On the Optimality of IPR Protection with Blocking Patents*

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Abstract

This paper develops a Schumpeterian growth model with overlapping intellectual property rights to analyze the effects of patent protection that features two policy instruments: patent breadth and the profit-division rule between sequential innovators. The former determines the markup and profits of firms whereas the latter determines the degree to which a patent blocks the subsequent invention. Elastic labor supply and subsidies to intermediate goods and research are also considered. The main results are as follows. First, patents and subsidies are substitutable in eliminating the distortions of this model. Second, given the limited use of subsidies in practice, we study optimal patent protection with exogenous subsidies and derive the coordination of patent instruments attaining the social optimum. Third, optimizing only the profit-division rule retains the first-best R&D level and growth, but optimizing only patent breadth could lead to under- or over-investment in R&D; either of these cases is less welfare-enhancing than optimizing their mix. Finally, the model is calibrated to quantify the welfare gains from the decentralized equilibrium to the outcomes with optimal patent instrument(s), and we find that the welfare improvements can be substantially large. Hence, this study sheds some light on the optimal design and welfare implications of a patent system that is multi-dimensional and blocks future innovations.

JEL classification: O31; O34; O38.
Keywords: Blocking patents; Economic growth; Optimal patent instruments; Subsidies

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1 Introduction

This study provides a growth-theoretic analysis to explore the optimality and welfare implications of intellectual property rights (IPR) policy that protects an invention against imitations and subsequent innovations. The latter dimension of protection, which is known as *blocking patents* in the literature, is a crucial component in the current patent system and is ubiquitous in the existing patent law. For example, in the review of the case *Standard Oil Co. v. United States* in relation to the issue of patent protection addressed by the US Supreme Court, patents are considered to be in a (one-way) blocking relationship if the practice of patent B requires a license from patent A, given that B improves (and infringes) A in some capacity. Another typical example of blocking patents is the 1948 case *United States v. Line Material Co.*, in which the patent held by Southern States Equipment Corporation blocked the patent held by Line Material Company, because the latter patent improved on the basic patent. In both cases, the Court recognized the necessity of a licensing agreement providing for the right to practice the new invention and for the division of "royalties" (i.e., the benefit of the technology) between the involved parties (Gilbert (2004)).[1] Therefore, one main feature of blocking patents is that it gives rise to overlapping IPR across sequential innovators due to infringement, and these rights are transferred by licensing as a vehicle (Chu, Cozzi, and Galli (2012)).

The provisions of licensing under blocking patents state that the current researcher must gain permission from previous inventors by sharing a proportion of the profit in order to undertake further innovations (Shapiro (2001)). How this profit is shared between consecutive innovators specifies the degree of blocking patents and is hence referred to as the *backloading effect*.[2] Recently, economists have paid close attention to the impacts of blocking patents on investment in research and development (R&D), economic growth, and social welfare.[3] The pioneering work of O'Donoghue and Zweimüller (2004) presents an endogenous growth model to analyze the backloading effect of blocking patents on innovations. Chu (2009) subsequently quantifies this effect in the US. He shows that eliminating the backloading effect increases R&D incentives and reduces investment distortions, which promotes economic growth and enhances social welfare.[4] In addition, Chu, Cozzi, and Galli (2012) find that a tightening of blocking patents stimulates variety expansion but stifles quality improvement, thus increasing social welfare despite a lower rate of overall growth.[5]

In fact, multiple dimensionality has emerged as a common attribute of the IPR system. Therefore, a comprehensive analysis of the backloading effect of blocking patents should take into account

[1] See more discussion on the use of blocking patents in Merges and Nelson (1990), pp.860-862.

[2] Boldrin and Levine (2009) argue that patents (or intellectual monopoly) have served to prevent future innovations with the following instructive examples: "Boulton and Watt's steam engine patent most likely delayed the industrial revolution by a couple of decades. Selten's automobile patent set back automobile innovation in the United States by roughly the same amount of time. The Wright Brothers airplane patent forced innovative work on airplane technology out of the United States to France. The patent system of England and France forced the chemical industry to move to Germany and Switzerland, where chemical patents did not exist or were much weaker."

[3] Chu (2009) reviews the studies of blocking patents in the literature on patent design and endogenous growth.

[4] Nonetheless, differing from the present paper, O'Donoghue and Zweimüller (2004) and Chu (2009) do not examine the optimal design of patent instruments.

[5] Cozzi and Galli (2014) study a quality-ladder model with endogenous skill acquisition in which the degree of blocking patents determines the share of applied patent value assigned to the basic (upstream) patent holder. They examine the optimal share of basic research and of applied research correcting for the negative externality of the R&D market size.

its connection to other forms of patent protection.[6] In particular, the cases documented by Gilbert (2004) (such as *Standard Oil*, *Line Material*, *Carpet Seaming*, and *New Wrinkle*) show that the Court (namely, patent authority) has been very concerned about the relationship between the use of blocking patents and the level of market power against competitive fringes; patents with a blocking relationship are usually found how their execution does not violate antitrust laws. Moreover, O'Donoghue and Zweimüller (2004) discuss the strong implications of blocking patents on market power consolidation across sequential innovators who face potential imitations. Given that multiple dimensionality of the patent system implies that it features multiple patent-policy levers that can be employed by the policymaker (Chu and Furukawa (2011)), the above examples motivate an in-depth study of the coordinated use of patent levers that determine the degree of blocking patents and market power. However, this topic is relatively less explored in the existing literature on growth and patents.[7]

To make the above argument more specific, we focus on an environment with sequential innovations along the quality ladder. Every innovator is initially an entrant who undertakes successful R&D that upgrades the technology. She then becomes an incumbent once her patent rights are infringed as a result of the arrival of the next innovator, and the entrant and the incumbent enter into a licensing agreement to transfer the patent right. In this situation, the analysis of IPR protection focuses on two policy instruments: *patent breadth* and *the profit-division rule*. On the one hand, the strength of patent breadth indicates the degree of protection for the inventor against potential imitations, thus determining the monopolistic markup and the *amount* of profits generated by the invention. This instrument captures the positive *market-power effect* on R&D efforts. On the other hand, because of the infringement, the profit-division rule under the licensing agreement specifies the distribution of profits between the entrant and the incumbent, thus determining the degree of blocking patents. This instrument delays the income stream of the inventor, which affects the *present value* of profits; it captures the negative backloading effect on R&D efforts. By adjusting the markup and the inventor's profit share, these patent levers can be adopted to affect the resources (i.e., labor in this study) on R&D. At the same time, in the presence of resource constraint, these patent levers also affect the resources on other input factors. As a result of resource reallocation, the growth rate of technology and in turn social welfare will be altered. More importantly, since the allocative effects are different and mixed, the combination of these patent levers can be further utilized for removing more distortions due to misallocation rather than a single lever, which serves a better purpose of restoring the social optimum. Consequently, an accurate investigation of the effects of patent protection should include the optimal use of the above two policy instruments, based on their potentially important implications for growth and welfare.

To properly characterize the above environment, we construct a Schumpeterian-type R&D-based growth model with sequential innovations, where IPR are overlapping and the incumbent extracts from the entrant a proportion of profits in each monopolistic pricing industry. By affecting

[6] Patent reforms in many countries usually involve policy changes for more than one patent lever. See Gallini (2002), Jaffe (2000), and Jaffe and Lerner (2002) for the changes in the US patent policy and Yang and Yen (2009) and Yu (2013) for the changes in the Amendments to the Chinese Patent Laws. Hence, we believe that the patent-policy instruments chosen in this paper and their coordination reflect the usual characteristics of the current patent system and serve as a plausible example, featuring multiple dimensionality for analytical purposes.

[7] Chu (2009) defines patent breadth as the positive dimension of patents and the backloading effect as the negative dimension. Nevertheless, his focus is on the welfare effect of eliminating the backloading. Therefore, his analysis considers only a fixed level of patent breadth without a mixed control on both dimensions of patent protection.

the entrant's R&D incentives, the use of the profit-sharing rule has an impact on the frequency of the arrival of innovations, which captures the *effective patent life* of an innovation. It is believed that this tool serves a better role than the tool *patent length*, as O'Donoghue, Scotchmer, and Thisse (1998) provide survey evidence that in many industries the majority of patents are normally terminated before the end of their statutory lives.[8] Furthermore, unlike the existing studies on patents and growth with fixed labor and no taxes, this model allows for elastic labor supply and lump-sum taxes that finance subsidies to the production of intermediate goods and R&D. In particular, considering elastic labor supply yields one more distortion on the leisure decision (relative to manufacturing) in addition to the distortion on R&D that stems from the externalities commonly discussed in the growth literature. Nonetheless, these distortions can be (completely) alleviated by the setting of dual patent levers in the presence of the subsidy levers. These specifications are critical in this study because they make the optimization of the market-equilibrium outcome possible, enriching the welfare analysis of the policy variables.

Previous studies in economic-growth patent policy have examined the welfare effects of blocking patents and patent breadth separately, but not their combination in one policy regime. These studies have also investigated the optimal coordination of other patent instruments, but not in relation to blocking patents.[9] Therefore, the novel contribution of this paper is to (a) review the welfare-maximizing design for a two-dimensional patent-protection policy in which the patent rights feature both the negative backloading effect and the positive market-power effect on innovations, in order to fully capture the growth and welfare implications of blocking patents; and (b) quantitatively decompose and compare the respective welfare gain of optimizing the profit-sharing rule and patent breadth.

The findings of this study are summarized as follows. First, we show how patent-policy instruments and subsidy-policy instruments interact to achieve the first-best optimal outcome. Specifically, patent breadth and intermediate-goods subsidies are substitutable in eliminating the distortion on the ratio of leisure and production labor, whereas the profit-division rule and research subsidies are substitutable in eliminating the distortion on the relative allocation of R&D labor. Second, given that subsidy policy is more difficult to implement than patent policy in reality and is thereby taken as exogenous, we derive an optimal mix of patent breadth and the profit-division rule to restore the first-best allocations. Third, optimizing only the rule of profit division in equilibrium induces the economy to reach the first-best growth rate, because it helps to allocate the first-best level of R&D labor against other labor inputs. Nevertheless, optimizing only patent breadth may lead to a higher or lower R&D level (and growth rate) than the first-best one. As expected, the underlying welfare level of optimizing a single patent-policy lever would be strictly lower than the counterpart of optimizing both levers. Additionally, our quantitative analysis reveals that starting from the decentralized equilibrium, the welfare improvement by optimizing only the degree of blocking patents is relatively small, but the welfare gain by optimizing a coordination of the patent instruments is substantial and can be as large as 3.4% of consumption. Therefore, most of the welfare gain comes from optimizing patent breadth. These results highlight the importance of the

[8]Chu (2010) also shows that effects of patent length on R&D may be negligible in an innovation-based growth model.

[9]See, for instance, Kwan and Lai (2003) and Iwaisako and Futagami (2013) for the effects of patent breadth. See also the Literature Review in this study for the effects of blocking patents and the optimal mix of patent instruments without the blocking feature.

coordinated use of the profit-division rule and patent breadth in increasing social welfare. Finally, extensions are considered with a *fishing-out* effect on innovations and with physical capital accumulation in a framework of semi-endogenous growth put forward by Jones (1995) and Segerstrom (1998). With these modifications that dispose of *scale effects*, the above theoretical results would be robust, but the sizes of welfare comparisons become much more significant; the patent instruments are even more welfare-enhancing under these realistic settings.

1.1 Literature Review

This study contributes to two strands of theoretical literature discussing the optimal mix of patent instruments. In the literature on patent design, Gilbert and Shapiro (1990) and Klemperer (1990) examine optimal patent breadth and length, and demonstrate that optimal patents should be either very narrow and long-lived or very broad and short-lived. In the literature on economic growth and patent protection, Iwaisako and Futagami (2003) investigate the impact of patent-policy levers, including compulsory licensing and patent length, on social welfare in an endogenous growth model. Chu and Furukawa (2011) analyze the optimal mix of patent instruments in a case where R&D firms undertake innovations in research joint ventures (RJVs). They show that optimizing both patent breadth and the profit-division rule between the R&D partner firms is necessary to attain the social optimum. However, in order to characterize the infringement between sequential innovators caused by overlapping patent rights, our paper revisits a mix of patent instruments in relation to the backloading effect that interferes with future innovations. Specifically, our analysis of patent levers focuses on the profit-sharing rule between the incumbent and the entrant along the quality ladder in addition to patent breadth against imitations from competitive fringes. To the best of our knowledge, this is the first study that analyzes the optimal coordination of patent levers in a growth-theoretic framework deterring subsequent inventions.

Furthermore, this paper is closely related to some recent research on blocking patents, such as Chu, Cozzi, and Galli (2012) and Chu and Pan (2013), but there are significant differences between their studies and ours. First, in a quality-ladder fashion, Chu, Cozzi, and Galli (2012) investigate the growth and welfare implications of blocking patents that have asymmetric effects on vertical and horizontal innovations. They find that a welfare-maximizing profit-division rule would exist and both a gradual increase and an immediate increase in the degree of blocking patents could improve social welfare. Their focus, however, is on the optimality of only one patent-policy tool (namely, the profit-division rule). In contrast, this study expands the dimensionality of the IPR system that involves blocking patents by controlling an extra patent instrument (i.e., patent breadth). We then highlight the underlying benefit, which is reflected by the welfare improvement from optimizing only one patent-policy lever (i.e., the second-best outcome) to optimizing both patent-policy levers (i.e., the first-best outcome). Second, Chu and Pan (2013) consider both the degree of blocking patents and patent breadth as policy variables in the case of an endogenous step size of innovation in a Schumpeterian growth model. In particular, they identify the interesting (escape-)infringement effect (i.e., a non-monotonic effect) of blocking patents on innovations and economic growth. Apart from the growth effect, it is also important to explore the optimal mix and the welfare effects of these patent-policy levers. Our study fills this gap, both analytically and quantitatively, by examining the design of the welfare-maximizing patent instruments and their policy implications based on the effects on remedying the distorted input allocations. Third, this study takes into account an elastic labor supply, which plays a critical role in our model in affecting the use of policy instruments that leads to the social optimum. Nevertheless, the impact of this factor is neglected in the above studies. Thus, this paper serves to complement the quality-ladder growth models by allowing for a thorough welfare analysis of IPR protection policy with blocking patents.

This paper is also related to the existing studies of R&D-based growth models with subsidies to intermediate goods and research.[10] In a semi-endogenous growth framework with variety expansion, Grossmann, Steger, and Trimborn (2013) prove that the optimal growth path can be implemented as a market equilibrium with a suitable choice of a constant subsidy to (intermediate-goods) production and a time-varying subsidy to R&D. They also compare the welfare losses of implementing the long-run optimal R&D subsidies rather than the dynamically optimal ones, which are found to be negligible.[11] Nuño (2011) analyzes optimal long-run subsidies financed by a lump-sum tax on households in a Schumpeterian growth model with business cycles, showing that subsidies to capital costs (i.e., costs of intermediate-goods production) and those to research are two complementary tools to recover the first-best allocations. This paper thereby differs from these studies by revealing the relationship between subsidy policy and patent policy (namely perfect substitutability) in the social optimum. In particular, our results show that when it is difficult to adjust subsidy policy for practical reasons, patent policy with a blocking relationship for sequential innovations is an alternative regime that can fully steer the market economy to achieve the first-best outcome in the presence of subsidization. Moreover, this study undertakes a policy experiment that demonstrates that subsidy policy is numerically less effective than patent policy in terms of increasing social welfare.

Lastly, given that the backloading effect of blocking patents in this analysis is embodied by the transfer of permission for production from the previous innovator to the current one by means of licensing, the present paper relates to the literature dealing with patent licensing in models of sequential innovations. Early studies, such as Green and Scotchmer (1995) and Scotchmer (1996), consider a two-stage model in which patent rights that are infringed operate through licensing to transfer profits to the prior inventor. However, as the value of innovations in their models is simply determined by the statutory patent life, the division of profit between consecutive innovators in the licensing agreement is not their focus. The importance of the profit-division rule in the present study differs from theirs, because this rule in our model affects the effective patent life, which is critical to pin down the value of innovations. In a more recent study, Bessen and Maskin (2009) analyze the effects of patent protection through licensing on sequential innovations where inventions are in the form of differentiated products building on their predecessors. Nonetheless, our study analyzes this effect where inventions are in the form of an infinite sequence of quality improvements, as in O'Donoghue, Scotchmer, and Thisse (1998). In addition, the current paper adds to this literature by focusing on the role of the transfer of patent licensing in a (dynamic-) general-equilibrium framework instead of in a partial-equilibrium framework, as in the above interesting studies.

[10]Barro and Sala-I-Martin (2003) and Acemoglu (2009) claim that in the Romer (1990) model with expanding varieties but nonadjustable patent policy, subsidies to research and those to intermediate goods are Pareto-improving interventions, by stimulating economic growth and by eliminating allocation inefficiencies, respectively.

[11]See also Zeng and Zhang (2007) for the growth and welfare implications in the Barro and Sala-I-Martin (2003) type of expanding-variety model with elastic labor supply, in which the purchase of intermediate goods and the costs of research are subsidized by labor income taxes. However, neither a single subsidy nor the mix of subsidies can achieve the social optimum in their analysis.

The rest of this paper is organized as follows. Section 2 introduces the model setup. Section 3 characterizes the decentralized equilibrium. Section 4 derives the first-best optimal outcome and explores the interrelation of all policy instruments in the social optimum. Section 5 studies first-best patent protection in which a mix of patent levers is used and the second-best outcomes in which some patent levers are fixed. Section 6 quantitatively compares the welfare differences between the decentralized equilibrium and the optimal outcomes in a calibrated economy. Section 7 considers extensions with semi-endogenous growth and physical capital accumulation. Section 8 concludes this study.

2 The Model

To analyze the growth and welfare implications of patent protection in a multiple-dimensional form, we extend the quality-ladder model of Grossman and Helpman (1991) to incorporate two patent instruments. The first is patent breadth, which determines the price-marginal-cost markup in each intermediate-goods sector, and the second is a profit-division rule between sequential innovators along the quality ladder. In order to obtain the compulsory licensing for producing, the current innovator (i.e., the entrant) must transfer a share $s \in [0, 1]$ of her profit to the previous innovator (i.e., the incumbent) as a licensing fee. This rule of profit division is assumed to be an exogenous bargaining agreement between the innovators, which is affected by patent policy, as in the existing literature (e.g., O'Donoghue and Zweimüller (2004), Chu (2009), and Chu, Cozzi, and Galli (2012)). The model also introduces a leisure-consumption decision and a lump-sum tax financing subsidies to intermediate goods and research. The policymaker (i.e., the government) is firstly allowed to control a mix of policy levers (including patents and subsidies) in order to close the gap between the decentralized equilibrium and the optimal allocations. Then, special cases are considered to explore optimal patent protection by taking some policy variables as exogenous.

2.1 Households

The economy admits a unit continuum of identical households, whose lifetime utility is

$$U = \int_0^\infty e^{-\rho t} \left(\ln C_t + \phi \ln L_t \right) dt, \tag{1}$$

where $\rho > 0$ is the discount rate, C_t is households' consumption of final goods, and L_t is the leisure at time t. The parameter $\phi > 0$ determines the intensity of leisure preference relative to consumption. There is no population growth in the economy, and each household is endowed with one unit of time that is allocated between leisure and labor supply. Thus, the law of motion for households' total assets is

$$\dot{V}_t = R_t V_t + W_t(1 - L_t) - P_t C_t - T_t, \tag{2}$$

where W_t denotes the wage rate that will be normalized to unity, V_t is the value of households' assets, R_t is the nominal interest rate, P_t is the price of final goods, and T_t is a lump-sum tax imposed by the government for subsidization.[12] Maximizing households' utility subject to (2)

[12]Note that the tax specified in (2) and (16) is a non-distortionary transfer that neither creates qualitative alterations on the dynamic behavior of the economy in (4) nor changes households' leisure-consumption decision (3). In

yields

$$W_t L_t = \phi P_t C_t. \tag{3}$$

Households' optimization problem also implies the usual Euler equation

$$\frac{\dot{E}_t}{E_t} = R_t - \rho, \tag{4}$$

where $E_t \equiv P_t C_t$ is the (nominal) consumption expenditure. Moreover, the households own a balanced portfolio of all firms in the economy. Finally, the transversality condition, $\lim_{t\to\infty}[V_t \exp(-\int_0^t R_\vartheta d\vartheta)] = 0$, implies that neither asset nor debt will remain at the end of the planning horizon.

2.2 Final Goods

Final goods Y_t are produced competitively using a unit continuum of fully depreciated intermediate goods indexed by $i \in [0,1]$, according to the standard Cobb-Douglas production function

$$\ln Y_t = \int_0^1 \ln X_t(i) di, \tag{5}$$

where $X_t(i)$ is the quantity of intermediate good i. With free entry and profit maximization, (5) implies the following demand for intermediate i:

$$X_t(i) = \frac{P_t Y_t}{P_t(i)}, \tag{6}$$

where $P_t(i)$ is the price of intermediate i and $P_t = \exp\left(\int_0^1 \ln P_t(i) di\right)$ is the price of final goods.

2.3 Intermediate Goods

The intermediate good in each industry i is produced by a monopolistic leader holding a patent on the latest innovation. Because of the *Arrow replacement effect*, the industries of the intermediate goods are replaced by innovations that are conducted by new entrants. The current leader's production function is

$$X_t(i) = z^{q_t(i)} L_{x,t}(i), \tag{7}$$

where the parameter $z > 1$ measures the step size of each quality improvement, $q_t(i)$ is the number of innovations between time 0 and time t, and $L_{x,t}(i)$ is production labor in industry i. Then, (7) implies that the marginal cost of producing an intermediate good is

$$MC_t(i) = \frac{\alpha_t W_t}{z^{q_t(i)}}, \tag{8}$$

contrast, if subsidization is financed by a tax on either consumption or labor incomes, then the tax rate is an extra endogenous variable in (3) and (4), and it needs to be determined when solving for the equilibrium labor allocations. This setting will further complicate the optimal design of patent instruments without adding new insight on their roles in removing the allocative distortions in this model. See also Footnote 20 for more discussion.

where $1 - \alpha_t \in (0, 1)$ is a subsidy rate proportional to the marginal cost. In our framework, subsidization of intermediate-goods producers will play a similar role as in Grossmann, Steger, and Trimborn (2013, 2016) where monopolistic profits are subsidized due, e.g., to the presence of a cost deduction system.[13]

Next consider our first patent instrument. Following Li (2001), Goh and Olivier (2002), and Iwaisako and Futagami (2013), we assume that current leader's markup $\mu_t > 1$ is a policy instrument that can be set by the policymaker as patent breadth. Therefore, standard Bertrand price competition leads to the monopolistic price given by

$$P_t(i) = \mu_t MC_t(i), \tag{9}$$

which is the limit price of the current leader against competitive fringes who undertake potential imitations.[14] In the original Grossman-Helpman setting, μ_t is assumed to equal one step size z of innovation, and α_t is assumed to be unity implying that there are no subsidies to intermediate goods. Finally, the leader's profit is given by

$$\Pi_t(i) = \left(1 - \frac{1}{\mu_t}\right) P_t(i)X_t(i) = (\mu_t - 1)\alpha_t W_t L_{x,t}(i), \tag{10}$$

where the second equality is obtained by substituting (7)-(9) into $\Pi_t(i)$. Observe that patent breadth μ_t and intermediate-goods subsidies α_t are the policy instruments that directly affect the *amount* of the monopolistic profits created by innovations.

2.4 Innovations and R&D

Following Chu, Cozzi, and Galli (2012) and Chu and Pan (2013), we assume that the most recent innovator (i.e., the entrant) infringes the second most recent innovator (i.e., the incumbent). Thus, the second patent instrument, a profit-division rule s_t, is introduced to serve as an agreement for transferring the licensing for production between two sequential innovators.[15] Specifically, this profit-sharing agreement, which is also affected by patent policy, allows the most recent innovator and the second most recent innovator to bargain over the profits accrued from the invention in (10).

First, the value of owning the second most recent innovation in industry i is denoted as $V_{2,t}(i)$.

[13] See also Leith and Wren-Lewis (2013) for an employment subsidy to intermediate-goods firms that eliminates the steady-state distortions associated with monopolistic competition and helps evaluate optimal monetary and fiscal policy in a New Keynesian economy, assuming that a lump-sum tax finances such a subsidy.

[14] As in Howitt (1999) and Segerstrom (2000), it is assumed that, once the incumbent stops production and leaves the market, she cannot threaten to reenter. Therefore, under incomplete patent breadth, the presence of monopolistic profits attracts potential imitations by competitive fringes, whose marginal cost of producing the same intermediate good is assumed to be higher than current leaders'. As a result, to prevent competitive fringes' entry, the current leaders' price is limited by the cost of imitations under Bertrand competition; thus, stronger patent breadth effectively raises the fringes' marginal cost for imitations, allowing monopolistic producers to charge a higher markup without the threat of imitations. See Chu and Cozzi (2014) for more discussion on this standard assumption of patent breadth in a quality-ladder growth model.

[15] O'Donoghue and Zweimüller (2004) and Chu (2009) study a more general set of profit-sharing rules assuming that the current innovator may infringe the patents of numerous former innovators. However, in this study, considering the simple case of profit division between the entrant and the incumbent yields a closed-form solution for the first-best degree of blocking patents, facilitating the analysis of the optimal coordination with patent breadth and that of interactions with subsidies.

Following the standard literature, we focus on a symmetric equilibrium (see, for example, Cozzi, Giordani, and Zamparelli (2007)) such that $\Pi_t(i) = \Pi_t$ and $V_{2,t}(i) = V_{2,t}$ for $i \in [0,1]$. Denote by λ_t the *aggregate-level* Poisson arrival rate of innovations, which determines the effective patent life of an innovation. Then, the Hamilton-Jacobi-Bellman (HJB) equation for $V_{2,t}$ is

$$R_t V_{2,t} = s_t \Pi_t + \dot{V}_{2,t} - \lambda_t V_{2,t}, \tag{11}$$

which is the no-arbitrage condition for the asset value. In equilibrium, the return on this asset $R_t V_{2,t}$ equals the sum of the flow payoffs $s_t \Pi_t$ due to infringement, the capital gain $\dot{V}_{2,t}$, and the potential losses $\lambda_t V_{2,t}$ because of creative destruction.

Similarly, denote by $V_{1,t}(i)$ the value of holding the most recent innovation in industry i. Then, the law of motion for $V_{1,t}$ in equilibrium is the following no-arbitrage condition:[16]

$$R_t V_{1,t} = (1 - s_t)\Pi_t + \dot{V}_{1,t} - \lambda_t \left(V_{1,t} - V_{2,t}\right). \tag{12}$$

The difference between the no-arbitrary condition (11) and (12) is that when the next innovation arises, the current innovator loses $V_{1,t}$ but gains $V_{2,t}$, since she becomes the second most recent innovator.

New innovations in each industry are invented by a unit continuum of R&D firms indexed by $j \in [0,1]$. Each of these firms employs R&D labor $L_{r,t}(j)$ for producing inventions subject to subsidization. The expected profit of the j-th R&D firm is

$$\pi_t(j) = V_{1,t}\lambda_t(j) - \sigma_t W_t L_{r,t}(j), \tag{13}$$

where $1 - \sigma_t \in (0,1)$ is a subsidy rate proportional to the research cost given that Impullitti (2010) shows that the subsidy rate to R&D investment in many OECD countries is positive. The *firm-level* arrival rate of innovations $\lambda_t(j)$ is given by

$$\lambda_t(j) = \varphi L_{r,t}(j), \tag{14}$$

where $\varphi > 0$ is R&D productivity. In equilibrium, the aggregate-level arrival rate of innovations equals the firm-level counterpart, namely $\lambda_t = \lambda_t(j)$. Then, free entry into the R&D sector implies the following zero-expected-profit condition:

$$\varphi V_{1,t} = \sigma_t W_t, \tag{15}$$

This equation is one condition pinning down the labor allocations among leisure, production, and R&D. Observing (11), (12), and (15) reveals that the profit-division rule s_t and R&D subsidies σ_t are the policy instruments that affect the *present value* of profits created by innovations.

2.5 Government Budget

Suppose that the policymaker can also have access to subsidies to intermediate goods and to research by choosing the subsidy rates $1 - \alpha_t$ and $1 - \sigma_t$, respectively. These subsidies are financed

[16] See Chu (2009) for the proof of how the value of blocking patents evolves when there are multiple innovations along the quality ladder.

by the lump-sum tax levied on the households such that

$$T_t = (1-\alpha_t)W_t L_{x,t} + (1-\sigma_t)W_t L_{r,t}, \tag{16}$$

where the left-hand side is the tax revenues collected from the households and the right-hand side is the expenditures on subsidizing production of intermediate goods and research. Hence, in this model the government can implement the IPR-policy instruments through patent authority and the subsidy-policy instruments through fiscal authority in order to affect the input allocations and steer the market economy.

3 Decentralized Equilibrium

An equilibrium consists of a sequence of allocations $[C_t, Y_t, X_t(i), L_t, L_{x,t}, L_{r,t}]_{t=0,i\in[0,1]}^{\infty}$ and a sequence of prices $[P_t(i), R_t, P_t, W_t, V_{1,t}, V_{2,t}, V_t]_{t=0,i\in[0,1]}^{\infty}$. Moreover, in each instant of time,

- households choose $[C_t, L_t]$ to maximize their utility given $[R_t, P_t, W_t]$;
- final-goods producers choose $[Y_t]$ to maximize profits given $[P_t, P_t(i)]$;
- monopolistic leaders for intermediate goods produce $[X_t(i)]$ and choose $[P_t(i), L_{x,t}]$ to maximize profits given $[W_t]$;
- R&D firms choose $[L_{r,t}]$ to maximize profits given $[W_t, V_{1,t}]$;
- the goods market clears such that $C_t = Y_t$;
- the labor market clears such that $L_t + L_{x,t} + L_{r,t} = 1$; and
- the values of innovations add up to households' assets value such that $V_{1,t} + V_{2,t} = V_t$.

3.1 Equilibrium Allocations

In this subsection, we define the decentralized equilibrium and show that the economy jumps to a uniquely stable balanced growth path (BGP). To ensure that R&D labor is nonnegative, we impose a lower bound on the R&D productivity parameter φ for an arbitrary path of patent breadth and the profit-division rule $[\mu_t, s_t]_{t=0}^{\infty}$ and an arbitrary path of subsidy rates $[\alpha_t, \sigma_t]_{t=0}^{\infty}$, such that

Assumption 1. $\varphi > \frac{\rho\sigma_t(1+\alpha_t\phi\mu_t)}{\alpha_t(\mu_t-1)(1-s_t)}$.

Hence, we obtain the following result.

Proposition 1. *Suppose that Assumption 1 holds. Then holding constant μ, s, α, and σ, the economy jumps to a unique and stable balanced growth path.*

Proof. See Appendix A. □

Proposition 1 demonstrates that, given a stationary time path of the policy levers, the consumption expenditure and the equilibrium labor allocations are stationary along the BGP. Then, using (3) and (10) in the labor-market-clearing condition yields the equilibrium level of leisure and of labor inputs such that

$$L = \phi E, \tag{17}$$

$$L_x = E/(\alpha\mu), \tag{18}$$

$$L_r = 1 - (\phi + 1/(\alpha\mu))\, E, \tag{19}$$

where the detailed derivations for E and the above expressions are shown in Appendix A.

Using (5) and (7), we derive $Y_t = Z_t L_{x,t}$, where Z_t is defined as the aggregate technology, such that $\ln Z_t \equiv \ln z \int_0^1 q_t(i)di = \ln z \int_0^t \lambda_\iota d\iota$. Differentiating this equation with respect to time yields the growth rate of technology, namely $g_t = \dot{Z}_t/Z_t = \lambda_t \ln z$.

3.2 Growth Effects of Policy Instruments

To facilitate the subsequent welfare analysis, we investigate the effects of policy variables/parameters on economic growth by examining the equilibrium arrival rate of innovations (λ) as in Chu and Pan (2013). Suppose that the instruments μ, s, α, and σ are imposed exogenously. From Proposition 1, we know that $\dot{V}_{1,t} = \dot{V}_{2,t} = \dot{V}_t = 0$. Using this fact with the stationarity of E_t yields that $V_2 = s\Pi/(\rho+\lambda)$ and $V_1 = ((1-s)\Pi + \lambda V_2)/(\rho+\lambda)$ in equilibrium. Combining (10) and (15) gives

$$\frac{\varphi}{\sigma}\left[(1-s) + \frac{s\lambda}{\rho+\lambda}\right]\frac{\alpha(\mu-1)}{\rho+\lambda} = \frac{1}{L_x}. \tag{20}$$

Further, using (3), (16), and the fact $\dot{V}_t = 0$ implies that (2) becomes $L = \phi\left[\frac{\rho\sigma}{\varphi}\left(1+\frac{V_2}{V_1}\right) + \alpha L_x + \sigma L_r\right]$. Substituting (11), (12), (14) and $L + L_x + L_r = 1$ into this equation yields

$$L_x = \frac{1}{\varphi(1+\alpha\phi)}\left[\varphi - \phi\sigma\rho\left(\frac{\rho+\lambda(1+s)}{\lambda+\rho(1-s)}\right) - \lambda(1+\sigma\phi)\right]. \tag{21}$$

Therefore, combining (20) and (21) and rearranging it gives the expression that determines the equilibrium level of λ:

$$-\frac{\alpha}{\sigma(1+\alpha\phi)}\left[(1+\sigma\phi)\lambda^2 + (-\varphi - \rho(1-s+2\sigma\phi))\,\lambda - \rho(-\sigma\phi\rho + \varphi(1-s))\right] = \frac{(\rho+\lambda)^2}{\mu-1}, \tag{22}$$

where both the left-hand side (LHS) and the right-hand side (RHS) are quadratic functions of λ. In Appendix A, we show that the uniqueness of the equilibrium level of λ is ensured by Assumption 1. Then, by illustrating the changes of LHS and RHS in (22), we obtain the following result.

Lemma 1. *Suppose that μ, s, α, and σ are given exogenously. Then the equilibrium growth rate is increasing in μ and α but decreasing in s and σ.*

Proof. See Appendix A. □

Intuitively, on the one hand, a larger patent breadth μ increases the monopoly markup that a current leader in the intermediate-goods sector can charge over the marginal cost, which raises the profits of innovations and yields more incentives to invest in R&D. A lower subsidy rate $1-\alpha$ increases the marginal cost of producing intermediate goods in (8). With a fixed markup, this effectively increases the monopolistic price $P_t(i)$ in (9), and consequently reduces the demand for intermediate goods $X_t(i)$ in (7). These changes imply a reallocation in labor from manufacturing to R&D. On the other hand, a lower σ decreases the cost of research, and a lower level of profit division s implies a reduction in the backloading effect of blocking patents with smaller licensing fees transferred from the current innovator to the previous one. These changes raise the incentives for R&D, and again more labor is shifted to conducting research activities. Based on the above analysis, there will be a higher level of R&D labor in equilibrium as a result. Hence, (14) implies that the economy exhibits a higher arrival rate of innovations leading to a higher rate of economic growth. These comparative statics for μ and s are consistent with those in O'Donoghue and Zweimüller (2004) and Chu and Pan (2013), and the counterparts for α and σ are analogous to those in Barro and Sala-I-Martin (2003) and Zeng and Zhang (2007).

4 Optimal Combination of Policy Instruments

In this section, we discuss the interrelations of the policy instruments (including patents and subsidies) when the government (i.e., the policymaker/social planner) uses their mix to replicate the first-best optimal outcome.

Given the saddle-point stability of the model under a stationary path of policy variables shown in Proposition 1, the economy is always on a BGP, along which the equilibrium labor allocations $\{L, L_x, L_r\}$ are stationary and the growth rate of technology $g = \lambda \ln z$ is also stationary. Furthermore, consumption, final goods, and technology grow at the same rate. Therefore, imposing the BGP, we integrate the households' lifetime utility in (1) to reexpress it as follows:

$$U = \frac{1}{\rho}\left(\ln C_0 + \phi \ln L + \frac{g}{\rho}\right), \tag{23}$$

where $C_0 = Z_0 L_x$ and $g = \lambda \ln z$. Dropping the exogenous term Z_0 and maximizing (23) subject to the labor-market-clearing condition $L + L_x + L_r = 1$ yields the first-best optimal labor allocations $\{L^*, L_x^*, L_r^*\}$:

$$L^* = \frac{\rho\phi}{\varphi \ln z}, \tag{24}$$

$$L_x^* = \frac{\rho}{\varphi \ln z}, \tag{25}$$

$$L_r^* = 1 - (1+\phi)\frac{\rho}{\varphi \ln z}, \tag{26}$$

where $\varphi > \rho(1+\phi)/\ln z$ ensures that the optimal R&D labor is positive. In Appendix A, it is shown that saddle-point stability is still satisfied when the economy attains the social optimum. Accordingly, the policy instruments $\{\mu, s, \alpha, \sigma\}$ can be applied to adjust the equilibrium labor allocations in (17)-(19) to restore the first-best allocations in (24)-(26).

Comparing the equilibrium labor allocations to the first-best counterparts reveals that the inefficiencies in the decentralized setting arise from two layers of distortions. The first distortion is present in the ratio of leisure and production labor L/L_x (which also defines the inverse supply of labor in manufacturing terms and thereafter relative labor supply). This ratio equals the leisure preference parameter ϕ in the social optimum where no policy interventions are involved, whereas it equals $\mu\alpha\phi$ in equilibrium where both patent breadth μ and the (inverse) subsidy rate for intermediate inputs α are involved in addition to ϕ. Specifically, when $\mu\alpha > (< 1)$, the ratio L/L_x in equilibrium becomes higher (lower) than in the first-best outcome, giving rise to the allocative inefficiencies. Therefore, setting these policy levers to satisfy $\mu\alpha = 1$ eliminates this layer of distortion. Notice that this distortion does not appear when labor supply is inelastic (i.e., $\phi = 0$).

Upon the removal of this distortion, the optimal interaction of patent breadth and intermediate-goods subsidies demonstrates that they are perfectly substitutable, in the sense that a larger μ has the opposite impact as a higher $1-\alpha$. On the one hand, a lower (inverse) subsidy α decreases the marginal cost of intermediate-goods production, increasing the demand for manufacturing labor and making the ratio of L/L_x below the first-best level ϕ. Hence, using a larger patent breadth μ to price at a higher monopoly value helps to decrease the excessive demand in L_x by reducing the labor-income share of output. In this circumstance, granting more monopoly rights by reinforcing the market-power effect of patent protection preserves the appropriate incentives for inventors to create products with higher qualities, and it suffices to correct the efficiency loss resulting from labor misallocation in leisure relative to production. On the other hand, a larger patent breadth μ enlargers the difference between the monopoly price of the intermediate goods and the marginal cost of production (i.e., the markup), thus decreasing the supply of manufacturing labor and making the ratio of L/L_x above the first-best level. Thus, the government can engineer a higher subsidy $1-\alpha$ to induce marginal-cost pricing for increasing L_x, which counteracts the impact of patent breadth and removes this distortion.[17] The latter policy implementation on subsidy reflects the conventional view in the endogenous-growth literature for eliminating the monopolistic distortion (e.g., as in Barro and Sala-I-Martin (2003) and Zeng and Zhang (2007)).

The second distortion is present in the allocation of research labor L_r relative to other labor inputs. Given that setting $\mu\alpha = 1$ holds the optimal ratio of L and L_x, the profit-division rule s and the (inverse) R&D subsidy rate σ are the feasible policy instruments that can adjust the equilibrium level of R&D labor. Specifically, when the values of s and σ induces $L_r(s,\sigma)|_{\mu\alpha=1} > (<) L_r^*$, too much (little) R&D investment is realized in the decentralized equilibrium, again giving rise to allocative inefficiencies. Therefore, if the choices of these policy levers satisfy $L_r(s,\sigma)|_{\mu\alpha=1} = L_r^*$, then this layer of distortion is also eliminated.

Upon the removal of this distortion, the optimal interaction of the profit-division rule and R&D subsidies demonstrates that they are again perfectly substitutable, in the sense that a higher s has the opposite impact as a higher $1-\sigma$. The reason is straightforward. A lower σ increases the subsidization on the research cost, and Lemma 1 implies that this will lead the equilibrium level of R&D to rise above the first-best level. To remove the inefficiency due to this bias on R&D, a higher s is imposed to increase the payoffs transferred from the entrant to the incumbent in the licensing agreement, thus reducing the research incentives and the resulting R&D level. In contrast, a higher s amplifies the backloading effect of blocking patents, which will lead the equilibrium level of R&D to fall below the first-best level. In this case, more research subsidies are needed to raise the R&D investment that corrects this distortion by placing a lower value of σ. Notice that the

[17] With regard to the evidence of subsidies to intermediate goods, Edge, Laubach, and Williams (2010) argue that in practice there may not exist such subsidies that correct distortions associated with firms' market power, because "in the real world the steady-state level of output is inefficient." They instead use the time-varying elasticity of substitution between the production inputs in their model to identify the subsidy rate to intermediate inputs. However, Gómez and Sequeira (2014) claim that these subsidies are analogous to subsidies to physical capital costs supported by the investment tax credit. Similar systems remain in countries such as United Kingdom and Australia, although this scheme (of 10% capital costs deduction) was abolished in the US in 1986. Grossmann, Steger, and Trimborn (2013, 2016) then provide quantitative evidence of a behaviorally relevant subsidy to capital costs in the US (namely, a positive subsidy rate to intermediate-goods production) to restore the first-best outcome in the steady state. Nuño (2011) also finds a similar result for the optimality of subsidies to intermediate goods by calibrating his model to match key empirical evidence for the US economy in the postwar period 1950–2007.

remedy for this R&D distortion can also be related to the interaction of all policy instruments since it is possible to adjust all instruments simultaneously to achieve L_r^*. For instance, when a higher μ or α increases L_r according to Lemma 1, a higher s or σ can depress L_r to meet the optimal level L_r^* and vice versa. Nonetheless, unless $\mu\alpha = 1$ is satisfied, the combination of all these tools that eliminates the R&D distortion does not lead to the first-best outcome because L/L_x is distorted.

To summarize, the policymaker can implement the policy tools that affect the amount of monopolistic profits created by innovations (i.e., μ and α) and the policy tools that affect the value of these profits (i.e., s and σ) to satisfy $\mu\alpha = 1$ and $L_r(s,\sigma)|_{\mu\alpha=1} = L_r^*$ to remove the inefficiencies present in the decentralized equilibrium. However, the substitutablity of patent policy and subsidy policy implies that fixing one policy regime, an optimal mix of either patent instruments or subsidy instruments will suffice to help recover the first-best outcome, given that a pair of the instruments in these regimes plays effectively the same role in allocating the labor inputs.

5 Optimal Patent Protection

In this section, first, we show evidence that the use of subsidization is more restricted than the use of patent protection. Second, we study first-best optimal patent protection in which an appropriately joint choice on patent breadth and the profit-division rule is made. Third, we analyze the second-best cases in which only one patent lever can be varied whereas the other patent lever is fixed at some predetermined level. Finally, we consider optimal patent policy under the special cases with inelastic labor supply and with a complete frontloading profit-sharing agreement, respectively.

5.1 Patents vs. Subsidies

In the remaining analysis, fiscal authority (and its subsidy policy) is taken as given, whereas the role of patent authority (and its patent policy) is on the main focus. This choice is supported by three caveats around the use of subsidies.

First, the financing system for subsidies in this model relies on a non-distorting lump-sum tax. However, in a more realistic situation, the tax T_t in (2) may be limited by an upper bound, implying that the subsidy rates would remain at a relatively low level to balance the governmental budget in (16). Furthermore, when some targets, such as fiscal commitments and international agreements, have to be met (see Woodford (2001) for the example of "the Maastricht treaty"), these constraints would make subsidy policy hard to alter either. Therefore, interventions in subsidies would be more difficult for the government to execute than those in patents, especially when tax revenues are scarce.[18]

Second, O'Donoghue and Zweimüller (2004) argue that in more practical terms, R&D subsidies may be inferior to patents because of asymmetric information between researchers and governments; the successful execution of research-input subsidies would be difficult if it is flexible for firms to claim their R&D costs (see Lichtenberg (1992)). Nevertheless, patent rights are better specified, given that they are awarded once a firm obtains an actual invention.[19]

[18]See Acemoglu, Aghion, and Zilibotti (2006) for a similar argument in a distance-to-frontier model about the disadvantages of using subsidies rather than (anti)competitive policy, where the latter policy plays a similar role in affecting market power as patent breadth does in our context.

[19]Lump-sum subsidies for firms via prizes may also suffer from an analogous problem when the value of innovations

Third, direct government supports can become problematic for political economy reasons. If some small groups who are better organized politically than others are strongly affected by particular government decisions, they may be able to lobby to change these decisions and distort the supporting (or subsidy) expenditures (see Cohen and Noll (1991) and Romer (1993)). For example, Kremer (1998) reveals that lobbying by defense contractors and AIDS activists has generated distortions on the pattern of the expenditures in military and medical research.

In summary, there are problems and limitations with the use of subsidies, which seems less prevalent than the use of patents. Thus, this study follows O'Donoghue and Zweimüller (2004) to take the perspective that subsidies are not effective policy instruments. Under such an environment, the subsidy rates may be first fixed at some (arbitrary) levels to meet the aforementioned requirements when necessary. The policymaker then implements the (adjustable) IPR levers through patent authority to manipulate labor allocations to achieve optimal outcomes, while taking existing subsidies as given.

5.2 Optimal Coordination of Patent Instruments

Given the exogenous subsidy rates, the equilibrium labor allocations can replicate the first-best allocations by applying the patent-policy instruments. In this setup, on the one hand, it is the use of patent breadth μ that determines the optimal ratio of leisure and production L/L_x. On the other hand, it is the use of the profit-division rule s that pins down the optimal allocation on L_r relative to other labor inputs. Therefore, under a scheme of patent protection blocking future inventions, which stands in contrast to the patent schemes in Iwaisako and Futagami (2003) and Chu and Furukawa (2011), this model also invokes two patent instruments that the policymaker requires in steering the market economy towards the social optimum.

Next, consider the optimal design of the patent tools. As for optimal patent breadth, it can be obtained by directly comparing the ratio of L and L_x in the market equilibrium to the one in the first-best outcome, such that

$$\mu^* = \frac{1}{\alpha}, \tag{27}$$

which negatively depends on the (inverse) subsidy rate for intermediate goods. Essentially, given that α is set at some fixed level (due to the reasons in Section 5.1), L/L_x can deviate from its first-best value without adjusting the markup to a suitable level. Hence, using optimal μ^* ensures to remove the allocative distortion on leisure relative to manufacturing labor. It is obvious that μ^* is decreasing in α; a higher subsidy to intermediate-goods production implies a larger optimal patent breadth. If subsidies to intermediate goods are absent (i.e., $\alpha = 1$), then optimal patent breadth becomes unattainable since $\mu > 1$. This implies that granting monopoly rights alone will lead to a distortion on L/L_x being too large.

Furthermore, substituting (27) into (19) and equating (19) and (26) yields the optimal profit-division rule:

$$s^* = \frac{\left((1+\varphi/\rho)\ln z - (1+\phi)\right)\left((1-\alpha+\sigma(1+\phi)) - \sigma(1+\varphi/\rho)\ln z\right)}{(1-\alpha)\ln z}, \tag{28}$$

cannot be accurately observed (Wright (1983)). In this case, Scotchmer (1999) shows that patents are able to serve as an advantageous revelation device.

which is used to eliminate the distortion on R&D as will be discussed. Hence, (27) and (28) in unison give the optimal coordination of patent instruments. Notice that the optimality of patent breadth and the profit-division rule relies on the existence of subsidies to production of intermediate goods, whereas subsidies to research only affect the level of s^* but do not affect its optimality (e.g., (28) may still hold when $\sigma = 1$).[20]

Since this study focuses on overlapping patent rights, it is important to ensure that the optimal s^* is bounded between zero and one. Then, we impose the following assumptions:

Assumption 2.
(2.1). $\varphi \in \left(\frac{\rho\left(1-\alpha+2\sigma(1+\phi-\ln z)+\sqrt{(1-\alpha)(1-\alpha-4\sigma\ln z)}\right)}{2\sigma\ln z}, \frac{\rho(1-\alpha+\sigma(1+\phi-\ln z))}{\sigma\ln z} \right)$;
(2.2). $\ln z < \frac{1-\alpha}{4\sigma}$,

where Assumption 2.1 ensures that s^* is between 0 and 1 and Assumption 2.2 implies that Assumption 2.1 is a non-empty set.[21] Denote the lower bound (the upper bound) of Assumption 2.1 as φ^- (φ^+), which is the boundary for s^* to be less than 1 (greater than 0).

To elaborate Assumption 2.1, notice that, given optimal patent breadth $\mu^* = 1/\alpha$, the equilibrium level of R&D labor L_r at $s = 0$ indeed equals the optimal level L_r^* if φ is on φ^+. When $\varphi < \varphi^+$, there is R&D overinvestment at $s = 0$ (i.e., $L_r|_{s=0} > L_r^*$). This implies that a higher s is desired, since a larger backloading effect can reduce L_r to achieve the optimal level. Hence, the optimal s^* exceeds 0. Contrarily, when $\varphi > \varphi^+$, R&D underinvestment occurs in the zero-profit-division-rule equilibrium and a smaller backloading effect is required to increase L_r for optimality, but this is infeasible because s cannot be negative. Finally, it can be verified that the assumption $\varphi > \rho(1+\phi)/\ln z$ that makes L_r^* positive is between φ^- and φ^+ due to Assumption 2.2. Hence, the optimal s^* is less than 1. In summary, we obtain the following result.

Proposition 2. *Suppose that Assumption 2 holds. Then the economy achieves the first-best outcome in equilibrium with optimal patent breadth μ^* in (27) and the optimal profit-division rule s^* in (28). Moreover, s^* increases in ϕ and ρ but decreases in φ, z, α, and σ.*

Proof. See Appendix A. □

Proposition 2 implies that the distortion on R&D is created by a wedge between the optimal level and the equilibrium level of research labor when the parameters in s^* change. As φ and

[20] We assume a non-distorting lump-sum tax to simplify our setup. If the government alternatively taxes households' consumption or labor incomes with a rate $1-\tau$, the leisure-consumption decision (3) will have one more policy variable to be determined. The analytical solutions for the underlying equilibrium labor allocations and thereby optimal patent instruments become much more complicated, since τ would be a function of all parameters. However, the implication for the effect of μ (s) on optimizing L/L_x (the relative L_r) continues to hold. In this case, μ^* will depend on τ in addition to α, and the existence of s^* remains unaffected by σ. When $\sigma = \alpha = 1$ (namely, no subsidies), τ should also equal one in order to balance the governmental budget in (16). Again, optimal patent breadth becomes unavailable because the assumption $\mu > 1$ will be violated, leading to a distortion on L/L_x. This means that in this formulation the optimality of patent breadth still relies on the existence of subsidies.

[21] In fact, there exists another interval of φ that ensures that s^* lies between 0 and 1, which is given by $\varphi \in \left(\frac{\rho(1+\phi-\ln z)}{\ln z}, \frac{\rho\left(1-\alpha+2\sigma(1+\phi-\ln z)-\sqrt{(1-\alpha)(1-\alpha-4\sigma\ln z)}\right)}{2\sigma\ln z} \right)$. However, this interval is excluded by the assumption $\varphi > \rho(1+\phi)/\ln z$. Moreover, if $\ln z$ is sufficiently small (i.e., $\ln z < \min\left\{ \frac{\alpha(1+\phi)(\mu-1)(1-s)}{\sigma(1+\alpha\phi\mu)}, \frac{1-\alpha}{4\sigma} \right\}$), the assumption $\varphi > \rho(1+\phi)/\ln z$, whose right-hand side is decreasing in z, becomes more likely to be consistent with (greater than) the lower bound of φ in Assumption 1.

z (ϕ and ρ) increase, this wedge increases (decreases), so it is optimal to reduce (increase) s for stimulating (depressing) the equilibrium R&D.[22]

This result is associated with various sources of R&D externalities as follows. According to (23) and the optimal labor allocations, R&D activities have a positive impact on welfare through economic growth. A rise in φ or z reinforces this impact more in the social optimum than in the decentralized setting making L_r^* exceed L_r (e.g., a worsening of the surplus-appropriability problem), which is a positive externality. Thus, a higher incentive for R&D is needed, which can be satisfied by a lower level of s^*. In contrast, a higher ϕ or ρ tends to dampen the benefit of growth on welfare. In this case, leisure and consumption are preferred for welfare and R&D activities are less desired. This impact becomes stronger in the social optimum making L_r^* smaller than L_r (e.g., a strengthening of the preference-spillover and intertemporal-spillover effects), which is a negative externality. Therefore, s^* increases to reallocate labor from R&D to leisure and manufacturing. The above effects of these parameters appear clearer in the first-best growth rate, which is obtained by combining $g = \varphi \ln z L_r$ with the optimal R&D labor, such that

$$g^* = \varphi \ln z - (1+\phi)\rho. \tag{29}$$

Additionally, a higher α stifles the positive impact of optimal patent breadth μ^* on growth, whereas a higher σ raises the costs for conducting R&D. These effects decrease the equilibrium R&D labor according to Lemma 1, but they do not affect the optimal labor allocations and the optimal growth rate (e.g., a worsening of the business-stealing effect), which is a positive externality. Consequently, it is optimal to decrease s^* for restoring the first-best outcome.

5.3 Optimal Rule of Profit Division

In this subsection, we analyze the policy implications of a second-best outcome in which the profit-division rule s is optimized for a given level of patent breadth μ. One reason to consider this formulation is that as argued by Chu, Cozzi, and Galli (2012), it would be difficult to reinforce the monopoly's market power by charging a higher markup because of antitrust laws (or by increasing competitive fringes' imitation costs).

Combining (17) and (18) shows that the equilibrium ratio of L and L_x equals $\phi\alpha\mu$. Using the labor-market-clearing condition, we can express L and L_x as a function of L_r, respectively, such that $L = \frac{\phi\alpha\mu}{1+\phi\alpha\mu}(1-L_r)$ and $L_x = \frac{1}{1+\phi\alpha\mu}(1-L_r)$. Consequently, substituting these labor relations into the households' lifetime utility given by (23) and rearranging it yields

$$U = \frac{1}{\rho}\left[(1+\phi)\ln(1-L_r) + \frac{\varphi \ln z}{\rho} L_r + \ln\left(\frac{(\phi\alpha\mu)^{\phi}}{(1+\phi\alpha\mu)^{1+\phi}}\right)\right], \tag{30}$$

where L_r follows its equilibrium value in (19). It is shown that optimizing only the profit-division rule separates the welfare effects of patent breadth on the R&D labor from those on leisure and

[22] It is obvious that $L_r^* = L_r$ when the social optimum arises. Given this condition, s can be considered as a function of the parameters φ, z, ϕ, and ρ. Then the implicit function theorem implies $\partial s/\partial k = -[\partial(L_r^* - L_r)/\partial k]/[\partial(L_r^* - L_r)/\partial s]$, where k denotes a single parameter as abovementioned and $\partial(L_r^* - L_r)/\partial s > 0$ according to Lemma 1. Thus, the comparative statics of s^* in Proposition 2 are necessitated by a wedge between L_r^* and L_r caused by the variation of these parameters as specified in the text.

the manufacturing labor, as shown in (30). Next, we derive the optimal profit-division rule by the following first-order condition:

$$\frac{\partial U}{\partial s} = \frac{1}{\rho}\frac{\partial L_r}{\partial s}\left[-\frac{1+\phi}{1-L_r} + \frac{\varphi \ln z}{\rho}\right], \tag{31}$$

where $\partial L_r/\partial s < 0$ according to Lemma 1. Thus, (31) implies that the second-best profit-division rule s^{**} balances a negative effect from final-goods production captured by $-(1+\phi)/(1-L_r)$ and a positive effect from the growth of technology captured by $\varphi \ln z/\rho$. Then, making (31) equal zero solves for the optimal profit-division rule s^{**} for any given level of μ. In this case, (31) shows that the second-best R&D labor coincides with the first-best counterpart given by $L_r^{**}|_{s=s^{**}} = L_r^*|_{s=s^*} = 1-(1+\phi)\frac{\rho}{\varphi \ln z}$. Accordingly, we obtain the following result.

Proposition 3. *When only the profit-division rule is chosen optimally, the equilibrium growth rate equals the first-best counterpart. However, the welfare would be lower than the case with the optimal mix of patent instruments.*

Proof. Using $L_r^{**} = L_r^*$ and the growth equation $g = \varphi \ln z L_r$, it is straightforward to show that, given any μ, the equilibrium (second-best) growth rate is identical to the first-best one, namely, $g^{**} = g^* = \varphi \ln z - (1+\phi)\rho$.

As for the comparison of welfare between the two outcomes, denote the welfare difference by $\Delta U = U^{**} - U^*$, where U^{**} is the social welfare when $s = s^{**}$ given any μ and U^* is the first-best welfare, respectively. Hence, we have

$$\Delta U = \frac{1}{\rho}\left[\ln\frac{1+\phi}{1+\phi\alpha\mu} + \phi\ln\frac{\alpha\mu(1+\phi)}{1+\phi\alpha\mu})\right]. \tag{32}$$

To see the sign of ΔU when μ varies, taking the derivative of (32) with respect to μ yields

$$\frac{\partial \Delta U}{\partial \mu} = \frac{\phi}{\rho(1+\phi\alpha\mu)}\left(\frac{1}{\mu} - \alpha\right). \tag{33}$$

Thus, there exists a threshold $\mu^* = 1/\alpha$, such that when $\mu < 1/\alpha$, ΔU is increasing in μ, whereas when $\mu > 1/\alpha$, ΔU is decreasing in μ. In addition, $\partial\Delta U/\partial\mu$ is decreasing in μ, implying that ΔU is a concave function (inverted U-shaped) of μ and reaches the maximum if $\mu = 1/\alpha$. Therefore, $\Delta U \leq \Delta U|_{\mu=1/\alpha} = 0$. □

In other words, the welfare level under the optimal mix of patent instruments would always be greater than under only the optimal profit-division rule, but the equilibrium growth rate arising from the second-best outcome remains socially optimal.[23] Intuitively, on the one hand, varying the profit-division rule helps to set optimally the relative R&D labor. Therefore, this characteristic is unaffected by optimizing merely s given any μ, which suffices to generate the optimal rate of growth. On the other hand, the equilibrium ratio L/L_x cannot be optimized by choosing patent

[23] Chu (2011) shows that in a two-sector quality-ladder growth model, uniform optimal patent breadth (i.e., optimizing one patent instrument across both sectors) achieves the optimal growth, but sector-specific optimal patent breadth (i.e., optimizing the patent instrument in each sector, respectively) yields welfare gains in addition to the optimal growth.

breadth μ, which leaves the distortion on the relative labor supply in the model, unless μ coincides with its socially optimal level. Precisely, if patent breadth is relatively narrow (i.e., $\mu < 1/\alpha$), too little leisure and too much production labor is assigned, yielding a lower equilibrium level of L/L_x as compared to the first-best level; otherwise, the equilibrium level of L/L_x becomes socially higher by relatively broad patent breadth (i.e., $\mu > 1/\alpha$). Either situation results in suboptimal labor allocations and a welfare loss for the economy.

5.4 Optimal Patent Breadth

In this subsection, we investigate another second-best outcome in which patent breadth μ is optimized for a given degree of blocking patents s. Differentiating the households' lifetime utility along the BGP (30) with respect to μ yields

$$\frac{\partial U}{\partial \mu} = \frac{1}{\rho}\left[\frac{\partial L_r}{\partial \mu}\left(-\frac{1+\phi}{1-L_r} + \frac{\varphi \ln z}{\rho}\right) + \underbrace{\frac{\phi(1-\alpha\mu)}{\mu(1+\phi\alpha\mu)}}_{\Omega}\right], \tag{34}$$

where $\partial L_r/\partial \mu > 0$ according to Lemma 1. The above first-order condition implies that the optimal (second-best) μ^{**} balances the two welfare effects as presented in (31) in addition to an extra welfare effect from the ratio of leisure and labor supply captured by $\Omega \equiv \phi(1/\mu - \alpha)/(1 + \phi\alpha\mu)$, which is positive (negative) when $\mu < (>) 1/\alpha$. Specifically, according to our previous analysis, when $\mu^{**} < (>) 1/\alpha = \mu^*$, the ratio L/L_x is below (above) the first-best optimal level. In this case, given that the additional welfare effect Ω becomes positive (negative), there is over- (under-)investment in R&D in this second-best outcome that strengthens (mitigates) the first negative welfare effect, implying that $L_r^{**}|_{\mu=\mu^{**}} > (<) L_r^*|_{\mu=\mu^*}$.

Thus, unlike the scenario under the second-best profit-division rule, optimizing merely patent breadth may not fully correct the distortion on the relative allocation of R&D labor, as second-best patent breadth μ^{**} has to take into consideration the welfare effects on all the labor inputs (which is reflected by the presence of the welfare effect Ω in (34)). Therefore, the resulting equilibrium growth rate could be higher or lower than the first-best counterpart, depending on the level of μ^{**}, which is a function of the given level of s. If μ^{**} is close to the first-best level $1/\alpha$, the distortion on L/L_x becomes less significant. In particular, when μ^{**} coincides with $1/\alpha$, this distortion becomes absent because L^{**}/L_x^{**} is socially optimized. This implies that the term Ω in (34) disappears, meaning that the distortion on the relative allocation of L_r will simultaneously be eliminated by optimizing patent breadth. Notice that this case only occurs when the equilibrium profit-division rule happens to be at its first-best level, namely $s = s^*$. As a result, the "second-best" μ^{**} attains the first-best growth rate in addition to the first-best allocations.

One can expect that the welfare level of optimizing only patent breadth would also be lower than that of optimizing both patent instruments due to the possibility of suboptimal R&D investment.[24] Nevertheless, the welfare comparison between the first-best outcome and the second-best outcome in this case is analytically difficult, and we leave this discussion in the numerical analysis.

[24] In a similar quality-ladder model with competitive RJVs, Chu and Furukawa (2011) also reveal that a suboptimal outcome is brought about by optimizing solely patent breadth. However, Yang (2013) shows that if RJVs are cooperative, then optimizing only patent breadth can still lead the economy to retain the first-best outcome in equilibrium.

5.5 Discussion

Inelastic Labor Supply. As mentioned in Section 4, introducing the elastic supply of labor imposes an extra distortion on the allocation of L/L_x together with a distortion on the relative allocation of R&D labor L_r, and the latter is the usual allocative distortion in R&D-based growth models. In the presence of these distortions, our model needs two policy levers to steer the market equilibrium towards the first-best outcome. However, if labor is supplied inelastically instead, then the former distortion no longer exists and labor is distributed to only production and research (i.e., L_x and L_r), implying that merely a single patent instrument is required to remedy the R&D distortion. Thus, applying $\phi = 0$ to L_r in (19) and equating it to L_r^* in (26) yields optimal patent breadth μ^* holding s constant or the optimal profit-division rule s^* holding μ constant, either of which helps attain the socially optimal allocations. Specifically, with an analogous parameter space limiting the range of φ and of z as in Assumption 2, the (first-best) optimal patent breadth is $\mu^* = \frac{\varphi \ln z/\rho + (-1+(1+s)\ln z)}{(1+\varphi/\rho)\ln z((-1+\ln z)+\varphi \ln z/\rho)+s\ln z}$ given any s. Alternatively, the (first-best) optimal degree of blocking patents is $s^* = \frac{((1+\varphi/\rho)\ln z-1)(1-\mu(1+\varphi/\rho)\ln z)}{(\mu-1)\ln z}$ given any μ.

Complete Frontloading. A complete frontloading profit-sharing agreement implies the highest incentives for R&D for any given level of patent breadth, since the profits received by an entrant are maximized. Chu (2009) shows that complete frontloading has positive impacts on R&D and welfare. This is also the usual assumption in the existing patents-and-growth studies such as Chu and Furukawa (2011). Under this setting, patent breadth is left available for steering the market economy. In fact, this is just a special case of Section 5.4 in which $s = 0$.

The setup with $s = 0$ implies $V_2 = 0$, and the equilibrium labor allocations are still characterized by (17)-(19), where the stationary consumption expenditure is reduced to $E = \frac{\alpha\sigma\mu(\varphi+\rho)}{\varphi(\sigma+\alpha(\mu-1+\sigma\phi\mu))}$. Hence, our model behaves similarly to the Grossman-Helpman quality-ladder model without the backloading effect of blocking patents. If labor supply is inelastic, by limiting the range of φ, a suitable level of patent breadth $\mu^* = \frac{\sigma-\alpha}{\alpha(1-\sigma(1+\varphi/\rho)\ln z)}$ would suffice to eliminate the distortion on R&D, recovering the social optimum. In contrast, if labor supply is elastic, then the analysis returns to Section 5.4, with a possibility of distorting R&D investment. Accordingly, optimal patent policy that would lead to the second-best allocations (i.e., μ^{**}) is given by (34). For instance, under $\alpha = \sigma = 1$ (i.e., no subsidies), the (locally) welfare-maximizing level of patent breadth is $\mu^{**} = \frac{(1+\varphi/\rho)\ln z}{1+\phi}$.[25]

6 Quantitative Analysis

In this section, we calibrate the model to the US economy in order to numerically evaluate the welfare differences from the case with equilibrium levels of patent instruments to (a) the scenario without policy interventions, (b) optimizing both patent-policy tools (i.e., the first-best outcome), and (c) optimizing only a single policy instrument (i.e., the second-best outcomes), respectively.

6.1 Calibration

To perform this numerical analysis, the strategy is to assign steady-state values to the following structural parameters $\{\rho, z, \varphi, \phi, \alpha, \sigma, s, \mu\}$. We follow Chu and Pan (2013) in choosing a conven-

[25] The derivation can be seen in the complementary *Mathematica* file and it is available upon request.

tional value of 0.04 for the discount rate ρ. To calibrate the R&D productivity parameter φ and the leisure preference parameter ϕ, we follow Acemoglu and Akcigit (2012) in setting the step size of innovation z to 1.05 and follow Chu, Cozzi, and Galli (2012) in choosing the empirical long-run growth rate of GDP per capita in the US, which is 1.5%. Furthermore, Comin (2004) considers that the contribution of R&D investment drives only a fraction of long-run economic growth in the US. Hence, we set this fraction to 0.4236 for our estimation, which is similar to the counterpart in Chu and Cozzi (2014) (i.e., 0.4).

To identify the (inverse) subsidy rate to intermediate-goods production (α), we set $\alpha = 0.75$; this value equals the one used in Grossmann, Steger, and Trimborn (2013) for the US economy, which restores the first-best outcome in their analysis.[26] To identify the (inverse) subsidy rate to R&D (σ), we again follow Grossmann, Steger, and Trimborn (2013), who use a tax credit system with the average US corporate income tax base of 0.25, to assume that innovating firms are allowed to deduct $1+(1-\tau_c)(1-\sigma)/\tau_c = 2.6941$ times their R&D costs from sales revenues.[27] Accordingly, we approximately have $\sigma = 0.4353$.[28] This deduction rate is lower than the estimation in Grossmann, Steger, and Trimborn (2013) (i.e., 3.4 times) and is consistent with that in Grossmann, Steger, and Trimborn (2016) (i.e., 2.5 times), but it is required to be higher than the current US policy (which is 1.1–1.2 times).[29]

As for the profit-division rule s, we use the arrival rate of innovations λ for its estimation. Lanjouw (1998) suggests the estimated probability of obsolescence ranging from 7% to 12%, whereas Caballero and Jaffe (2002) estimate a mean rate of creative destruction of roughly 4%. Thus, we consider the values $\lambda \in [0.04, 0.13024]$ to cover these empirical estimates. We choose the upper bound of λ as the market level for matching the empirical/standard moments. This value implies a duration of 7.68 years between the arrival of innovations, which is close to the empirical findings of Hughes, Moore, and Snyder (2002) who show that the US new chemical entities possess about 8 years of effective patent life. Then, s can be varied to change the degree of the backloading effect within the range of λ. As for patent breadth μ, we focus on values $\mu = \{1.1, 1.2, 1.3, 1.35, 1.4\}$ by taking into account the empirical estimates of the markup reported in Jones and Williams (2000) (i.e., 1.05–1.4) and in Laitner and Stolyarov (2004) (i.e., approximately 1.1). We use $\mu = 1.1$ as the market level and then increase μ to strengthen the market-power effect of patent breadth. Therefore, setting $\lambda = \varphi L_r$ to 0.13024, $g = \lambda \ln z$ to the equilibrium growth rate, and the time fraction of L to 0.6871 with the aid of the labor allocations in (16) and (19) yields $s = 0.15$, $\varphi = 3.68927$ and $\phi = 3$ (see Footnote 25), which suggests that the backloading effect is present in the market equilibrium.[30] In addition, the range of λ pins down the values $s \in [0.15, 1]$. The combination of

[26] Edge, Laubach, and Williams (2010) consider an inverse subsidy rate of 0.8 to intermediate-goods production.

[27] The federal US statutory corporate income rate is 35% for large corporations and 15% for small corporations.

[28] Nuño (2011) shows that with the optimal subsidy to intermediate goods, the optimal R&D subsidy for restoring the first-best steady state can even reach 0.69 in a stochastic version of Schumpeterian endogenous growth. In fact, some developed countries provide an R&D subsidy that can be as high as in our calibration, such as the average R&D subsidy rate in Portugal (0.55) and in France (0.51) to their small and medium enterprises (OECD (2013)).

[29] Applying the deduction rate in the current US policy of 1.2 times with $\tau_c = 0.25$ delivers $\sigma = 0.934$, which is in accordance with the reports in OECD (2009) and OECD (2013). However, under this set of calibrated parameter values, we cannot find an interior solution for s^* bounded between 0 and 1, implying that Assumption 2 will be violated. Note that the main focus of this exercise is not on the size of the R&D subsidy, but on the welfare comparisons between the decentralized equilibrium and the outcomes with optimal patent instruments in the presence of subsidization.

[30] In Appendix B of Chu (2009), a backloading discount factor is derived to capture the fraction of the total amount

the above calibrated values ensures that the first-best profit-division rule s^* is bounded between 0 and 1; it is calculated to be around 0.5. Finally, the welfare difference is expressed as the usual equivalent variation in consumption flow denoted by $\xi \equiv \exp(\rho\Delta U) - 1$.

6.2 Numerical Results

This analysis starts from the welfare comparison between the decentralized equilibrium in which realistic values are calibrated and an extreme case in which no policy tools are introduced. The purpose of this exercise is to quantify the welfare losses (or gains) of the equilibrium level in our model as compared to that in the original quality-ladder model. In this no-policy outcome, we follow Grossman and Helpman (1991) to choose $\mu = z = 1.05$ (i.e., one step size of quality improvement), $s = 0$ (i.e., no backloading effect), and $\alpha = \sigma = 1$ (i.e., no subsidies). Denote the welfare differences by ξ_0 as shown in Table 1-(1). It can be seen that as compared to our benchmark case, there is a welfare loss of 0.11% when all policy interventions are dismantled. Notice that the equilibrium degree of blocking patents is slightly higher than 0 in this comparison. Also, recall that the interaction of α and μ affects the input distortion on L/L_x. Relative to the first-best level of this interaction (i.e., 1), the benchmark level of $\mu\alpha$ (i.e., 0.825) is only a little further away than the one under the no-policy outcome (i.e., 1.05). Thus, the welfare difference in ξ_0 is mainly driven by the presence of subsidies to R&D, which effectively increases the level of R&D labor and the growth rate in equilibrium (namely, $L_r = 0.035 > L_r|_{original} = 0.001$ and $g = 0.006 > g|_{original} = 0.0002$). As the backloading effect becomes more prominent through raising s in the decentralized equilibrium, the growth rate decreases and the positive welfare effect of R&D subsidies is undermined. As a result, the equilibrium level of welfare U gradually declines and finally becomes smaller than the level under the no-policy outcome.

6.2.1 Optimizing a Mix of Patent Instruments

The analysis now turns to quantify the welfare improvements from the equilibrium level U to the first-best level U^*, which are denoted as ξ_1. Using our calibration and making use of Proposition 2, the first-best optimal mix of patent instruments is given by $\mu^* = 1/\alpha = 4/3$ and $s^* = 0.5$. Since the equilibrium labor allocations are altered by the choices of μ and s, the underlying welfare differences rely on the combination of deviations of these patent-policy tools as compared to their first-best levels. Table 1-(1) displays the welfare gains accordingly. When μ is fixed at 1.1, varying the equilibrium level of s from 0.15 to 1 induces a significant welfare gain in ξ_1, which increases from 2.85% under $s = 0.15$ to 3.40% under $s = 1$. Intuitively, the first-best outcome is restored by adjusting two policy instruments to remedy the two distortions occurring in the decentralized equilibrium. Given $\alpha = 0.75$, $\mu = 1.1$ implies that patent breadth is devised to be less compatible with subsidies to intermediate goods in the sense that μ is smaller than its optimal value ($1/\alpha$). This is the first distorting impact on the equilibrium ratio of leisure and production labor L/L_x. In addition, there is another bias in the relative allocation of R&D labor L_r due to the suboptimal profit-division rule at $s = 0.15$. When $\mu = 1.1$ and $s = 0.15$ are raised to their first-best levels μ^*

of monopolistic profits created by an invention (i.e., $1-s$ in our context), which is intuitively equivalent to the inverse measure of the backloading effect of blocking patents. This backloading discount factor is calibrated between 0.48 and 0.85, but infringement on multiple previous inventors is possible. Hence, the market level of $s = 0.15$ in our estimation is consistent with the lower-bound calibrated value in Chu (2009) to allow for the smallest degree of backloading.

and s^*, respectively, the above two layers of distortions are eliminated by reallocating the labor inputs.[31] The increment in welfare is considerable because of the rise in the growth rate from 0.6% to 2% and much less distortion in the relative labor supply.

As the equilibrium level of s increases, more welfare improvements can be explored by optimally setting the combination of patent instruments. The largest welfare gain, which equals approximately 3.40%, is obtained where $s = 1$. In this case, given the distortion on L/L_x determined by the equilibrium level of μ, $s = 1$, which lies on the upper bound of the calibrated range, implies the largest backloading effect and thereby generates the most severe R&D underinvestment. Specifically, relative to the first-best allocations, much less R&D labor is assigned in equilibrium (i.e., $L_r = 0.021 < L_r^* = 0.111$), so the potential size of welfare gains enlarges due to more leisure and consumption production at the cost of a lower growth rate (i.e., $L = 0.697 > L^* = 0.667$, $L_x = 0.281 > L_x^* = 0.222$, and $g = 0.0038 < g^* = 0.02$). Hence, in addition to adjusting μ to $1/\alpha$ that corrects the distortion on L/L_x, reducing the degree of backloading to s^* will correct the distortion from the above R&D underinvestment, yielding the most significant welfare gain. Moreover, in the case of $\mu = 1.1$, the welfare difference between the two extreme scenarios (namely, the no-policy outcome and the first-best outcome) is approximately 2.96% (which equals $\xi_1 - \xi_0$), implying that removing the policy interventions also results in a considerable welfare loss.

6.2.2 Optimizing the Profit-Division Rule

Next, we quantify the welfare differences between the equilibrium level U and the second-best level U^{**} in which s is optimized given the level of μ. These differences are denoted as ξ_2. First of all, given the benchmark choice $\mu = 1.1$, it is found that the equilibrium level U is decreasing in s, so the second-best s^{**} is given by a corner solution such that $s^{**} = 0$ (i.e., complete frontloading). Therefore, a rise in s magnifies the welfare difference between U and U^{**}. To gain the intuition, recall that L_r is correlated negatively by s but positively by μ as shown in Lemma 1. Suppose $s = s^*$. If μ is fixed by a level that is lower than μ^*, then this generates a negative welfare effect because the ratio L/L_x is distorted. Moreover, in this case, a relatively low level of L_r is initially assigned as compared with L_r^*, and lowering s (i.e., reducing the backloading effect) causes a positive welfare effect through increasing L_r to mitigate this allocation problem. Notwithstanding, the latter positive effect tends to overwhelm the former negative effect. Then, balancing the gains and losses yields the fact that the welfare difference ξ_2 is increasing in s when μ is relatively small. This analysis implies that in (31), the equilibrium R&D labor L_r is smaller than the optimal counterpart L_r^* because of a small μ, yielding $\partial U/\partial s < 0$. Hence, a lower s prompts L_r to approach L_r^* and monotonically increases the equilibrium welfare level, so that the second-best outcome is given by a corner solution.

Table 1 shows that, across the equilibrium levels of s, the welfare gains are much less significant in ξ_2 relative to ξ_1, with the upper bound decreasing from 3.40% to 0.59% of consumption and the mean decreasing from 3.07% to 0.28%. Intuitively, in contrast to restoring the first-best outcome, one degree of policy freedom in equilibrium is restricted in achieving the second-best outcome since the value of μ cannot be altered. Thus, the distortion on the equilibrium ratio of leisure and production labor still exists, which attenuates the welfare improvements through optimizing only

[31] More precisely, in this situation we have $L = L^* = 0.667$, $L_x = L_x^* = 0.222$, and $L_r = L_r^* = 0.111$, which induces the optimal growth rate to be 2%.

the profit-division rule. In the equilibrium cases, except for the bias in the relative allocation of R&D labor due to the suboptimal choices of instruments, the ratio of leisure and production labor is equivalent to that in the second-best outcome, whereas it is lower than in the first-best outcome as $\mu = 1.1 < \mu^*$. Because of the difference in the dimensions of optimal policy to correct these distortions, ξ_1 substantially exceeds ξ_2 by approximately 2.79% of consumption. This pattern can be seen in ξ, which displays the welfare differences between the first-best outcome and the second-best outcome. This analysis confirms our finding implied by Proposition 3, such that the welfare level of optimizing only s can be considerably lower than that of optimizing both s and μ.

Furthermore, we conduct similar exercises to estimate the welfare differences from the market equilibrium to the no-policy outcome and to the optimal outcomes by varying patent breadth μ to 1.2, 1.3, 1.35, and 1.4, respectively.[32] Parts (2)-(4) in Table 1 present the welfare differences ξ_0, ξ_1, ξ_2, and ξ according to the changes in μ. It is worthwhile noting that the equilibrium welfare level U is decreasing in s under $\mu = 1.2$ and $\mu = 1.3$ while it is increasing in s under $\mu = 1.4$, so the second-best profit-division rule s^{**} equals 0 and 1, respectively. In contrast, U is an inverted-U shape with respect to s under $\mu = 1.35$, implying that s^{**} becomes an interior solution (which is 0.916); it is obtained as discussed in the situation of Section 5.3, where the equilibrium level of R&D labor in (19) achieves the optimal level in (26).[33] [34]

It is observed that in the variations of μ, the magnitude of the welfare gains under the optimal outcomes become much smaller, especially in the case of $\mu = 1.3$ and $\mu = 1.35$. This is the consequence of the values of μ being quantitatively close to its optimum μ^* in the presence of other calibrated parameters. Given α and the values of μ in our consideration, the equilibrium ratio of leisure and the manufacturing labor ($\phi\alpha\mu$) is not very different from the optimal ratio (ϕ). As for ξ_1, the impact of optimizing μ on correcting the distortion on L/L_x would not be too large. Moreover, the impact of optimizing s on reallocating the R&D labor L_r in both ξ_2 and ξ_1 is also quite limited, which can be seen in Figures 1(b)–1(d) in terms of changes in the magnitude of U. These impacts together result in a small size of welfare gains from the decentralized equilibrium to the optimal outcomes. If the value of μ deviated further from μ^* (as in the benchmark), a much larger allocative distortion between L and L_x would arise and the effect of reallocating the labor inputs would become more evident, yielding even more significant welfare improvements. A similar explanation applies to the welfare comparisons between the decentralized equilibrium and the no-policy outcome; as μ approaches μ^*, the magnitude of welfare losses in ξ_0 enlarges since

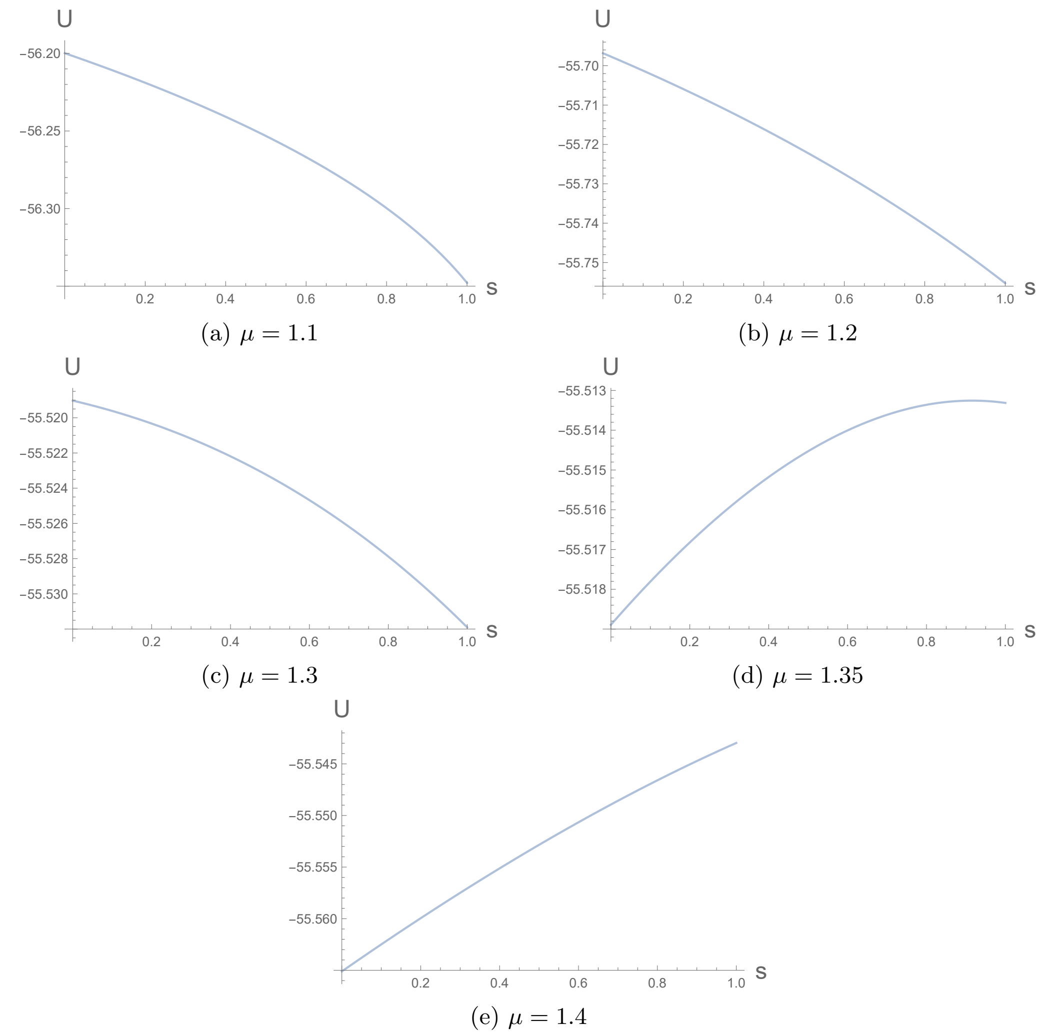
[32] To minimize the changes in the current calibrated parameters, under these levels of patent breadth, the equilibrium growth rate that is used for matching key empirical features (as in Section 6.1) is allowed to exceed 1.5%. The largest growth rate in use is 2.4%, which is still in line with some empirical estimates for the US economy. See, for example, Zeng and Zhang (2007) (3%) and Grossmann, Steger, and Trimborn (2013) (2%). Correspondingly, the equilibrium arrival rate of innovations is also allowed to exceed 0.13024 and the largest value in use is 0.486. This implies that the expected duration of time between two consecutive inventions is about 2 years, which is close to the estimate in Acemoglu and Akcigit (2012) (namely 3 years).

[33] Under the current set of calibrated parameters, the second-best profit-division rule s^{**} is bounded between 0 and 1 if $\mu \in (1.31529, 1.35358)$.

[34] Again, assume that $s = s^*$. If μ is relatively larger than μ^* (e.g., $\mu = 1.4$), then a higher level of L_r is allocated than L_r^*. Hence, raising s causes a positive welfare effect through decreasing the level of L_r, which overwhelms a negative welfare effect due to the distorted ratio of L/L_x, making the welfare difference decreasing in s. Of course, when μ is close to μ^* (e.g., $\mu = 1.35$), the positive effect first governs but finally becomes weaker than the negative effect, leading to an inverted-U shape with respect to s. A similar reasoning also applies to the fact that the welfare difference between the decentralized equilibrium and the no-policy outcome (i.e., ξ_0) is decreasing in s under a sufficiently high level of μ.

the welfare level under the decentralized equilibrium becomes closer to that under the first-best allocations. In this situation, it is shown that the equilibrium choice of s does not considerably change the sizes of welfare differences either.

Overall, most of the welfare losses in the decentralized equilibrium and in the above second-best outcome are inclined to stem from the presence of suboptimal choices of patent breadth in the intermediate-goods sector. This finding is in line with the counterpart in the steady-state welfare analysis of Nuño (2011). This fact yields one of the key results: starting from the decentralized equilibrium, the increase in welfare gains by optimizing a mix of patent-policy levers rather than by optimizing only the degree of blocking patents can be non-negligible. Patent breadth tends to more effective than the profit-division rule in terms of raising social welfare, even though optimizing the profit-division rule attains the first-best growth rate. Therefore, this argument reveals the important policy implication of the coordination of the profit-division rule and patent breadth, and thereby complements Chu, Cozzi, and Galli (2012) who consider the welfare effect of only the backloading of blocking patents.



(a) $\mu = 1.1$

(b) $\mu = 1.2$

(c) $\mu = 1.3$

(d) $\mu = 1.35$

(e) $\mu = 1.4$

Figure 1: Second-best welfare level and the profit-division rule

Table 1: The welfare differences for adjusting the degree of blocking patents.

(1) $\mu = 1.1$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ξ_0	-0.11	-0.09	-0.05	0.00	0.05	0.10	0.17	0.24	0.32	0.43
ξ_1	2.85	2.87	2.91	2.96	3.01	3.07	3.13	3.20	3.29	3.40
ξ_2	0.06	0.08	0.12	0.16	0.21	0.27	0.33	0.40	0.48	0.59
ξ	2.79	2.79	2.79	2.80	2.80	2.80	2.80	2.80	2.81	2.81

(2) $\mu = 1.2$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ξ_0	-2.13	-2.12	-2.10	-2.08	-2.06	-2.03	-2.00	-1.98	-1.95	-1.92
ξ_1	0.77	0.78	0.80	0.82	0.84	0.87	0.89	0.92	0.95	0.98
ξ_2	0.03	0.04	0.06	0.08	0.10	0.12	0.15	0.17	0.20	0.23
ξ	0.74	0.74	0.74	0.74	0.74	0.75	0.74	0.75	0.75	0.75

(3) $\mu = 1.3$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ξ_0	-2.842	-2.841	-2.838	-2.834	-2.829	-2.824	-2.818	-2.812	-2.804	-2.796
ξ_1	0.033	0.034	0.037	0.042	0.046	0.051	0.058	0.064	0.072	0.080
ξ_2	0.004	0.005	0.009	0.013	0.017	0.023	0.029	0.035	0.043	0.052
ξ	0.029	0.029	0.028	0.029	0.029	0.028	0.029	0.029	0.029	0.028

(4) $\mu = 1.35$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ξ_0	-2.853	-2.854	-2.858	-2.861	-2.863	-2.865	-2.867	-2.868	-2.868	-2.868
ξ_1	0.022	0.020	0.017	0.013	0.011	0.009	0.007	0.006	0.006	0.006
ξ_2	0.016	0.014	0.011	0.008	0.005	0.003	0.001	0.000	0.000	0.000
ξ	0.006	0.006	0.006	0.005	0.006	0.006	0.006	0.006	0.006	0.006

(5) $\mu = 1.4$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ξ_0	-2.68	-2.69	-2.70	-2.71	-2.71	-2.72	-2.73	-2.74	-2.75	-2.75
ξ_1	0.20	0.19	0.18	0.17	0.16	0.16	0.15	0.14	0.13	0.12
ξ_2	0.07	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.01	0.00
ξ	0.13	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.12	0.12

Notes: Suppose that s is the control variable. ξ_0, ξ_1, ξ_2, and ξ denote the welfare gains (losses) in percentage between the equilibrium and the no-policy outcome, between the equilibrium and the first-best outcome, between the equilibrium and the second-best outcome, and between the second-best outcome and the first-best outcome, respectively. The benchmark parameter set is $\rho = 0.04$, $z = 1.05$, $\varphi = 3.68927$, $\phi = 3$, $\alpha = 0.75$, $\sigma = 0.4353$, $\mu = 1.1$, and $s = 0.15$. For matching the empirical/standard moments, the equilibrium growth rate alters to 0.0133, 0.0190, 0.0214, and 0.0237, the arrival rate of innovations alters to 0.2721, 0.3886, 0.4394, 0.4859, whereas the leisure moment alters to 0.6759, 0.6667, 0.6627, and 0.659 in (2), (3), (4), and (5), respectively.

6.2.3 Sensitivity

To examine the sensitivity of the above numerical analysis, we now consider the following exercises with respect to the inverse subsidy to intermediate-goods production α and that to R&D σ.

First, we vary α and σ separately so that s^* equals the boundary values (i.e., complete backloading and complete frontloading in the social optimum, respectively). Second, α and σ are altered together such that s^* is still maintained at 0.5. Additionally, we recalibrate the model by using the discount rate $\rho = 0.03$ and the step size of innovation $z = 1.04$, respectively. Table 2 accordingly presents the average gains for the welfare differences ξ_0, ξ_1, ξ_2, and ξ under the alternative sets of structural parameters over the range of $s \in [0.15, 1]$.

In general, the main results of our numerical exercises seem quite robust in terms of the qualitative pattern and the quantitative magnitude. First, the welfare comparisons in ξ_0 can be either positive or negative, because of the differences between μ and μ^* as well as the opposing effects of R&D subsidies $(1-\sigma)$ and the backloading (s) on growth. Second, the welfare gains in ξ_1 continue to substantially exceed the counterparts in ξ_2. In particular, the largest changes in ξ_1 and ξ occur under the setting of $\rho = 0.03$, with the mean increasing to 3.87% and 3.58%, respectively. The reason is as follows. A decline in ρ implies a worsening of the intertemporal-spillover effect, which is a positive R&D externality. Also, the underlying calibrated value of φ (σ) decreases, and it implies a strengthening of the surplus-appropriability problem (the business-stealing effect), which is a negative R&D externality. The former effect outweighs the latter effect(s) leading to further deviations in the equilibrium allocations of L_r and L; the equilibrium welfare level becomes much lower than the first-best one (and also the second-best one since ξ_2 rises to 0.29%). Consequently, more significant welfare improvements from the decentralized equilibrium to the first-best outcome can be explored relative to the benchmark case, where most of the gains still come from the impact of the first-best μ^*.

Table 2: The welfare differences for adjusting the degree of blocking patents: Sensitivity Checks.

Parameter set	ξ_0	ξ_1	ξ_2	ξ
Benchmark	0.11	3.07	0.28	2.79
$\alpha = 0.7378 \Rightarrow s^* = 1$	0.38	3.35	0.28	3.07
$\alpha = 0.7611 \Rightarrow s^* = 0$	-0.11	2.85	0.27	2.58
$\sigma = 0.4150 \Rightarrow s^* = 1$	0.02	2.98	0.26	2.72
$\sigma = 0.4555 \Rightarrow s^* = 0$	0.20	3.16	0.29	2.87
$\alpha = 0.7378, \sigma = 0.4565$	0.47	3.44	0.29	3.15
$\alpha = 0.7611, \sigma = 0.4160$	-0.19	2.76	0.26	2.50
$\rho = 0.03 \Rightarrow \varphi = 3.0744, \phi = 3.3333^{\ddagger}$	-0.86	3.87	0.29	3.58
$z = 1.04 \Rightarrow \varphi = 4.5894^{\S}$	0.11	2.97	0.20	2.77

Notes: Suppose that s is the control variable. ξ_0, ξ_1, ξ_2, and ξ denote the welfare differences between the equilibrium and the no-policy outcome, between the equilibrium and the first-best outcome, between the equilibrium and the second-best outcome, and between the second-best outcome and the first-best outcome, respectively. The welfare differences display the average gain in percentage over the range of values for s. The benchmark parameter set is $\rho = 0.04$, $z = 1.05$, $\varphi = 3.68927$, $\phi = 3$, $\alpha = 0.75$, $\sigma = 0.4353$, $\mu = 1.1$, and $s = 0.15$. The indicated parameters show the respective deviations from the benchmark set of parameters, along with corresponding changes in the leisure moment and the contribution of R&D investment to long-run economic growth for matching the key empirical features, following the calibration strategy as specified in Section 6.1. ‡ and § represent that σ changes to 0.3375 and 0.4468, respectively, to maintain $s^* = 0.5$.

6.2.4 Optimizing Patent Breadth

To further justify the argument that the welfare effect of patent breadth is more effective than that of the profit-division rule, this subsection considers another policy regime where μ is optimized given the level of s. This exercise corresponds to Section 5.4. We estimate the welfare gains from the decentralized equilibrium to this regime, which are denoted by ξ_2 in Table 3. Additionally, ξ_0, ξ_1, and ξ, which are similarly denoted as in the preceding subsections, are quantified. We focus on the values of $\mu \in \{1.1, 1.2, 1.3, 1.4\}$ and the values of $s \in \{0, 0.15, 0.5, 0.8, 1\}$, respectively.

In each combination of the equilibrium values of μ and s, Table 3 displays that the welfare comparisons between the decentralized equilibrium and the no-policy outcome follow the same pattern as those shown in Table 1. At our benchmark degree of blocking patents $s = 0.15$, Table 3-(2) shows that the welfare improvements from the equilibrium to the social optimum (i.e., ξ_1) are significant, ranging from 2.848% under $\mu = 1.1$ to 0.198% under $\mu = 1.4$. More importantly, these welfare gains are almost entirely contributed by optimizing patent breadth (i.e., ξ_2), varying from 2.847% under $\mu = 1.1$ to 0.196% under $\mu = 1.4$. Again, these results are consistent with our previous findings. In addition, because optimizing solely μ yields the second-best patent breadth given by $\mu^{**} = 1.32725 < \mu^*$ in the case of $s = 0.15$, according to Section 5.4, there is R&D over-investment resulting in a suboptimal growth rate (i.e., $L_r^{**}|_{\mu=\mu^{**}} = 0.1130 > L_r^* = 0.1111$ and $g^{**}|_{\mu=\mu^{**}} = 0.0203 > g^* = 0.02$). This distortion is completely removed by setting s optimally leading to the first-best outcome.[35]

As the equilibrium level of s deviates, the magnitudes of welfare improvements by optimizing only μ and by optimizing a mix of patent instruments do not change substantially; for $s > (<)0.15$, the gains increase (decrease) when $\mu < \mu^*$ and decrease (increase) when $\mu > \mu^*$. Moreover, Table 3 indicates that varying the equilibrium choice of s does not affect the fact that ξ_2 accounts for the majority of ξ_1. This fact becomes more obvious in the case of $s = 0.5$ (which coincides with the first-best level s^*). In this case, setting μ optimally captures all the welfare gains of adjusting the distorted labor allocations, so $\xi_1 = \xi_2$.

6.2.5 Optimizing Subsidies: the Size of ξ_2

In this subsection, we consider a policy experiment in which subsidies to intermediate-goods production and those to R&D are optimized, respectively, given that the patent-policy instruments are fixed at their realistic values. The welfare comparisons between this policy experiment (which is also a second-best outcome) and the decentralized equilibrium are then quantified. The objective of conducting this exercise is to evaluate the welfare effects of optimizing the subsidy tools and contrast these effects to those of optimizing the patent tools (i.e., the sizes of ξ_2 in Tables 1 and 3), in order to reveal the fact that subsidy policy is less prevalent than patent policy in terms of affecting social welfare. This analysis justifies our earlier perspective that the use of subsidies is restricted, and thereby supports the rationale that the use of patents is on the main focus in this study by taking subsidy rates as exogenous.

Table 4 shows the results for these welfare differences accordingly. As for the (second-best) optimal R&D subsidies σ^{**}, the welfare comparisons are undertaken in the way, such that the

[35] Even if complete frontloading (i.e., $s = 0$) is assumed, as shown in Table 3-(1), optimizing μ alone continues to achieve only the second-best allocations in the presence of elastic labor supply, which results in a small difference between ξ_2 and ξ_1 (i.e., ξ is slightly greater than 0). This validates our discussion in Section 5.5.

Table 3: The welfare differences for adjusting patent breadth.

(1) $s = 0$

μ	1.1	1.2	1.3	1.4	average
ξ_0	-0.164	-2.152	-2.846	-2.667	-2.231
ξ_1	2.791	0.743	0.029	0.213	0.662
ξ_2	2.787	0.739	0.026	0.210	0.659
ξ	0.004	0.004	0.003	0.003	0.003

(2) $s = 0.15$

μ	1.1	1.2	1.3	1.4	average
ξ_0	-0.107	-2.126	-2.842	-2.682	-2.214
ξ_1	2.848	0.770	0.033	0.198	0.679
ξ_2	2.847	0.768	0.031	0.196	0.677
ξ	0.001	0.002	0.002	0.002	0.002

(3) $s = 0.5$

μ	1.1	1.2	1.3	1.4	average
ξ_0	0.050	-2.055	-2.829	-2.714	-2.168
ξ_1	3.010	0.843	0.046	0.164	0.727
ξ_2	3.010	0.843	0.046	0.164	0.727
ξ	0.000	0.000	0.000	0.000	0.000

(4) $s = 0.8$

μ	1.1	1.2	1.3	1.4	average
ξ_0	0.2359	-1.982	-2.812	-2.739	-2.115
ξ_1	3.202	0.919	0.064	0.139	0.780
ξ_2	3.200	0.917	0.063	0.138	0.781
ξ	0.002	0.002	0.001	0.001	0.001

(5) $s = 1$

μ	1.1	1.2	1.3	1.4	average
ξ_0	0.429	-1.924	-2.796	-2.753	-2.069
ξ_1	3.401	0.978	0.080	0.125	0.828
ξ_2	3.397	0.974	0.076	0.121	0.824
ξ	0.004	0.004	0.004	0.004	0.004

Notes: Suppose that μ is the control variable. ξ_0, ξ_1, ξ_2, and ξ denote the welfare gains (losses) in percentage between the equilibrium and the no-policy outcome, between the equilibrium and the first-best outcome, between the equilibrium and the second-best outcome, and between the second-best outcome and the first-best outcome, respectively.

profit-division rule s is fixed at 0.15 in equilibrium and the level of patent breadth μ is altered from 1.1 to 1.2, 1.3, 1.35 and 1.4, respectively. Also, a reasonable welfare comparison against optimizing s is made under the same parameter space that guarantees Assumption 2. This implies that the value of σ in consideration is limited to $[0.4150, 0.4555]$, because it is the interval for σ

that satisfies the first-best relation between σ and s in (28) where s is bounded between 0 and 1 in the presence of other calibrated values.[36] Our estimation shows that as σ increases, more (less) welfare improvements are realized if μ is smaller (greater) than μ^*; optimizing σ has an identical impact on welfare differences as optimizing s in terms of the qualitative patterns, since σ and s play the same role in correcting the distortion on R&D (see Section 4). However, for a wider range of μ (namely, for $\mu < 1.328$), the (average) magnitude of the welfare gains by adjusting σ turns out to be smaller than those by adjusting s in Table 1. Since the second-best σ^{**} is given by a corner solution (i.e., 0.4150) and the equilibrium values of σ are not far away from this second-best level, the welfare effect of optimizing σ through R&D is abated, especially when μ is sufficiently small.

As for the (second-best) optimal subsidies to intermediate-goods production α^{**}, the welfare differences are estimated by fixing μ at 1.1 and varying s from 0 to 0.15, 0.5, 0.8 and 1, respectively. Similar to the range of σ used in the above analysis, the value of α in consideration is limited to $[0.7378, 0.7611]$, given that it is the interval for α that satisfies the first-best relation between α and s in (28) where s is bounded between 0 and 1. The second-best α^{**} in all the cases displayed by Table 4-(2) is given by the corner solution of 0.7611. Accordingly, the welfare gain declines as α rises, and the largest gain is present under $\alpha = 0.7378$. Recall that α and μ have an analogous impact on correcting the distortion arising from the relative allocation in leisure over production labor. Nevertheless, the size of welfare gains by optimizing μ (in Table 3) is larger than optimizing α, implying that patents tend to be more effective than intermediate-goods subsidies. It is also observed that for the same level of α, the welfare difference is approximately the same across various levels of s. This result again indicates that adjusting the profit-division rule is less useful than adjusting the policy instruments in the intermediate-goods sector.

In summary, policy instruments that determine the amount of the monopolistic profits (i.e., μ and α) are more effective than those that determine the value of these profits (i.e., s and σ) in terms of raising social welfare. Furthermore, as for correcting the distortion on R&D, optimizing s turns to be more welfare-enhancing than optimizing σ under a wide range of calibrated value (when μ does not exceed 1.328). Finally, as for correcting the distortion on L/L_x, the use of μ is on average more welfare-increasing than that of α.

7 Extensions

In this section, we investigate the generality of the baseline model by considering two extended versions. First, we modify the model from fully endogenous growth to semi-endogenous growth, which eliminates scale effects. Second, we introduce physical capital as a factor input into the production of both intermediate goods and innovations. Our analytical results show that the first-best optimal design for the patent instruments is robust to these realistic extensions. However, the quantitative results differ in terms of the magnitudes of welfare comparisons between the decentralized equilibrium and the optimal outcomes.

[36]Of course, using a wider value range for σ amplifies the distortion on the equilibrium allocation of R&D, which may overestimate the welfare improvements after setting σ optimally. Additionally, this will violate the parameter space in Assumption 2.

Table 4: The welfare differences for optimal subsidies.

(1)

σ	0.416	0.425	0.435	0.445	0.455	average
$\mu = 1.1$	0.004	0.039	0.076	0.113	0.148	0.077
$\mu = 1.2$	0.003	0.032	0.065	0.098	0.131	0.067
$\mu = 1.3$	0.000	0.003	0.008	0.016	0.027	0.010
$\mu = 1.35$	0.043	0.027	0.014	0.006	0.000	0.017
$\mu = 1.4$	0.121	0.085	0.052	0.024	0.001	0.055

(2)

α	0.738	0.74	0.745	0.75	0.76	average
$s = 0$	0.483	0.437	0.326	0.220	0.021	0.236
$s = 0.15$	0.483	0.438	0.327	0.220	0.021	0.236
$s = 0.5$	0.484	0.439	0.327	0.221	0.021	0.236
$s = 0.8$	0.486	0.441	0.329	0.222	0.021	0.237
$s = 1$	0.491	0.445	0.332	0.224	0.021	0.240

Notes: Suppose that σ and α are the control variable in sub-tables (1) and (2), respectively. All the entry numbers denote the welfare gains in percentage between the equilibrium and the second-best outcome. The range in consideration for σ is $[0.415046, 0.455546]$, whereas the counterpart for α is $[0.7378, 0.761111]$.

7.1 Semi-Endogenous Growth

In our baseline setting, the population size is normalized to unity and the equilibrium growth rate of technology depends on the size of research labor, implying that scale effects are present as in the early endogenous Schumpeterian growth models (e.g., Grossman and Helpman (1991) and Aghion and Howitt (1992)). Nonetheless, Jones (1995) and the subsequent studies argue that scale effects are not consistent with the observations of growth in modern industrialized economies. Hence, following Segerstrom (1998) and Chu and Furukawa (2011), we reexamine optimal patent policy and its implications in a quality-ladder model with semi-endogenous growth by incorporating population growth and a fishing-out effect on innovations.

The population in this economy (or the representative household) now grows at a constant rate $n \in (0, \rho)$, and its size evolves according to $\dot{N}_t = nN_t$. Consequently, the labor-market-clearing condition becomes $L_t + L_{x,t} + L_{r,t} = N_t$ if each individual is still endowed with one unit of time. The household's preference is given by

$$U = \int_0^\infty e^{-(\rho-n)t} \left[\ln C_t + \phi \ln(L_t/N_t)\right] dt, \tag{35}$$

where C_t is the per capita consumption. Furthermore, the law of motion for per capita assets is $\dot{V}_t = (R_t - n) V_t + W_t (1 - L_t/N_t) - E_t - T_t$, where $E_t \equiv P_t C_t$ and T_t are the nominal consumption expenditure and the lump-sum tax for each individual, respectively. Other notations are the same as before.

Suppose that the arrival rate of innovations is subject to a fishing-out effect: innovations become more difficult as the aggregate technology advances, and the formulation is given by $\lambda_t = (\varphi/Z_t) L_{r,t}$. Therefore, along the BGP, the growth rate of technology requires a constant arrival rate of innovations, implying that this steady-state (equilibrium) growth rate equals the population growth rate, namely, $g = \lambda \ln z = n$. In this case, the steady-state growth rate of technology becomes independent of the population size, and thus scale effects do not occur. Additionally, in Appendix B we derive the steady-state equilibrium labor allocations as follows:

$$L_t = \phi\alpha\mu L_{x,t}, \tag{36}$$

$$L_{x,t} = \left[1 + \phi\alpha\mu + \frac{\alpha\lambda(\mu-1)}{\sigma}\left(\frac{1-s}{\rho-n+\lambda} + \frac{s\lambda}{(\rho-n+\lambda)^2}\right)\right]^{-1} N_t, \tag{37}$$

$$L_{r,t} = \frac{\alpha\lambda(\mu-1)}{\sigma}\left[\frac{1-s}{\rho-n+\lambda} + \frac{s\lambda}{(\rho-n+\lambda)^2}\right] L_{x,t}, \tag{38}$$

where $\lambda = n/\ln z$.

As for the social optimum shown in Appendix B, maximizing (35) subject to the constraints $C_t = Z_t L_{x,t}/N_t$, $L_t + L_{x,t} + L_{r,t} = N_t$, and $\dot{Z}_t = \varphi \ln z L_{r,t}$ yields the optimal labor allocations along the BGP:

$$L_t^* = \phi L_{x,t}^*, \tag{39}$$

$$L_{x,t}^* = \left(1 + \phi + \frac{n}{\rho}\right)^{-1} N_t, \tag{40}$$

$$L_{r,t}^* = \frac{n}{\rho} L_{x,t}^*. \tag{41}$$

Comparing the ratio of $L_t/L_{x,t}$ between the steady-state equilibrium in (36) and the social optimum in (39) reveals that the distortion in the relative supply of labor is eliminated by optimal patent breadth remaining as $\mu^* = 1/\alpha$, which is still determined by the exogenous subsidy rate for intermediate-goods production. Moreover, using optimal patent breadth and comparing (37)-(38) and (40)-(41) generates the optimal profit-division rule such that

$$s^* = \left(1 + \frac{n/\rho}{(1-n/\rho)\ln z}\right)\left[1 - \frac{\sigma}{1-\alpha}\left(\ln z\left(1 - \frac{n}{\rho}\right) + \frac{n}{\rho}\right)\right]. \tag{42}$$

In the absence of scale effects on economic growth, s^* continues to decrease in z, α, and σ, as in the baseline model. However, (42) implies that n/ρ has both a positive and a negative effect on s^*, and that which effect dominates is ambiguous. Intuitively, the profit-division rule pins down the relative allocation of research labor against other labor inputs, which helps remove the distortion on R&D. We investigate this relation by comparing the ratio of R&D-production labor between the steady-state equilibrium and the social optimum. Combining (38) and (41) and rearranging it yields

$$\frac{L_{r,t}/L_{x,t}}{L_{r,t}^*/L_{x,t}^*} = \frac{\frac{1-\alpha}{\sigma}\left[\frac{1-s}{\ln z(\rho/n-1)+1} + \frac{s}{(\ln z(\rho/n-1)+1)^2}\right]}{n/\rho}, \tag{43}$$

where both the numerator and the denominator increase as n/ρ increases. One the one hand, for a relatively small $\ln z$, a higher n/ρ tends to increase $\frac{L_{r,t}/L_{x,t}}{L_{r,t}^*/L_{x,t}^*}$;[37] as compared to in the social

[37]See Appendix B for the level of $\ln z$ at which the positive effect of n/ρ on s^* dominates the negative one.

optimum, in the steady-state equilibrium too much R&D labor is allocated to overcome the fishing-out effect caused by the increasing complexity of innovations. Thus, s^* rises to offset this impact by depressing the equilibrium R&D.[38] On the other hand, for a relative large $\ln z$, a higher n/ρ would decrease $\frac{L_{r,t}/L_{x,t}}{L^*_{r,t}/L^*_{x,t}}$; there is too much manufacturing labor allocated in equilibrium. In this case, s^* declines to stimulate R&D, which promotes innovations and maintains the steady-state equilibrium growth.

For the analytical welfare comparison between the first-best outcome through optimizing both patent instruments and the second-best outcome with optimizing only the profit-division rule s as in Section 5.3, in Appendix B we show that under semi-endogenous growth, the equilibrium level of R&D labor in the second-best outcome still equals the counterpart in the first-best outcome. The welfare difference between these two outcomes is captured similarly by (32), except that the welfare loss is amplified because the utility function comes from the representative household, whose population grows at the rate n. Moreover, the analysis for the effect of optimizing only patent breadth μ on R&D investment in this extension is analogous to that in Section 5.4.

Accordingly, we can perform a quantitative analysis to evaluate the welfare differences between the decentralized equilibrium and the optimal outcomes in this extension. The second-best outcomes are obtained by optimizing s under $\mu = 1.3$ in Table 5-(1) and by optimizing μ under $s = 0.8$ in Table 5-(2), respectively.[39] The parameters are calibrated by the same values as previously, and we set $n = 0.0216$, which is close to the long-run average growth rate of labor force in the US (roughly 0.02).[40] Thus, the first-best patent breadth is still given by $\mu^* = 4/3$ and the first-best profit-division rule is maintained at $s^* = 0.5$.

It can be seen that the qualitative patterns of the welfare differences do not differ considerably from those in the baseline model: (a) optimizing a mix of patent instruments yields more welfare improvements than optimizing either of them; (b) optimizing μ is much more welfare-enhancing than optimizing s, because the majority of the welfare gains in both Tables 5-(1) and 5-(2) are contributed by adjusting μ.[41] More importantly, the welfare effects of the patent instruments become more significant. In contrast to the limited impact of optimizing s in the previous analysis as shown by ξ_2 in Table 1-(3), under this setting the welfare gain of using the second-best optimal s^{**} rises from 0.004% to 0.26% for the market equilibrium with $\mu = 1.3$ and $s = 0.15$ and from 0.023% to 1.04% on average. Also, within the same ranges of the calibrated values, the welfare gains from the second-best outcomes to the first-best outcome enlarge drastically under semi-endogenous growth (i.e., the average is 2.89% for s^{**} and 0.53% for μ^{**}) as compared to under fully endogenous growth (i.e., the average is 0.029% for s^{**} and 0.001% for μ^{**}). This result verifies our prediction

[38] Notice that in the numerator of (43), the marginal change of $\frac{-s}{\ln z(\rho/n-1)+1}$ with respect to s outweighs that of $\frac{s}{(\ln z(\rho/n-1)+1)^2}$. Hence, a decrease (increase) in the numerator requires a higher (lower) s.

[39] In this numerical exercise, the welfare function relies on the BGP utility function of the household given by (B.16). Under semi-endogenous growth, the welfare effect of patent instruments mainly depends on the channel of R&D labor, which dominates the welfare effect through the channels of consumption production and leisure. Thus, if μ is too small (large), which makes the equilibrium R&D level low (high), the welfare improvement of optimizing a coordination of patent instruments is overstated (undermined). To balance these impacts on allocations, we focus on a medium range of $\mu \in [1.24, 1.3333]$ with a high level of s in Table 5-(2) and $\mu = 1.3$ is chosen as the steady-state equilibrium value in Table 5-(1) for making reasonable comparisons against the baseline setting.

[40] Data source: the Bureau of Labor Statistics.

[41] Notice that the second-best policy instruments in this exercise are given by corner solutions. Specifically, we obtain the second-best optimal $s^{**} = 0$ in Table 5-(1) and the second-best optimal $\mu^{**} = 1.3333$ in Table 5-(2), respectively.

in Appendix B.

Table 5: The welfare differences for semi-endogenous growth.

(1) $\mu = 1.3$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	average
ξ_1	3.13	3.23	3.41	3.60	3.78	3.98	4.17	4.36	4.56	4.76	3.93
ξ_2	0.26	0.35	0.53	0.71	0.90	1.08	1.27	1.46	1.65	1.84	1.04
ξ	2.87	2.87	2.88	2.88	2.89	2.89	2.90	2.90	2.91	2.92	2.89

(2) $s = 0.8$

μ	1.24	1.25	1.26	1.27	1.28	1.29	1.30	1.31	1.32	1.33	average
ξ_1	14.82	12.66	10.69	8.89	7.25	5.74	4.36	3.09	1.92	0.84	6.70
ξ_2	14.24	12.10	10.14	8.35	6.71	5.22	3.84	2.58	1.41	0.34	6.17
ξ	0.57	0.56	0.55	0.54	0.54	0.53	0.52	0.51	0.51	0.50	0.53

Notes: ξ_1, ξ_2, and ξ denote the welfare gains (losses) in percentage between the equilibrium and the first-best outcome, between the equilibrium and the second-best outcome, and between the second-best outcome and the first-best outcome, respectively.

7.2 Physical Capital Accumulation

In this subsection, we follow Chu (2009) to consider an extension of the semi-endogenous model in the previous analysis by incorporating both capital and labor as factor inputs into the production of intermediate goods and innovations.

The population size of the household is N_t growing at the rate $n \in (0, \rho)$, and the household's utility function is still given by (35). However, the final goods now can be either consumed by the household or invested in physical capital accumulation. To ensure the balanced growth (which will be described below), we choose final goods as the numeraire implying that $P_t = 1$. Therefore, the law of motion for per capita asset becomes $\dot{A}_t = (R_t - n)\, A_t + W_t\,(1 - L_t/N_t) - C_t - T_t$, in which A_t denotes the value of risk-free financial assets in the form of patents and physical capital owned by each household member. The familiar Euler equation derived from the household's optimization yields the growth rate of per capita consumption such that $g_c \equiv \dot{C}_t/C_t = R_t - \rho$.

The final-goods production follows (5), but the demand for intermediate-goods in industry i becomes $X_t(i) = Y_t/P_t(i)$. The intermediate-goods production function is altered to $X_t(i) = z^{q_t(i)} K^{\theta}_{x,t}(i) L^{1-\theta}_{x,t}(i)$, where $K_{x,t}(i)$ is the capital inputs for producing intermediate goods i and θ is the capital share in production. From cost minimization, the marginal cost of production for the current leader in industry i is $MC_t(i) = \left(\alpha/z^{q_t(i)}\right) (Q_t/\theta)^{\theta}\, (W_t/(1-\theta))^{1-\theta}$, where Q_t is the rental price of capital and $1-\alpha$ is the fixed rate of subsidy to intermediate-goods production that applies to the cost of both capital and labor. Using the definition of patent breadth μ_t, the monopolistic price is given by (9), and the leader's profit is $\Pi_t(i) = (1 - 1/\mu_t)\, P_t(i) X_t(i) = (1 - 1/\mu_t)\, Y_t$. The factor payments for labor and capital are $\alpha W_t L_{x,t} = ((1-\theta)/\mu_t)\, Y_t$ and $\alpha Q_t K_{x,t} = (\theta/\mu_t)\, Y_t$, respectively.

With the infringement of sequential innovations along a quality ladder and the profit-division rule s_t for the licensing agreement between the entrant and the incumbent, the no-arbitrage conditions for the values of the second-most recent innovation and the most recent innovation are respectively given by (11) and (12). In the R&D sector, the arrival rate of innovations for an R&D firm j is now a function of capital and labor such that $\lambda_t(j) = \bar{\varphi}_t K_{r,t}^{\theta}(j) L_{r,t}^{1-\theta}(j)$, where $\bar{\varphi}$ represents R&D productivity that the R&D firm takes as given. Hence, in the symmetric equilibrium, the first-order conditions for an R&D firm's profit maximization are given by $(1-\theta)V_{1,t}\bar{\varphi}_t \left(K_{r,t}/L_{r,t}\right)^{-\theta} = \sigma W_t$ and $\theta V_{1,t}\bar{\varphi}_t \left(K_{r,t}/L_{r,t}\right)^{\theta-1} = \sigma Q_t$, where $1-\sigma$ is the fixed rate of subsidy to R&D that applies to the cost of both capital and labor. To remove scale effects, the R&D productivity takes a formulation of $\bar{\varphi} = \varphi (K_{r,t}^{\theta} L_{r,t}^{1-\theta})^{\gamma-1}/Z_t^{1-\beta}$, where $\gamma \in (0,1)$ captures the negative duplication externality and $\beta \in (-\infty, 1)$ captures the knowledge spillovers externality, respectively. Consequently, the aggregate-level arrival rate of innovations is given by $\lambda_t = \varphi (K_{r,t}^{\theta} L_{r,t}^{1-\theta})^{\gamma}/Z_t^{1-\beta}$, and the growth rate of technology is $g_z \equiv \dot{Z}_t/Z_t = \lambda_t \ln z$.

In the decentralized equilibrium, the final-goods market clears such that $Y_t = C_t N_t + I_t$, where I_t denotes the level of capital investment. The capital market and the labor market clear such that $K_t = K_{x,t} + K_{r,t}$ and $N_t = L_t + L_{x,t} + L_{r,t}$, respectively. The capital stock evolves according to $\dot{K}_t = Y_t - C_t N_t - \delta K_t$, where δ is the depreciation rate that satisfies the no-arbitrage condition for capital such that $R_t = Q_t - \delta$. Using the intermediate-goods demand and (5) yields the aggregate production function of final goods such that $Y_t = Z_t K_{x,t}^{\theta} L_{x,t}^{1-\theta}$. On the balanced growth path, each labor allocation grows at the rate n. Then, the growth rate of technology is given by $g_z = \gamma(\theta g_k + (1-\theta)n)(1-\beta)^{-1}$, where g_k denotes the growth rate of capital. Also, the growth rate of consumption per capita is $g_c = g_y - n$, where g_y denotes the growth rate of final goods. Imposing the BGP on the aggregate production function Y_t yields $g_k = g_y = n + g_z/(1-\theta)$, which implies $g_z = [(1-\beta)/\gamma - \theta/(1-\theta)]^{-1} n$. Thus, the steady-state arrival rate of innovations is given by $\lambda = g_z/\ln z$, and the steady-steady real interest rate is $R_t = R = \rho + g_c = \rho + g_y - n$.

Denote the rate of capital investment by $i_t \equiv I_t/Y_t$. In Appendix C, we derive the steady-state equilibrium labor allocations as follows:

$$L_t = (1-i)\frac{\phi\alpha\mu}{1-\theta} L_{x,t}, \tag{44}$$

$$L_{x,t} = \left[1 + (1-i)\frac{\phi\alpha\mu}{1-\theta} + \frac{\alpha\lambda(\mu-1)}{\sigma}\left(\frac{1-s}{\rho-n+\lambda} + \frac{s\lambda}{(\rho-n+\lambda)^2}\right)\right]^{-1} N_t, \tag{45}$$

$$L_{r,t} = \frac{\alpha\lambda(\mu-1)}{\sigma}\left[\frac{1-s}{\rho-n+\lambda} + \frac{s\lambda}{(\rho-n+\lambda)^2}\right] L_{x,t}, \tag{46}$$

and the steady-state equilibrium R&D share of factor inputs and the rate of capital investment as follows:

$$\frac{m}{1-m} = \frac{\alpha\lambda(\mu-1)}{\sigma}\left[\frac{1-s}{\rho-n+\lambda} + \frac{s\lambda}{(\rho-n+\lambda)^2}\right], \tag{47}$$

$$i = \frac{\theta}{\alpha\mu}\left(1 + \frac{m}{1-m}\right)\frac{g_k+\delta}{R+\delta}, \tag{48}$$

where $m = m_k = m_l$, $m_k \equiv K_{r,t}/K_t$, and $m_l \equiv L_{r,t}/L_t$.

As for the first-best allocations, the social planner chooses leisure L_t, the ratio of R&D allocation m_t, and the rate of capital investment i_t to maximize the households' utility subject to the constraints of final-goods production, labors, capital accumulation, and technology. The derivation is shown in Appendix C. This optimization yields the optimal R&D share of factor inputs given by

$$\frac{m^*}{1-m^*}=\frac{L^*_{r,t}}{L^*_{x,t}}=\frac{K^*_{r,t}}{K^*_{x,t}}=\frac{\gamma g_z}{\rho-n+(1-\beta)g_z}, \tag{49}$$

the optimal rate of capital investment given by

$$i^*=\theta\left(1+\frac{m^*}{1-m^*}\right)\frac{g_k+\delta}{R+\delta}, \tag{50}$$

and the optimal ratio of leisure and production labor such that

$$L^*_t=(1-i^*)\frac{\phi}{1-\theta}L^*_{x,t}. \tag{51}$$

Notice that the mix of μ and s suffices to equate $m/(1-m)$ and i_t under the steady-state equilibrium (i.e., (47) and (48)) to $m^*/(1-m^*)$ and i^*_t under the social optimum (i.e., (49) and (50)). Hence, comparing $L_t/L_{x,t}$ in (44) and $L^*_t/L^*_{x,t}$ in (51) reveals that optimal patent breadth continues to be $\mu^*=1/\alpha$. In addition, using optimal patent breadth and comparing (47) and (49) yields the optimal profit-division rule such that

$$s^*=\frac{1}{1-1/\Phi}\left[1-\frac{\sigma}{1-\alpha}\left(\frac{\gamma}{(1-1/\Phi)/\ln z+(1-\beta)/\Phi}\right)\right], \tag{52}$$

where $\Phi\equiv\ln z[(1-\beta)/\gamma-\theta/(1-\theta)](\rho/n-1)+1$. Obviously, s^* in (52) reduces to the one without capital accumulation in (42) when $\beta=0$, $\gamma=1$, and $\theta=0$.

Comparing the steady-state equilibrium and the social optimum indicates that this extended model features four layers of allocative distortions: (a) the distortion on the relative supply of labor given by L/L_x; (b) the distortion on the relative allocation of R&D in labor given by L_r/L_x; (c) the distortion on the relative allocation of R&D in capital given by K_r/K_x; and (d) the distortion on the investment for capital accumulation given by i. However, the above result demonstrates that the optimal coordination of patent instruments is sufficient for correcting all these distortions and reaching the first-best outcome. The reason is straightforward. Along the BGP, the R&D share of factor inputs is identical under both labor and capital, implying that the distortions (b) and (c) coincide with each other and can be removed simultaneously. Moreover, as shown in Chu (2009), the discrepancy between the steady-state rate of capital investment and the first-best rate (i.e., distortion (d)) stems from the markup μ in addition to the difference between the equilibrium R&D share of capital and its socially optimal share (i.e., distortion (b)). More importantly, the discrepancy between the steady-state ratio of leisure and production labor and its first-best counterpart (i.e., distortion (a)) stems from the markup μ in addition to the difference between the steady-state rate of capital investment and the first-best rate (i.e., distortion (d)). In other words, once the wedges in allocations caused by the markup and the R&D share of factor inputs (i.e., $m/(1-m)$) are eliminated, all of the above four distortions are simultaneously remedied; thus, the first-best outcome can be restored. This objective is achievable by employing patent breadth together with the profit-division rule to adjust the effects of markup and $m/(1-m)$.

As for the comparative statics of the optimal profit-sharing rule, observing (52) shows that s^*

continues to decrease in α and σ, as in the prior two models. In addition, combining (46) and (49) yields the labor allocation in R&D relative to manufacturing such that

$$\frac{L_{r,t}/L_{x,t}}{L^*_{r,t}/L^*_{x,t}} = \frac{1-\alpha}{\sigma}\left[\frac{1-s}{\tilde{\Phi}\ln z(\rho/n-1)+1} + \frac{s}{(\tilde{\Phi}\ln z(\rho/n-1)+1)^2}\right]\left[\frac{\tilde{\Phi}(\rho/n-1)+1-\beta}{\gamma}\right], \tag{53}$$

where $\tilde{\Phi} \equiv (1-\beta)/\gamma - \theta/(1-\theta)$. Therefore, like the similar reasoning applied in (43), s^* is strictly decreasing in z, but it could be increasing or decreasing in n/ρ, β, and θ, depending on the size of $\ln z$.

Denote $l_t \equiv L_t/N_t$, $l_{x,t} \equiv L_{x,t}/N_t$, and $k_{x,t} \equiv K_{x,t}/N_t$. To quantify the welfare comparisons between the decentralized equilibrium and the optimal outcomes in this extension, imposing balanced growth on (35) yields the steady-state equilibrium welfare function given by

$$U = \frac{1}{\rho-n}\left[\ln C_0 + \phi \ln l + \frac{g_c}{\rho-n}\right], \tag{54}$$

where $C_0 = (1-i)Z_0(1-m)^\theta k_0^\theta l_x^{1-\theta}$, $Z_0 = [\varphi \ln z(N_0 m^\theta k_0^\theta l_r^{1-\theta})^\gamma/g_z]^{1/(1-\beta)}$, $g_c = g_z/(1-\theta) = [(1-\beta)/\gamma - \theta/(1-\theta)]^{-1}n/(1-\theta)$, and the exogenous terms N_0 and k_0 will be dropped. As in Section 7.1, we continue to focus on the second-best outcomes by optimizing s under $\mu = 1.3$ in Table 6-(1) and by optimizing μ under $s = 0.8$ in Table 6-(2), respectively. The parameters are again calibrated as previously, except that we set $n = 0.01$ to match the long-run average growth rate of US population.[42] There are four structural parameters $\{\theta, \delta, \gamma, \beta\}$ that are new in this numerical exercise. First, we set the annual depreciation rate δ on physical capital and the capital-share parameter θ to their conventional values of 0.08 and 0.3, respectively. As for $\{\gamma, \beta\}$, we use $R = 0.08$ as the market level of real interest rate, which is in line with the historical rate of return on the US stock market. Accordingly, maintaining the first-best value of s^* at 0.5 in our previous analysis, the calibrated value of R&D duplication externality γ and that of knowledge spillovers externality β are approximately 0.98 and 0.23, respectively. The first-best patent breadth μ^* remains at 4/3 in this case.

As shown in Table 6, the qualitative patterns of the numerical results are analogous to those in the previous analysis. It is worthwhile noticing that the size of welfare gains in this exercise is even much larger than those in the previous two models. For example, in Table 6-(1) where s is optimized given the level of μ, starting off with the market equilibrium in which $\mu = 1.3$ and $s = 0.15$, ξ_1, ξ_2, and ξ increases from 0.033%, 0.004%, and 0.029% in the baseline model (or 3.13%, 0.26%, 2.87% in semi-endogenous growth without physical capital) to 6.99%, 0.44%, 6.55% with physical capital. Furthermore, as compared to the setting of no capital, optimizing s in the presence of physical capital accumulation is more prominent to enhance welfare, with the upper bound of the welfare gain increasing from 1.84% to 3.19% and the average increasing from 1.04% to 1.78%. The range of these welfare sizes thus becomes in line with that in Chu (2009). This pattern on the changes in the welfare sizes is also true for the scenario where μ is optimized given the level of s, as shown in Table 6-(2).

[42] Data source: World Development Indicators. Although the choice of n is changed in this exercise, the implied growth rate of technology is $g_z = 0.028$, which is only slightly lower than the counterpart in the previous subsection (i.e., 0.0216).

The reasons for the above changes are twofold as follows. First, the current model follows the foregoing semi-endogenous growth model to use a utility function stemming from the representative household instead of an individual. Second, four layers of distortions exist in this model as above-mentioned. Specifically, in addition to the distortions in labor allocations $\{L, L_x, L_r\}$, distortions in the rate of capital investment i and the R&D share of capital inputs $\{K_r, K_x\}$ are introduced. Due to these extra distortions, the steady-state welfare level in the current model is considerably lower than in the first-best outcome and in the second-best outcomes, and optimizing one patent instrument alone or their mix effectively remedy two more layers of distortions in this variant than in the previous ones. Thus, it is not surprising that welfare improvements in this exercise turn out to be more remarkable when appropriate policy interventions are executed.

Table 6: The welfare differences for semi-endogenous growth with physical capital accumulation.

(1) $\mu = 1.3$

s	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	average
ξ_1	6.99	7.15	7.48	7.81	8.14	8.49	8.83	9.19	9.55	9.91	8.42
ξ_2	0.44	0.60	0.90	1.21	1.53	1.85	2.18	2.51	2.84	3.19	1.78
ξ	6.55	6.56	6.58	6.60	6.62	6.64	6.66	6.68	6.70	6.72	6.63

(2) $s = 0.8$

μ	1.24	1.25	1.26	1.27	1.28	1.29	1.3	1.31	1.32	1.33	average
ξ_1	31.82	27.11	22.84	18.96	15.41	12.17	9.19	6.45	3.92	1.58	14.24
ξ_2	30.73	26.06	21.82	17.97	14.46	11.24	8.28	5.56	3.06	0.73	13.29
ξ	1.09	1.05	1.02	0.99	0.96	0.93	0.90	0.88	0.86	0.84	0.95

Notes: ξ_1, ξ_2, and ξ denote the welfare gains (losses) in percentage between the equilibrium and the first-best outcome, between the equilibrium and the second-best outcome, and between the second-best outcome and the first-best outcome, respectively.

8 Conclusion

In this study, we explore the optimality and welfare implications of an IPR-policy regime in a quality-ladder endogenous growth model where IPR are overlapping. This IPR regime includes two policy instruments: patent breadth and the profit-division rule, to capture the market-power effect and the backloading effect of patent protection on R&D incentives, respectively. In addition, elastic labor supply and subsidies to the production of intermediate goods and research are taken into consideration. In this model, the equilibrium labor allocations are subject to two layers of distortions, namely, the distortion on the ratio of leisure and the manufacturing labor (i.e., relative labor supply) and the distortion on the relative allocation of R&D labor (against other labor inputs). The government can therefore adjust the labor allocations to mitigate these distortions by implementing the patent-policy tools and the subsidy-policy tools.

Our results show the interrelation between patent instruments and subsidy instruments that replicates the first-best optimal outcome. Specifically, patent breadth and intermediate-goods subsidies are substitutable in removing the distortion on relative labor supply, whereas the profit-division rule and R&D subsidies are substitutable in removing the distortion on the relative allocation of R&D. We then provide evidence to support the claim that in reality, the use of subsidies is more constrained than the use of patents. Thus, a pragmatic strategy for the policymaker is to manipulate resource allocations to steer the market economy with the aid of patent-policy levers, while taking subsidies as exogenous. Accordingly, the first-best optimal mix of patent levers is derived.

Moreover, we consider the second-best outcomes, in which one patent instrument is optimized and the other is fixed at a predetermined level. We found that optimizing the profit-division rule only is sufficient for obtaining the first-best R&D level and growth rate, but optimizing patent breadth only may lead to over- or under-investment in R&D. Due to the possibility of a suboptimal choice on the patent instrument that is fixed, the economy would still suffer welfare losses. Our numerical analysis shows that starting off with the decentralized equilibrium, the welfare gains by applying the optimal coordination of patent instruments can be substantial. We show that the welfare improvement by optimizing patent breadth is much larger than the counterpart by optimizing the profit-division rule, despite the fact that the first-best growth rate is attained in the latter case.

We also consider two extended versions of the baseline model by incorporating semi-endogenous growth and physical capital accumulation. The theoretical implications are quite robust to these extensions, but the magnitudes of the welfare comparisons turn out to be even more significant; the patent instruments are of more crucial use for welfare improvements in these realistic modifications.

This study thereby complements studies on policy applications for optimal patent protection that has a role in blocking future innovations, given that the choice in the profit-division rule critically determines the backloading effect of blocking patents. This study also presents an example in the growth and welfare analysis that reveals the characteristics of multiple dimensionality in a frequently perceived patent system. In particular, in sharp contrast to the existing literature that considers the welfare effect of blocking patents alone, the present study provides an important policy implication: the use of blocking patents is closely related to other forms of patent protection (e.g., patent breadth), and a comprehensive design of the welfare-policy regime should take into account more dimensions in the modern IPR system, because their effects on allocating resources and eliminating distortions can be very different.

Appendix A

Proof of Proposition 1

Suppose that the government chooses a stationary path of $[\mu_t, s_t, \alpha_t, \sigma_t]_{t=0}^{\infty}$. Define transformed variables $\Psi_{1,t} \equiv P_t C_t / V_{1,t}$ and $\Psi_{2,t} \equiv P_t Y_t / V_{2,t}$. First, differentiating $\Psi_{2,t}$ with respect to t yields

$$\frac{\dot{\Psi}_{2,t}}{\dot{\Psi}_{2,t}} \equiv \frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} - \frac{\dot{V}_{2,t}}{V_{2,t}} = \frac{\dot{E}_t}{E_t} - \frac{\dot{V}_{2,t}}{V_{2,t}} = R_t - \rho - \frac{\dot{V}_{2,t}}{V_{2,t}}, \tag{A.1}$$

where the definition of E_t is used in the second equality and the third equality follows the Euler equation in (4). Combining (10), (12), and (14), the no-arbitrary condition for $V_{2,t}$ can be expressed as

$$\frac{\dot{V}_{2,t}}{V_{2,t}} = R_t - s\left(1 - \frac{1}{\mu}\right)\Psi_{2,t} + \varphi L_{r,t}. \tag{A.2}$$

Substituting (A.2) into (A.1) yields

$$\frac{\dot{\Psi}_{2,t}}{\Psi_{2,t}} = s\left(1 - \frac{1}{\mu}\right)\Psi_{2,t} - \varphi L_{r,t} - \rho. \tag{A.3}$$

Similarly, using $C_t = Y_t$ along with (4), (10), (11), and (14) yields

$$\frac{\dot{\Psi}_{1,t}}{\Psi_{1,t}} = (1-s)\left(1 - \frac{1}{\mu}\right)\Psi_{1,t} - \varphi L_{r,t} + \varphi L_{r,t}\frac{V_{2,t}}{V_{1,t}} - \rho. \tag{A.4}$$

To derive the relationship between $L_{r,t}$, $\Psi_{1,t}$, and $\Psi_{2,t}$, we first use $W_t = 1$ in (15) to obtain $V_{1,t} = \sigma/\varphi$. Substituting this condition, $L_{x,t} = E_t/(\alpha\mu)$, and $L_t = \phi E_t$ into the labor-market-clearing condition yields

$$L_{r,t} = 1 - \frac{\sigma}{\varphi}\left(\frac{1}{\alpha\mu} + \phi\right)\Psi_{1,t}. \tag{A.5}$$

Then, using (A.5) in (A.3) and (A.4) yields

$$\frac{\dot{\Psi}_{1,t}}{\Psi_{1,t}} = \left[(1-s)\left(1 - \frac{1}{\mu}\right) + \sigma\left(\frac{1}{\alpha\mu} + \phi\right)\right]\Psi_{1,t} + \varphi\frac{\Psi_{1,t}}{\Psi_{2,t}} - \sigma\left(\frac{1}{\alpha\mu} + \phi\right)\frac{\Psi_{1,t}^2}{\Psi_{2,t}} - (\rho + \varphi), \tag{A.6}$$

$$\frac{\dot{\Psi}_{2,t}}{\Psi_{2,t}} = s\left(1 - \frac{1}{\mu}\right)\Psi_{2,t} + \sigma\left(\frac{1}{\alpha\mu} + \phi\right)\Psi_{1,t} - (\rho + \varphi), \tag{A.7}$$

where we use the fact that $V_{2,t}/V_{1,t} = \Psi_{1,t}/\Psi_{2,t}$. Linearizing (A.6) and (A.7) around the steady-state equilibrium yields

$$\begin{bmatrix} \dot{\Psi}_{1,t} \\ \dot{\Psi}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Psi_{1,t} - \Psi_1 \\ \Psi_{2,t} - \Psi_2 \end{bmatrix}, \tag{A.8}$$

where

$$\begin{aligned} a_{11} &= \rho + \varphi - \sigma\left(\tfrac{1}{\alpha\mu} + \phi\right)\tfrac{\Psi_{1,t}^2}{\Psi_{2,t}}, \quad & a_{12} &= -\tfrac{\Psi_{1,t}^2}{\Psi_{2,t}^2}\left[\varphi - \sigma\left(\tfrac{1}{\alpha\mu} + \phi\right)\Psi_{1,t}\right], \\ a_{21} &= \sigma\left(\tfrac{1}{\alpha\mu} + \phi\right)\Psi_{2,t} > 0, & a_{22} &= s\left(1 - \tfrac{1}{\mu}\right)\Psi_{2,t} > 0. \end{aligned} \tag{A.9}$$

To determine the signs of a_{11} and a_{12}, we assume that leisure intensity is sufficiently small such that $\phi < \min\left\{\frac{(V_{1,t}/V_{2,t})(\rho/\varphi+1)}{E_t} - \frac{1}{\alpha\mu}, \frac{1}{E_t} - \frac{1}{\alpha\mu}\right\}$. Accordingly, $\phi < \frac{(V_{1,t}/V_{2,t})(\rho/\varphi+1)}{E_t} - \frac{1}{\alpha\mu}$ implies $a_{11} > 0$, whereas $\phi < \frac{1}{E_t} - \frac{1}{\alpha\mu}$ implies $a_{12} < 0$. Notice that with (A.5), the latter inequality of ϕ conforms to Assumption 1 such that the R&D labor is nonnegative. Moreover, it can be shown that around the steady-state equilibrium, the former boundary of ϕ is always larger than the latter.[43] [44] Thus, Assumption 1 suffices to ensure this low level of ϕ in this proof.

Let υ_1 and υ_2 be the two characteristic roots of the dynamical system. The trace of the Jacobian is given by $\text{Tr} = \upsilon_1 + \upsilon_2 = a_{11} + a_{22} > 0$. Moreover, the determinant of the Jacobian is given by $\text{Det} = \upsilon_1\upsilon_2 = a_{11}a_{22} - a_{12}a_{21} > 0$. Therefore, the two characteristic roots are both positive. Given that $\Psi_{1,t}$ and $\Psi_{2,t}$ are jump variables, the above findings imply that the dynamical system displays saddle-point stability. As indicated in the phase diagram in Figure 2, where the $\dot{\Psi}_{2,t}$ locus is downward-sloping whereas the $\dot{\Psi}_{1,t}$ locus is upward-sloping, $\Psi_{1,t}$ and $\Psi_{2,t}$ must jump to their steady-state values in the unique equilibrium given by Point A.[45]

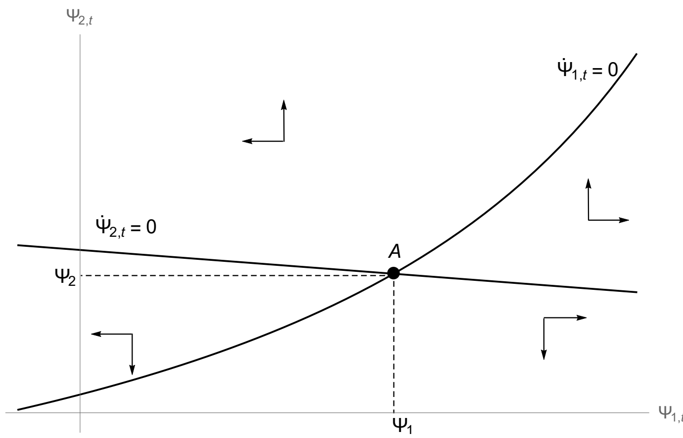


Figure 2: Phase diagram

Derivations for Consumption Expenditure and Equilibrium Labor Allocations

According to Proposition 1, normalizing $W_t = 1$ together along (15) and $V_{1,t} + V_{2,t} = V_t$ implies $\dot{V}_{1,t} = \dot{V}_{2,t} = \dot{V}_t = 0$. Thus, (11) becomes

$$V_{2,t} = \frac{s\Pi_t}{R_t + \lambda_t}. \tag{A.10}$$

[43] Restricting the range of ϕ simplifies the discussion for the dynamics of the model, which excludes the possibility of indeterminancy. In fact, the boundary of ϕ can be reexpressend as $E_t < \min\left\{\frac{\alpha\mu\left(V_{1,t}/V_{2,t}\right)(\rho/\varphi+1)}{1+\phi\alpha\mu}, \frac{\alpha\mu}{1+\phi\alpha\mu}\right\}$. As will be shown in the next subsection, the equilibrium value of E_t can only lie within this range. Around the steady-state equilibrium, $\dot{\Psi}_{1,t} = 0$ and $\dot{V}_{1,t} = 0$ together imply $\dot{E}_t = 0$, so the Euler equation in (4) yields $R_t = \rho$. Using this condition with the equilibrium value of E_t reveals that $(V_{1,t}/V_{2,t})(\rho/\varphi + 1) > 1$ always holds. Hence, $L_r > 0$ in (A.5) suffices to guarantee $E_t < \frac{\alpha\mu}{1+\phi\alpha\mu} < \frac{\alpha\mu\left(V_{1,t}/V_{2,t}\right)(\rho/\varphi+1)}{1+\phi\alpha\mu}$. This result confirms that the steady-state equilibrium is indeed saddle-point stable.

[44] *Mathematica* files for the derivation in this proof, which are available upon request.

[45] Specifically, the $\dot{\Psi}_{1,t}$ locus is hyperbolic, but it exhibits convexity using the assumption of $\phi < 1/E_t - 1/\alpha\mu$ to pin down $\Psi_1 > 0$ in the steady state, which ensures uniqueness by intersecting only once with the $\dot{\Psi}_{2,t}$ locus.

Furthermore, adding (11) to (12) gives

$$R_t = \frac{1}{V_t}\left(\Pi_t - \frac{\sigma\lambda_t}{\varphi}\right). \tag{A.11}$$

From (10), we know that $\Pi_t = (1 - 1/\mu)\, E_t$. In addition, substituting (3) and (14) into the labor-market-clearing condition yields $\lambda_t/\varphi = 1-(\phi + 1/(\alpha\mu))\, E_t$. Therefore, combining these conditions with (A.10) and (A.11) to substitute for V_t yields

$$R_t = \frac{\varphi\,[R_t + \varphi(1 - (\phi + 1/(\alpha\mu))\, E_t)]}{\sigma\,[R_t + \varphi(1 - (\phi + 1/(\alpha\mu))\, E_t)] + \varphi s\,(1 - 1/\mu)\, E_t}\left[\left(1 + \sigma\phi + \frac{1}{\mu}\left(\frac{\sigma}{\alpha} - 1\right)\right) E_t - \sigma\right]. \tag{A.12}$$

Solving the quadratic equation in (A.12) yields a solution of R_t as a function of E_t:

$$R_t(E_t) = \frac{\varphi}{2\alpha\sigma\mu}\left(M(E_t) + \sqrt{M(E_t)^2 - 4\sigma\left((\sigma E_t + \alpha(\mu + \sigma\phi\mu - 1)E_t - \alpha\sigma\mu)(E_t + \alpha\mu(\phi E_t - 1))\right)}\right). \tag{A.13}$$

where $M(E_t) \equiv -2\alpha\sigma\mu + (2\sigma + \alpha((u-1)(1-s) + 2\sigma\phi\mu))E_t$. Note that we exclude the other root of (A.12) because it can only be negative (see Footnote 44). Combining (A.13) and (4) yields a differential equation for E_t:

$$\frac{\dot{E}_t}{E_t} = \frac{\varphi}{2\alpha\sigma\mu}\left(M(E_t) + \sqrt{M(E_t)^2 - 4\sigma\left((\sigma E_t + \alpha(\mu + \sigma\phi\mu - 1)E_t - \alpha\sigma\mu)(E_t + \alpha\mu(\phi E_t - 1))\right)}\right) - \rho. \tag{A.14}$$

Along the BGP, Proposition 1 implies that E_t grows at the same rate as $V_{1,t}$ and $V_{2,t}$, which is constant. Moreover, it can be shown that $\dot{E}_t/E_t$ is a concave function of E_t.[46] The solution of setting (A.14) to zero indicates that two potential (low and high) steady-state equilibria may occur. However, only the low steady-state equilibrium is smaller than $\overline{E} \equiv \frac{\alpha\mu}{1+\phi\alpha\mu}$, which is consistent with the assumption imposed in the proof of Proposition 1, such that

$$E = \frac{\alpha\mu\left(Q - \sqrt{Q^2 - 4(\varphi+\rho)^2(1+\alpha\phi\mu)(\sigma + \alpha(\sigma\phi\mu + \mu - 1))}\right)}{2\varphi(1+\alpha\phi\mu)(\sigma + \alpha(\mu + \sigma\phi\mu - 1))}. \tag{A.15}$$

where $Q \equiv 2\sigma(\varphi+\rho) + \alpha\left[2\sigma\phi(\mu\varphi + \rho) + (\mu - 1)(\varphi + \rho(1-s))\right] > 0$ (see Footnote 44).

In fact, the high steady-state equilibrium of E is greater than $\overline{E}$, which violates the requirement of nonnegative R&D labor; so it is abandoned. Finally, the stationarity of E_t ensures that the labor allocations of L_t, $L_{x,t}$ and $L_{r,t}$ are stationary in equilibrium.

Proof of Lemma 1

Firstly, holding constant μ, s, α and σ and using Assumption 1, we derive that when $\lambda = 0$, the LHS of (22) $\left(\text{i.e., } \frac{\alpha\rho(-\sigma\phi\rho+\varphi(1-s))}{\sigma(1+\alpha\phi)}\right)$ is always greater than the RHS of (22) $\left(\text{i.e., } \frac{\rho^2}{\mu-1}\right)$. It is shown that the LHS of (22) is a concave function of λ, whereas the RHS of (22) is a convex function of λ. Thus, there exists only one intersection between LHS and RHS of (22), which solves for the

[46] Specifically, $\frac{\partial^2\left(\dot{E}_t/E_t\right)}{\partial E_t^2} = -\frac{2\varphi\alpha^3\sigma s^2\mu(\mu-1)^2}{\left[M(E_t)^2 - 4\sigma((\sigma E_t + \alpha(\mu+\sigma\phi\mu-1)E_t - \alpha\sigma\mu)(E_t + \alpha\mu(\phi E_t - 1)))\right]^{3/2}} < 0$.

equilibrium level of λ. This result is consistent with the uniqueness of the BGP in Proposition 1.

Then, it is clear that the equilibrium growth rate g is increasing in the arrival rate of innovations λ. An increase in μ shifts down the RHS of (22), yielding a higher level of λ and g. Denote the LHS of (22) as $-\alpha\Lambda/(\sigma(1+\alpha\phi))$, where $\Lambda \equiv (1+\sigma\phi)\lambda^2+(-\varphi-\rho(1-s+2\sigma\phi))\,\lambda-\rho(-\sigma\phi\rho+\varphi(1-s))<0$. An increase in α shifts up the LHS of (22) for any λ because $\partial\text{LHS}/\partial\alpha = -\frac{\Lambda}{\sigma(1+\alpha\phi)^2} > 0$, which also increases λ and g.

A rise in s or σ shifts down the LHS of (22) for any λ. This is because given that $\lambda = \varphi L_r$, we obtain $\partial\text{LHS}/\partial s = -\frac{\alpha\varphi\rho(1-L_r)}{\sigma(1+\alpha\phi)} < 0$ and $\partial\text{LHS}/\partial\sigma = -\frac{\alpha\varphi(1-L_r)(\lambda+\rho(1-s))}{\sigma^2(1+\alpha\phi)} < 0$, respectively, both of which imply a lower level of λ and g.

Stability in Social Optimum

For the first-best outcome, the social planner chooses allocations of L_t, $L_{x,t}$, and $L_{r,t}$ to maximize the households' lifetime utility given by (1), subject to the constraint of consumption production, technology, and labors. Then, the current-value Hamiltonian is given by

$$H_t = \ln Z_t + \ln L_{x,t} + \phi\ln L_t + \eta_{1,t}\left(1 - L_t - L_{x,t} - L_{r,t}\right) + \eta_{2,t}\left(\varphi\ln z Z_t L_{r,t}\right), \tag{A.16}$$

where $\eta_{1,t}$ and $\eta_{2,t}$ are the costate variables associated with the labor-market-clearing condition and the law of motion for technology, respectively, and also we use the fact that $C_t = Y_t = Z_t L_{x,t}$. Thus, the first-order conditions (FOCs) for the labor inputs are given respectively as follows:

$$\frac{\partial H_t}{\partial L_t} = 0 \Rightarrow L_t = \frac{\phi}{\eta_{1,t}}; \tag{A.17}$$

$$\frac{\partial H_t}{\partial L_{x,t}} = 0 \Rightarrow L_{x,t} = \frac{1}{\eta_{1,t}}; \tag{A.18}$$

$$\frac{\partial H_t}{\partial L_{r,t}} = 0 \Rightarrow \eta_{1,t} = \eta_{2,t}\varphi\ln z Z_t; \tag{A.19}$$

$$\frac{\partial H_t}{\partial Z_t} = \frac{1}{Z_t} + \eta_{2,t}\varphi\ln z L_{r,t} = \rho\eta_{2,t} - \dot{\eta}_{2,t}. \tag{A.20}$$

Using the definition of $\dot{Z}_t$ and (A.20), we obtain a differential equation such that $\dot{\eta}_{2,t}Z_t + \eta_{2,t}\dot{Z}_t = \rho\eta_{2,t}Z_t - 1$, implying that $\eta_{2,t}Z_t$ must jump to its steady-state value given by $1/\rho$. This implies that the dynamical behavior of the model in the social optimum is also characterized by saddle-point stability. Accordingly, combining this condition with (A.17)-(A.19) and the labor-market-clearing condition yields the first-best allocations that are specified in (24)-(26), which confirms the result of BGP utility maximization in the main text.

Proof of Proposition 2

Using the optimal profit-division rule s^* in (28), we obtain that

$$\frac{\partial s^*}{\partial\rho} = \frac{2\sigma\varphi\ln z - \rho(1-\alpha+2\sigma(1+\phi-\ln z))}{\rho^3(1-\alpha)}, \tag{A.21}$$

$$\frac{\partial s^*}{\partial \phi} = \frac{2\sigma\varphi \ln z - \rho(1-\alpha+2\sigma(1+\phi-\ln z))}{\rho(1-\alpha)\ln z}, \tag{A.22}$$

$$\frac{\partial s^*}{\partial \varphi} = \frac{\rho(1-\alpha+2\sigma(1+\phi-\ln z)) - 2\sigma\varphi\ln z}{\rho^2(1-\alpha)}. \tag{A.23}$$

Given $\varphi > \varphi^-$, it can be shown that since $\sqrt{(1-\alpha)(1-\alpha-4\sigma\ln z)} > 0$, $\partial s^*/\partial\phi$ and $\partial s^*/\partial\rho$ are positive but $\partial s^*/\partial\varphi$ is negative.

Furthermore, the impact of the innovation size z on the optimal s^* is given by

$$\frac{\partial s^*}{\partial \ln z} = \frac{\sigma\left(\varphi+\rho\right)^2\left(\ln z\right)^2 + \rho^2(1+\phi)(\alpha-1-\sigma(1+\phi))}{(\alpha-1)\rho^2\left(\ln z\right)^2}. \tag{A.24}$$

It is obvious that the denominator is negative. To determine the sign of the numerator, which can be considered as a function of φ, we find that the condition $1-\alpha+2\sigma(1+\phi) > 2\sqrt{\sigma(1+\phi)(1-\alpha+\sigma(1+\phi))}$ always holds. This condition together with $\sqrt{(1-\alpha)(1-\alpha-4\sigma\ln z)} > 0$ supports that φ^- is always greater than $\frac{\rho}{\ln z}\sqrt{\sigma(1+\phi)(1-\alpha+\sigma(1+\phi))} - 1$, which is the (larger) root of the numerator in (A.24). Therefore, the numerator is positive so that we obtain $\partial s^*/\partial \ln z < 0$.

Finally, it is easy to derive that

$$\frac{\partial s^*}{\partial \alpha} = -\frac{\left[\varphi\ln z - \rho(1+\phi-\ln z)\right]^2}{(1-\alpha)^2\rho^2\ln z}, \tag{A.25}$$

$$\frac{\partial s^*}{\partial \sigma} = -\frac{\left[\varphi\ln z\rho(1+\phi-\ln z)\right]^2}{(1-\alpha)\rho^2\ln z}, \tag{A.26}$$

both of which are clearly negative.

Appendix B

In this Appendix, we derive the labor allocations in the steady-state equilibrium and in the social optimum, respectively, under semi-endogenous growth. Additionally, we show the comparative statics of the optimal profit-division rule in (42). Finally, we investigate the second-best outcomes for optimizing only a single patent instrument.

Steady-State and Optimal Labor Allocations

With the arrival rate of innovations subject to the fishing-out effect, normalizing W_t to unity implies that V_t, $V_{1,t}$, and $V_{2,t}$ all grow at the rate n along the BGP. Therefore, using the logic in (11) and (12), we obtain that $V_{2,t} = \frac{s\Pi_t}{R_t-n+\lambda}$ and $V_{1,t} = \left[\frac{1-s}{R_t-n+\lambda} + \frac{s\lambda}{(R_t-n+\lambda)^2}\right]\Pi_t$. Moreover, combining the monopoly profit (i.e., $\Pi_t = (1-1/\mu)N_tE_t$), the FOC for the R&D labor (i.e., $V_{1,t} = \sigma L_{r,t}/\lambda$), and the labor-market-clearing condition, a few steps of simplification yield $\frac{V_{1,t}}{\Pi_t} = \frac{\sigma(1/E_t-1/(\alpha\mu)-\phi)}{\lambda(1-1/\mu)}$, where we also use the leisure-consumption relation (i.e., $W_tL_t = \phi E_tN_t$) and the production-labor share of output (i.e., $\alpha W_tL_{x,t} = E_tN_t/\mu$). Moreover, these conditions imply that E_t is constant, given that labor inputs grow at the rate n along the BGP.

Thus, we can derive the relationship between R_t and E_t as follows:

$$\frac{1-s}{R_t-n+\lambda}+\frac{s\lambda}{(R_t-n+\lambda)^2}=\frac{\sigma(1/E_t-1/(\alpha\mu)-\phi)}{\lambda(1-1/\mu)}. \tag{B.1}$$

From (B.1), we know that R_t is an increasing function of E_t, so the Euler equation implies that $\dot{E}_t/E_t$ is also increasing in E_t. In addition, when $E_t \to 0$, $R_t \to n-\lambda$ and $\dot{E}_t/E_t < 0$ since $n < \rho$. Then, the steady-state equilibrium value of E_t must be positive. Hence, setting the Euler equation to zero yields $E = \frac{\alpha\sigma\mu}{\alpha\lambda(\mu-1)[(\rho-n)(1-s)+\lambda]/(\rho-n+\lambda)^2+\sigma(1+\alpha\mu\phi)}$, which implies $R_t = \rho$.

Consequently, from the production-labor share of output and the FOC for R&D labor, (B.1) implies $\frac{L_{r,t}}{L_{x,t}} = \frac{\alpha\lambda(\mu-1)}{\sigma}\left[\frac{1-s}{\rho-n+\lambda}+\frac{s\lambda}{(\rho-n+\lambda)^2}\right]$. Making use of the leisure-consumption relation, it is easy to derive the steady-state equilibrium labor allocations in (36)-(38).

As for the optimal labor allocations, the social planner maximizes (35) subject to the constraints of consumption production, technology, and labors, yielding the following current-value Hamiltonian

$$H_t = \ln\frac{Z_t L_{x,t}}{N_t}+\phi\ln\frac{L_t}{N_t}+\eta_{1,t}n\left(L_t+L_{x,t}+L_{r,t}\right)+\eta_{2,t}\left(\varphi\ln z L_{r,t}\right), \tag{B.2}$$

where $\eta_{1,t}$ and $\eta_{2,t}$ are the costate variables associated with the law of motion for population and for technology, respectively. Therefore, the FOCs for L_t, $L_{x,t}$, and $L_{r,t}$ are given respectively as follows:

$$\frac{\partial H_t}{\partial L_t}=0 \Rightarrow L_t=-\frac{\phi}{n\eta_{1,t}}; \tag{B.3}$$

$$\frac{\partial H_t}{\partial L_{x,t}}=0 \Rightarrow L_{x,t}=-\frac{1}{n\eta_{1,t}}; \tag{B.4}$$

$$\frac{\partial H_t}{\partial L_{r,t}}=0 \Rightarrow n\eta_{1,t}=-\eta_{2,t}\varphi\ln z; \tag{B.5}$$

$$\frac{\partial H_t}{\partial N_t}=-\frac{1+\phi}{N_t}=\eta_{1,t}(\rho-n)-\dot{\eta}_{1,t}, \tag{B.6}$$

$$\frac{\partial H_t}{\partial Z_t}=\frac{1}{Z_t}=\eta_{2,t}(\rho-n)-\dot{\eta}_{2,t}. \tag{B.7}$$

Combining (B.3) and (B.4) yields (39). Then, manipulating (B.6) and using $\dot{N}_t = nN_t$ yields a differential equation such that $(\dot{-\eta_{1,t}})N_t+(-\eta_{1,t})\dot{N}_t = \rho(-\eta_{1,t})N_t-(1+\phi)$. Integrating this equation with respect to time implies that $(-\eta_{1,t})N_t = (1+\phi)/\rho$. Furthermore, the stationarity of $(-\eta_{1,t})N_t$ and (B.5) indicate $(\dot{-\eta_{1,t}})/(-\eta_{1,t}) = \dot{\eta}_{2,t}/\eta_{2,t} = -n$. Substituting this condition into (B.7) yields $\eta_{2,t}Z_t = 1/\rho$. Using this result with (B.4) and (B.5) implies that Z_t and all labor inputs grow at the rate of n. Therefore, applying the definition of λ_t in (B.7) yields (41), given $\lambda = n/\ln z$ in the steady-state equilibrium. Finally, combining (39), (41), and the labor-market-clearing condition yields (40).

Comparative Statics of the Optimal Profit-Division Rule

First, we find out the range of n/ρ and $\ln z$ to ensure that s^* in (42) lies between 0 and 1, that is $n/\rho < \frac{1-\alpha}{\sigma}$ and $\ln z \in (\ln z_1, \ln z_0)$, where $\ln z_1 \equiv \frac{-(n/\rho)+\sqrt{(1-\alpha)(n/\rho)/\sigma}}{1-n/\rho}$ is the boundary for s^* to be less than 1 and $\ln z_0 \equiv \frac{1-\alpha-\sigma(n/\rho)}{\sigma(1-n/\rho)}$ is the boundary for s^* to be greater than 0, respectively.

Next, from (42), it is straightforward to see that s^* decreases as z, σ, and/or α increases. Nevertheless, n/ρ has both a positive and a negative effect on s^*. To see the conditions under which the positive effect dominates the negative one, taking the derivative of s^* with respect to n/ρ yields

$$\frac{\partial s^*}{\partial(n/\rho)} = \frac{\sigma(\ln z - 1)^2 - \frac{\alpha+\sigma-1}{(1-n/\rho)^2}}{(1-\alpha)\ln z}, \tag{B.8}$$

where the denominator is positive. Thus, whether or not $\partial s^*/\partial(n/\rho)$ being positive depends on the sign of the numerator, which is determined by the sign of $\alpha+\sigma-1$ and the magnitude of $\ln z$. There are two cases to be considered.

(1) Suppose $\alpha+\sigma-1>0$. Then the numerator in (B.8) can be viewed as a function of $\ln z$, and the roots of the numerator are $\ln z^{\pm} \equiv 1 \pm \frac{\sqrt{\alpha+\sigma-1}}{\sqrt{\sigma}(1-n/\rho)}$. To guarantee that $\ln z^- > 0$, we assume $n/\rho < 1-\sqrt{\frac{\alpha+\sigma-1}{\sigma}} \equiv \overline{\chi}$, where $\overline{\chi} < \frac{1-\alpha}{\sigma}$. Also, it is easy to verify that $\ln z^+ > \ln z_0 > \max\{\ln z^-, \ln z_1\}$ and that $\ln z^- \gtreqless \ln z_1$ if and only if $n/\rho \lesseqgtr \frac{\alpha+2\sigma-1-2\sqrt{\sigma(\alpha+\sigma-1)}}{1-\alpha} \equiv \underline{\chi}$, where $0<\underline{\chi}<\overline{\chi}$. Thus, comparing $\ln z^-$ and $\ln z_1$ implies the following four possibilities for the effect of n/ρ on s^*.
(i) when $n/\rho \in \left(\underline{\chi}, \overline{\chi}\right)$, the set of $\ln z$ that makes s^* bounded between 0 and 1 and increasing in n/ρ is empty;
(ii) when $n/\rho \in \left(\underline{\chi}, \overline{\chi}\right)$, the set of $\ln z$ that makes s^* bounded between 0 and 1 and decreasing in n/ρ is $(\ln z_1, \ln z_0)$;
(iii) when $n/\rho \in \left(0, \underline{\chi}\right)$, the set of $\ln z$ that makes s^* bounded between 0 and 1 and increasing in n/ρ is $(\ln z_1, \ln z^-)$;
(iv) when $n/\rho \in \left(0, \underline{\chi}\right)$, the set of $\ln z$ that makes s^* bounded between 0 and 1 and decreasing in n/ρ is $(\ln z^-, \ln z_0)$.
In other words, the numerator and $\partial s^*/\partial(n/\rho)$ are positive (or negative) if $\ln z$ is sufficiently small (i.e., $\ln z < \ln z^-$ in (iii)) (or large (i.e., $\ln z > \ln z_1 > \ln z^-$ in (ii) and $\ln z > \ln z^-$ in (iv)).

(2) Suppose $\alpha+\sigma-1<0$. Then the numerator in (B.8) is positive, so s^* is strictly increasing in n/ρ, regardless of the level of $\ln z$. However, this case does not apply when subsidies to intermediate goods and research are relatively small (namely, α and σ are relatively large), which is more likely to occur.

Welfare Analysis of Second-Best Outcome

To derive the second-best labor allocations along the balanced growth path, first, it is known that the equilibrium ratio of L_t and $L_{x,t}$ equals $\phi\alpha\mu$ as shown in (36). Using $L_{x,t}+L_{r,t}+L_t=N_t$, we derive $L_t = \frac{\phi\alpha\mu}{1+\phi\alpha\mu}(N_t - L_{r,t})$ and $L_{x,t} = \frac{1}{1+\phi\alpha\mu}(N_t - L_{r,t})$. Holding μ and s constant, the current-value Hamiltonian is given by

$$H_t(\mu, s) = \ln\left(\frac{Z_t}{N_t}\frac{(N_t - L_{r,t}(\mu, s))}{1+\phi\alpha\mu}\right) + \phi\ln\left(\frac{1}{N_t}\frac{\phi\alpha\mu(N_t - L_{r,t}(\mu, s))}{1+\phi\alpha\mu}\right) + \eta_{1,t} n N_t + \eta_{2,t}(\varphi\ln z) L_{r,t}(\mu, s), \tag{B.9}$$

where $L_{r,t}(\mu, s)$ follows (38), and $\eta_{1,t}$ and $\eta_{2,t}$ are the costate variables. Therefore, the FOCs for s and Z_t are respectively given by

$$\frac{\partial H_t}{\partial s} = \frac{\partial H_t}{\partial L_{r,t}}\frac{\partial L_{r,t}}{\partial s} = 0 \Rightarrow \frac{1+\phi}{N_t - L_{r,t}} = \eta_{2,t}\varphi\ln z, \tag{B.10}$$

$$\frac{\partial H_t}{\partial N_t} = (1+\phi)\frac{L_{r,t}/N_t}{N_t - L_{r,t}} + \eta_{1,t} n = \eta_{1,t}(\rho - n) - \dot{\eta}_{1,t}, \tag{B.11}$$

$$\frac{\partial H_t}{\partial Z_t} = \frac{1}{Z_t} = \eta_{2,t}(\rho - n) - \dot{\eta}_{2,t}, \tag{B.12}$$

where in (B.10) we use the fact that for N_t at any time t, $\frac{\partial L_{r,t}}{\partial s} = -\frac{\frac{\alpha\lambda(\mu-1)}{\sigma}\left[(\rho-n)/(\rho-n+\lambda)^2\right]N_t}{1+\phi\alpha\mu+\frac{\alpha\lambda(\mu-1)}{\sigma}\left[\frac{1-s}{\rho-n+\lambda}+\frac{s\lambda}{(\rho-n+\lambda)^2}\right]} < 0$. Substituting (B.10) into (B.11) and using the definitions of $\dot{Z}_t$ and $\dot{N}_t$ yield

$$\eta_{2,t}\dot{Z}_t = (\rho - n)\eta_{1,t}N_t - \eta_{1,t}\dot{N}_t - \dot{\eta}_{1,t}N_t. \tag{B.13}$$

Similarly, (B.12) can be reexpressed as

$$\dot{\eta}_{2,t}Z_t = (\rho - n)\eta_{2,t}Z_t - 1. \tag{B.14}$$

Adding up (B.13) and (B.14) yields

$$\eta_{1,t}\dot{N}_t + \dot{\eta}_{1,t}N_t + \eta_{2,t}\dot{Z}_t + \dot{\eta}_{2,t}Z_t = (\rho - n)(\eta_{1,t}N_t + \eta_{2,t}Z_t) - 1. \tag{B.15}$$

(B.15) is a one-dimensional differential equation in $\eta_{1,t}N_t + \eta_{2,t}Z_t$. Integrating (B.15) with respect to time yields that $\eta_{1,t}N_t + \eta_{2,t}Z_t = 1/(\rho - n)$, implying that both $\eta_{1,t}N_t$ and $\eta_{2,t}Z_t$ are stationary along the BGP. Observing (B.11) with this result reveals that $L_{r,t}/N_t$ is constant, which implies that we can use the steady-state growth rate of technology such that $\dot{Z}_t/Z_t = n$ in (B.12) to obtain $\dot{\eta}_{2,t}Z_t + \eta_{2,t}\dot{Z}_t = \rho\eta_{2,t}Z_t - 1$. Integrating this differential equation yields $\eta_{2,t}Z_t = 1/\rho$. In addition, combining the arrival rate of innovations λ_t with (B.10) and (B.12) yields the second-best R&D labor given by $L^{**}_{r,t} = \left(\frac{n/\rho}{1+\phi+n/\rho}\right)N_t$, equaling the first-best counterpart in (41).

Next, we analytically investigate the welfare difference between the first- and second-best outcomes. Denote $l_{x,t} \equiv L_{x,t}/N_t$, $l_{r,t} = L_{r,t}/N_t$, and $l_t \equiv L_t/N_t$ as the per capital level of production labor, of R&D labor, and of leisure, respectively, which are stationary in both outcomes of optimal allocations. Imposing the balanced growth on (35) yields the household's lifetime utility as follows

$$U = \frac{1}{\rho - n}\left[\ln C_0 + \phi\ln l + \frac{n}{\rho - n}\right], \tag{B.16}$$

where $C_0 = Z_0 l_x$, $Z_0 = \varphi \ln z l_r N_0/n$, and the exogenous term N_0 will be dropped. With $l_r^{**} = \frac{n/\rho}{1+\phi+n/\rho}$, the second-best labor allocation of production and of leisure are $l_x^{**} = \frac{1}{1+\phi\alpha\mu}\left(\frac{1+\phi}{1+\phi+n/\rho}\right)$ and $l^{**} = \frac{\phi\alpha\mu}{1+\phi\alpha\mu}\left(\frac{1+\phi}{1+\phi+n/\rho}\right)$, respectively. Furthermore, substituting the first-best labor allocations (39)-(41) and the second-best counterparts into (B.16) yields the corresponding welfare (i.e., U^* and U^{**}), respectively. Hence, given that the R&D labors l_r^{**} and l_r^* are equal, the welfare difference is

$$U^{**} - U^* = \frac{1}{\rho - n}\left[\ln\frac{1+\phi}{1+\phi\alpha\mu} + \phi\ln\frac{\alpha\mu(1+\phi)}{1+\phi\alpha\mu}\right], \tag{B.17}$$

which is similar to (32) in the main text. The welfare loss in (B.17) is amplified, since the lifetime utility is measured in terms of the representative household whose population grows at the rate of n, making the discount factor larger.

Finally, as for the analysis of optimal patent breadth, taking the derivative of (B.9) with respect to μ yields

$$\frac{\partial H_t}{\partial \mu} = 0 \Rightarrow \left(-\frac{1+\phi}{N_t - L_{r,t}} + \frac{g}{\rho L_{r,t}}\right)\frac{\partial L_{r,t}}{\partial \mu} + \frac{\phi(\alpha - 1/\mu)}{1+\phi\alpha\mu} = 0, \tag{B.18}$$

where we use the fact that $g = \dot{Z}_t/Z_t = \varphi \ln z L_{r,t}/Z_t$ and that $\eta_{2,t}Z_t = 1/\rho$ in the steady-state equilibrium. It is obvious that the relation between the second-best μ^{**} and $1/\alpha$ determines the sign of the term $\phi(\alpha - 1/\mu)/(1+\phi\alpha\mu)$ in (B.18). Thus, given that $\partial L_{r,t}/\partial\mu > 0$, it can be inferred that $\mu^{**} < (>) 1/\alpha = \mu^*$ implies $L_{r,t}^{**}|_{\mu=\mu^{**}} > (<) L_{r,t}^*|_{\mu=\mu^*}$, and this result is consistent with that in Section 5.4. Again, the welfare difference between the second-best outcome with μ^{**} and the social optimum stems from losing one patent lever, and it is magnified by the change in the utility function as aforementioned.

Appendix C

In this Appendix, under the extended version of the blocking-patents model with physical capital accumulation, we derive the allocations of labor and capital in addition to the rate of capital investment in the steady-state equilibrium and in the first-best outcome, respectively.

The rate of capital investment is denoted by $i_t \equiv I_t/Y_t$. As for the steady-state equilibrium allocations, on the balanced growth path, using (11) and (12) yields $V_{1,t} = \left[\frac{1-s}{R_t - g_y + \lambda} + \frac{s\lambda}{(R_t - g_y + \lambda)^2}\right]\Pi_t$, where $R_t = g_c + \rho = g_y - n + \rho$. Moreover, combining this condition with the capital factor payment and the FOC for capital in R&D (i.e., $\alpha Q_t K_{x,t} = (\theta/\mu_t)\,Y_t$ and $\theta V_{1,t}\bar{\varphi}_t\,(K_{r,t}/L_{r,t})^{\theta-1} = \sigma Q_t$) yields (47), implying that $m_{k,t}$ is stationary given that μ, s, and λ are constant along the BGP. Then, using capital accumulation $i_t = (g_k + \delta)K_t/Y_t$, the capital factor payment, and the no-arbitrage condition for capital (i.e., $Q_t = R_t + \delta$) yields (48), implying that i_t depends on m and is thus stationary.

Next, using the labor factor payment (i.e., $\alpha W_t L_{x,t} = ((1-\theta)/\mu_t)\,Y_t$), the FOC for labor in R&D (i.e., $(1-\theta)V_{1,t}\bar{\varphi}_t\,(K_{r,t}/L_{r,t})^{-\theta} = \sigma W_t$), and the relation between $V_{1,t}$ and Π_t yields (45), which implies that $m_{l,t}$ is stationary and confirms $m = m_k = m_l$. Moreover, the final-goods resource constraint implies $(1-i_t)Y_t = C_t N_t$. Combining this constraint together along the labor factor payment and the leisure-consumption condition (i.e., $W_t L_t = \phi C_t N_t$) yields (44). Substituting (44) and (45) into the labor-market-clearing condition yields (46).

As for the first-best allocations, the social planner chooses L_t, m_t, and i_t to maximize the households' utility subject to the constraints of $Y_t = Z_t K_{x,t}^{\theta} L_{x,t}^{1-\theta}$, $\dot{K}_t = i_t Y_t - \delta K_t$, $N_t = L_t + L_{x,t} + L_{r,t}$, and $\dot{Z}_t = \varphi \ln z (K_{r,t}^{\theta} L_{r,t}^{1-\theta})^{\gamma} Z_t^{\beta}$, which yields the following current-value Hamiltonian

$$H_t = \ln(1 - i_t)\frac{Z_t(1 - m_t)K_t^{\theta}(N_t - L_t)^{1-\theta}}{N_t} + \phi \ln \frac{L_t}{N_t} + \zeta_{k,t}\left[i_t Z_t(1 - m_t)K_t^{\theta}(N_t - L_t)^{1-\theta} - \delta K_t\right] + \zeta_{z,t}\left[Z_t^{\beta} m_t^{\gamma} K_t^{\gamma\theta}(N_t - L_t)^{\gamma(1-\theta)} \varphi \ln z\right], \tag{C.1}$$

where $\zeta_{k,t}$ and $\zeta_{v,t}$ are the costate variables associated with the law of motion for capital and for technology, respectively. Therefore, the FOCs for L_t, $s_{r,t}$, and i_t are given respectively as follows:

$$\frac{\partial H_t}{\partial L_t} = 0 \Rightarrow \frac{\phi}{1-\theta}\frac{N_t - L_t}{L_t} = 1 + \zeta_{k,t} i_t Y_t + \zeta_{z,t}\gamma g_z Z_t; \tag{C.2}$$

$$\frac{\partial H_t}{\partial m_t} = 0 \Rightarrow \zeta_{z,t}\gamma g_z Z_t \frac{1 - m_t}{m_t} = 1 + \zeta_{k,t} i_t Y_t; \tag{C.3}$$

$$\frac{\partial H_t}{\partial i_t} = 0 \Rightarrow (1 - i_t)\zeta_{k,t} Y_t = 1; \tag{C.4}$$

$$\frac{\partial H_t}{\partial K_t} = (\rho - n)\zeta_{k,t} - \dot{\zeta}_{k,t} \Rightarrow \theta\left(1 + \zeta_{k,t} i_t Y_t + \zeta_{z,t}\gamma g_z Z_t\right) = (R_t + \delta)\zeta_{k,t} K_t; \tag{C.5}$$

$$\frac{\partial H_t}{\partial Z_t} = (\rho - n)\zeta_{z,t} - \dot{\zeta}_{z,t} \Rightarrow 1 + \zeta_{k,t} i_t Y_t = \left[\rho - n + (1 - \beta)g_z\right]\zeta_{z,t} Z_t, \tag{C.6}$$

where we use the fact that $g_{v_k} = -g_y = -g_c - n$ in (C.5) and that $g_{v_z} = -g_z$ in (C.6) along the BGP. Combining (C.3) and (C.6) immediately yields (49). Moreover, substituting (C.3) and (C.4) into (C.5) and applying the law of motion for capital yields (50). Finally, combining (C.2)-(C.4) and using the definition of $m^* = L_{r,t}^*/(N_t - L_t^*)$ yields (51).

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