that the comparison between R&D and capital productivity is a major determinant of the growth effects of IPR protection. However, the findings of this study suggest that such a comparison continues to be important for the welfare effects of IPR protection. In particular, our results show that when the relative productivity of R&D is low, economic growth and social welfare both decrease in the strength of IPR protection, implying that the growth- and welfare-maximizing degree of patent protection coincide with one another. By contrast, when the relative productivity of R&D is high, economic growth increases in the strength of IPR protection, whereas social welfare has an inverted-U shape, implying that the welfare-maximizing degree of patent protection would be below the growth-maximizing degree. Therefore, this study provides a novel channel for welfare analysis to explain why countries, with either high or low productivity, may not prefer a significantly high degree of patent protection, even though stricter enforcement of patent rights could be growth-stimulating.

[13] In the case of high φ/ϕ, the first-best optimal labor allocations are given by $L_x^* = 0.0150$, $L_r^* = 0.9850$, and $L_k^* = 0$, yielding R&D growth rate $g_n^* = 8.7744\%$, capital growth rate $g_k^* = 0$, output growth rate $g_y^* = 2.3079\%$, and welfare level 959.360.

Appendix

Proof of Lemma 1

In this proof, we examine the stability of this model given a stationary path of μ_t. First, we use the symmetry across varieties to use (4) to rewrite (5) as $\alpha Y_t/N_t = P_t(j)X_t$. Then, combining (10) and (14) yields

$$W_t = \varphi V_{n,t} N_t = \frac{\left(\frac{1-\gamma}{\mu}\right) P_t(j) X_t(j)}{L_{x,t}(j)} = \frac{\left(\frac{1-\gamma}{\mu}\right) \alpha Y_t}{N_t L_{x,t}(j)} = \frac{\left(\frac{1-\gamma}{\mu}\right) \alpha Y_t}{L_{x,t}}, \tag{A.1}$$

where in the third equality, we use (5) and in the fourth equality, we aggregate the production labor $L_{x,t}(j)$ across varieties. Differentiating (A.1) with respect to t yields

$$\frac{\dot{V}_{n,t}}{V_{n,t}} + \frac{\dot{N}_t}{N_t} + \frac{\dot{L}_{x,t}}{L_{x,t}} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = R_t - \rho, \tag{A.2}$$

where in the second equality, we use the fact that $Y_t = C_t$ and in the third equality, we use the Euler equation (3). Substituting (12) and $\dot{N}_t/N_t = \varphi L_{r,t}$ into (A.2) yields $-\Pi_{x,t}/V_{n,t} + \varphi L_{r,t} + \dot{L}_{x,t}/L_{x,t} = -\rho$, which can be rewritten as

$$\varphi L_{r,t} - \varphi \left(\frac{\mu - 1}{1-\gamma}\right) L_{x,t} + \frac{\dot{L}_{x,t}}{L_{x,t}} = -\rho, \tag{A.3}$$

where we use the facts that $\Pi_{x,t}(j) = (\mu - 1) W_t L_{x,t}(j)/(1-\gamma)$ from (9) and (10) and that $\varphi N_t V_{n,t} = W_t$ from (14).

Using the same logic, we combine (10) and (17) to derive $W_t = \phi V_{k,t} K_t = (1-\gamma)\alpha Y_t/(\mu L_{x,t})$. Differentiating it with respect to t yields

$$\frac{\dot{V}_{k,t}}{V_{k,t}} + \frac{\dot{K}_t}{K_t} + \frac{\dot{L}_{x,t}}{L_{x,t}} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = R_t - \rho. \tag{A.4}$$

Substituting (15), (17), and the law of motion for capital such that $\dot{K}_t/K_t = \phi L_{k,t}$ into (A.4) yields

$$\phi L_{k,t} - \phi \left(\frac{\gamma}{1-\gamma}\right) L_{x,t} + \frac{\dot{L}_{x,t}}{L_{x,t}} = -\rho, \tag{A.5}$$

where we use the fact that $Q_t K_t(j) = \gamma W_t L_{x,t}(j)/(1-\gamma)$ from (10) and (11).

Hence, we combine (A.3), (A.5), and the labor-market-clearing condition $L_{x,t} + L_{r,t} + L_{k,t} = 1$ to eliminate $L_{r,t}$ and $L_{k,t}$. After a few steps of manipulation, we obtain an autonomous dynamic system of $L_{x,t}$ as follows:

$$\frac{\dot{L}_{x,t}}{L_{x,t}} = \frac{\phi}{1 + \phi/\varphi} \left[\left(\frac{\mu}{1-\gamma}\right) L_{x,t} - \left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right) \right]. \tag{A.6}$$

Therefore, the dynamics of $L_{x,t}$ is characterized by saddle-point stability such that $L_{x,t}$ jumps immediately to its interior steady-state value given by

$$L_x = \frac{1-\gamma}{\mu}\left(1 + \frac{\rho}{\varphi} + \frac{\rho}{\phi}\right), \tag{A.7}$$

which reveals that the level of manufacturing labor L_x is stationary. Finally, (A.3) and (A.5) imply that if L_x is stationary, then L_r and L_k are also stationary.

Appendix B: Not for Publication

Socially Optimal Allocations

For the first-best optimal allocations in this model, we rewrite the final-goods constraint as $C_t = Y_t = (S_t)^{1-\alpha}(N_t)^{1-\alpha}[A(K_t)^{\gamma}(L_{x,t})^{1-\gamma}]^{\alpha}$. Using this equation, the welfare maximization problem for the social planner is given by the following current-value Hamiltonian:

$$\begin{aligned} H_t =& (1-\alpha)\ln S_0 + (1-\alpha)\ln N_t + \alpha\gamma \ln K_t + \alpha(1-\gamma)\ln L_{x,t} \\ &+ \eta_{1,t}(\varphi N_t L_{r,t}) + \eta_{2,t}(\phi K_t L_{k,t}) + \eta_{3,t}(1 - L_{x,t} - L_{r,t} - L_{k,t}), \end{aligned} \tag{B.1}$$

where $\eta_{1,t}$, $\eta_{2,t}$, and $\eta_{3,t}$ are the costate variables associated with the law of motion for R&D technology, capital production, and labor market, respectively, and we use the fact that $S_t = S_0$ over time. Then, the first-order conditions for $L_{x,t}$, $L_{r,t}$, and $L_{k,t}$ are given by

$$\frac{\partial H_t}{\partial L_{x,t}} = \frac{\alpha(1-\gamma)}{L_{x,t}} - \eta_{3,t} = 0; \tag{B.2}$$

$$\frac{\partial H_t}{\partial L_{r,t}} = \eta_{1,t}\varphi N_t - \eta_{3,t} \leq 0; \tag{B.3}$$

$$\frac{\partial H_t}{\partial L_{k,t}} = \eta_{2,t}\phi K_t - \eta_{3,t} \leq 0; \tag{B.4}$$

$$\frac{\partial H_t}{\partial N_t} = \frac{1-\alpha}{N_t} + \eta_{1,t}\varphi L_{r,t} = \rho\eta_{1,t} - \dot{\eta}_{1,t}; \tag{B.5}$$

$$\frac{\partial H_t}{\partial K_t} = \frac{\alpha\gamma}{K_t} + \eta_{2,t}\phi L_{k,t} = \rho\eta_{2,t} - \dot{\eta}_{2,t}. \tag{B.6}$$

Manipulating (B.5) and (B.6) yields two differential equations such that $\eta_{1,t}\dot{N}_t + \dot{\eta}_{1,t}N_t = \rho\eta_{1,t}N_t - (1-\alpha)$ and $\eta_{2,t}\dot{K}_t + \dot{\eta}_{2,t}K_t = \rho\eta_{2,t}K_t - \alpha\gamma$, implying that $\eta_{1,t}N_t$ and $\eta_{2,t}K_t$ must jump to their steady-state values given by $(1-\alpha)/\rho$ and $\alpha\gamma/\rho$. In this case, the above autonomous dynamic system of either $\eta_{1,t}N_t$ or $\eta_{2,t}K_t$ indicates that saddle-point stability is satisfied. Therefore, this saddle-point stability implies that the underlying welfare level under the present maximization problem is equivalent to the (steady-state) level of welfare under the maximization problem with respect to the lifetime utility of households along the balanced growth path (namely the maximization of (25) subject to the resource constraint of labors.).

If $\varphi/\phi > \alpha\gamma/(1-\alpha)$, then substituting the conditions that $\eta_{1,t}N_t = (1-\alpha)/\rho$ and $\eta_{2,t}K_t = \alpha\gamma/\rho$ into (B.3) and (B.4) yields

$$\eta_{3,t} = \eta_{1,t}\varphi N_t = \frac{\varphi(1-\alpha)}{\rho} > \frac{\alpha\gamma\phi}{\rho} = \eta_{2,t}\phi K_t. \tag{B.7}$$

Substituting (B.7) into (B.2) and (B.4) yields the first-best production labor $L_x^* = \rho\alpha(1-\gamma)/[\varphi(1-\alpha)]$ and the first-best capital labor $L_k^* = 0$. Combining L_x^* and L_k^* with $L_{x,t} + L_{r,t} + L_{k,t} = 1$ yields the first-best R&D labor $L_r^* = 1 - \rho\alpha(1-\gamma)/[\varphi(1-\alpha)]$.

Similarly, if $\varphi/\phi < \alpha\gamma/(1-\alpha)$, then substituting the conditions that $\eta_{1,t}N_t = (1-\alpha)/\rho$ and

$\eta_{2,t}K_t = \alpha\gamma/\rho$ into (B.10) and (B.11) yields

$$\eta_{1,t}\varphi N_t = \frac{\varphi(1-\alpha)}{\rho} < \frac{\alpha\gamma\phi}{\rho} = \eta_{2,t}\phi K_t = \eta_{3,t}. \tag{B.8}$$

Substituting (B.8) into (B.2) and (B.3) yields the first-best production labor $L_x^* = \rho(1-\gamma)/(\varphi\gamma)$ and the first-best R&D labor $L_r^* = 0$. Combining L_x^* and L_r^* with $L_{x,t} + L_{r,t} + L_{k,t} = 1$ yields the first-best capital labor $L_k^* = 1 - \rho(1-\gamma)/(\varphi\gamma)$.

Finally, if $\varphi/\phi = \alpha\gamma/(1-\alpha)$, then we obtain $\eta_{3,t} = \varphi(1-\alpha)/\rho = \alpha\gamma\phi/\rho$, implying that $L_{x,t}$ can equal either $\rho\alpha(1-\gamma)/[\varphi(1-\alpha)]$ or $\rho(1-\gamma)/(\varphi\gamma)$. In this case, the equation system (B.2)-(B.6) loses one condition to pin down the relationship between $L_{r,t}$ and $L_{k,t}$, such that any combination of $\{L_{r,t}, L_{k,t}\}$ satisfying $L_{r,t} + L_{k,t} = 1 - \alpha(1-\gamma)(\rho/\varphi)/(1-\alpha) = 1 - (1-\gamma)(\rho/\phi)/(1-\alpha)$ can be a solution. Without loss of generality, it is assumed that φ/ϕ is either large or small to facilitate the welfare analysis that follows, and thus, the possibility of $\varphi/\phi = \alpha\gamma/(1-\alpha)$ is excluded.

Consequently, the above labor allocations by dynamic welfare maximization imply a corner solution in the first-best allocations. The first-best allocations are efficient in terms of assigning labor to the growth engine that has a larger effect on welfare. In other words, when the impact of the growth engine through R&D (capital accumulation) on welfare is stronger, that is $(1-\alpha)\varphi >$ $(<)\alpha\gamma\phi$, no labor is allocated to the weaker growth engine, yielding $L_k^* = 0$ ($L_r^* = 0$).

References

Barro, R. J., and X. Sala-I-Martin (2004): *Economic Growth*. The MIT Press, Cambridge, MA.

Chen, P.-H., H. Chu, and C.-C. Lai (2015): "Do R&D Subsidies Necessarily Stimulate Economic Growth?," Working paper, Institute of Economics, Academia Sinica.

Chu, A. C. (2009): "Effects of Blocking Patents on R&D: A Quantitative DGE Analysis," *Journal of Economic Growth*, 14(1), 55–78.

Chu, A. C., G. Cozzi, and S. Galli (2012): "Does Intellectual Monopoly Stimulate or Stifle Innovation?," *European Economic Review*, 56(4), 727–746.

Chu, A. C., C.-C. Lai, and C.-H. Liao (2012): "A Tale of Two Growth Engines: The Interactive Effects of Monetary Policy and Intellectual Property Rights," Working paper, Institute of Economics, Academia Sinica.

Chu, A. C., and S. Pan (2013): "The Escape-Infringement Effect of Blocking Patents on Innovation and Economic Growth," *Macroeconomic Dynamics*, 17(04), 955–969.

Falvey, R., N. Foster, and D. Greenaway (2006): "Intellectual Property Rights and Economic Growth," *Review of Development Economics*, 10(4), 700–719.

Furukawa, Y. (2007): "The Protection of Intellectual Property Rights and Endogenous Growth: Is Stronger Always Better?," *Journal of Economic Dynamics and Control*, 31(11), 3644–3670.

Futagami, K., and T. Iwaisako (2007): "Dynamic Analysis of Patent Policy in an Endogenous Growth Model," *Journal of Economic Theory*, 132(1), 306–334.

Goh, A.-T., and J. Olivier (2002): “Optimal Patent Protection in a Two-Sector Economy,” *International Economic Review*, 43(4), 1191–1214.

Horii, R., and T. Iwaisako (2007): “Economic Growth with Imperfect Protection of Intellectual Property Rights,” *Journal of Economics*, 90(1), 45–85.

Iwaisako, T. (2013): “Welfare Effects of Patent Protection and Productive Public Services: Why Do Developing Countries Prefer Weaker Patent Protection?,” *Economics Letters*, 118(3), 478–481.

Iwaisako, T., and K. Futagami (2013): “Patent Protection, Capital Accumulation, and Economic Growth,” *Economic Theory*, 52(2), 631–668.

Jones, C. I., and J. C. Williams (2000): “Too Much of a Good Thing? The Economics of Investment in R&D,” *Journal of Economic Growth*, 5(1), 65–85.

Judd, K. L. (1985): “On the Performance of Patents,” *Econometrica*, 53(3), 567–85.

Laitner, J., and D. Stolyarov (2004): “Aggregate Returns to Scale and Embodied Technical Change: Theory and Measurement Using Stock Market Data,” *Journal of Monetary Economics*, 51(1), 191–233.

Li, C.-W. (2001): “On the Policy Implications of Endogenous Technological Progress,” *Economic Journal*, 111(471), C164–79.

O’Donoghue, T., and J. Zweimüller (2004): “Patents in a Model of Endogenous Growth,” *Journal of Economic Growth*, 9(1), 81–123.

Pan, S., M. Zhang, and H.-F. Zou (2017): “Status Preference and the Effects of Patent Protection: Theory and Evidence,” *Macroeconomic Dynamics*, Forthcoming.

Park, W. G. (2005): *Do Intellectual Property Rights Stimulate R&D and Productivity Growth? Evidence from Cross-National and Manufacturing Industries Data, In: J. Putnam (Ed.), Intellectual Property Rights and Innovation in the Knowledge-Based Economy*. University of Calgary Press, Calgary.

——— (2008): “International Patent Protection: 1960-2005,” *Research Policy*, 37(4), 761–766.

Romer, P. M. (1986): “Increasing Returns and Long-run Growth,” *Journal of Political Economy*, 94(5), 1002–37.

——— (1990): “Endogenous Technological Change,” *Journal of Political Economy*, 98(5), S71–102.

Thompson, M. A., and F. W. Rushing (1996): “An Empirical Analysis of the Impact of Patent Protection on Economic Growth,” *Journal of Economic Development*, 21(2), 700–719.