Unpublished Appendix

In this appendix, we first show that with a sufficiently large number of firms, $\underline{c}$ is always greater than the assumption $c > \frac{a(2n+3)}{(n+1)(n+2)}$ in Footnote 10. Second, we prove Proposition 2 and derive Proposition 3. Finally. we show the boundary for the slope of the marginal cost of domestic R&D investment (i.e., $\bar{\gamma}$) in Case 3.4, so that domestic cost reduction is always welfare-improving when the foreign firm's production strategy is changed from FDI to exporting. A numerical example for this case is also provided for illustration.

Derivation for $\underline{c} > \frac{a(2n+3)}{(n+1)(n+2)}$

First, we show how the assumption $c > \frac{a(2n+3)}{(n+1)(n+2)}$ in Footnote 9 is obtained. We have to ensure that $\overline{c_x} = \frac{c\gamma n(n+2)+a(\gamma(n+2)-4)}{\gamma(n+1)(n+2)-4} < \frac{c\gamma(n+2)^2-4a}{4}$ so that we always have $c > z_i^{x*}$. This condition is satisfied if $4c\gamma(n+2)+4a\gamma(n+2) < c\gamma^2(n+1)(n+2)^3 - 4c\gamma(n+2)^2 - 4a\gamma(n+1)(n+2)$. Then, this inequality can be simplified to

$$8c\gamma(n+1) + 4a\gamma(n+2) < c\gamma^2(n+1)(n+2)^2. \tag{A.1}$$

Since we assume that $c < a/2$, then (A.1) is satisfied if $4a(2n+3) < c\gamma(n+1)(n+2)^2$. Therefore, with $\gamma > 4/(n+2)$, the assumption $c > \frac{a(2n+3)}{(n+1)(n+2)}$ is imposed.

Second, according to Footnote 10, we need to ensure that $\underline{c}$ always satisfies the above assumption on c, which implies that $\underline{c} = \frac{an(\gamma(n+2)^2-4)}{\gamma(3n+2)(n+2)^2-16(n+1)} > \frac{a(2n+3)}{(n+1)(n+2)}$. This condition is sufficiently satisfied if $n(n+1)(n+2)(\gamma(n+2)^2-4) > 2(n+2)(\gamma(3n+2)(n+2)^2-16(n+1))$. Further simplification yields

$$\gamma(n+2)^2(n^2-5n-4) > 4(n+1)(n-8). \tag{A.2}$$

Then with $\gamma > 4/(n+2)$, (A.2) is obtained if $(n+2)(n^2-5n-4) > (n+1)(n-8)$, which can be satisfied by $n^2 - 6n + 4 > 0$ or $n > 3 + \sqrt{5}$. Finally, a relatively large number of firms, namely $n \geq 6$, suffices the above condition. Notice that $n \geq 6$ also ensures that $\frac{a(2n+3)}{(n+1)(n+2)} < \underline{c} < c < \frac{a}{2}$, which is required in the first part of this derivation.

Proof for Proposition 2

Given the production strategy of the foreign firm, we compare how R&D investment affects domestic firms' profits in the following cases.

Case A.1. Suppose that the foreign firm undertakes either exporting or FDI regardless of domestic R&D investment, namely, $K > \max\left\{K^N, K^I\right\}$ or $K < \min\left\{K^N, K^I\right\}$. Then, comparing (3) and (8) as well as (5) and (9), respectively, we know that each domestic firm's profit becomes higher with R&D investment than without R&D investment, which implies that domestic R&D incentives always increase in this case.

Case A.2. Suppose that domestic R&D investment prevents the foreign firm from undertaking FDI, namely, $K \in (K^I, K^N)$ for $c < c^*$ and $c_x > c_x^*$. Then, comparing (5) and (8) reveals that the profit of each domestic firm with R&D investment when the foreign firm exports is greater than the counterpart without R&D investment when the foreign firm undertakes FDI, which implies that domestic R&D incentives always increase in this case.

Case A.3. To investigate how domestic R&D incentives change when the foreign firm's incentives for FDI rise under domestic R&D investment–that is $K \in (K^N, K^I)$ for either $c > c^*$ and $c_x < \overline{c_x} < c_x^*$, or $c < c^*$ and $c_x < c_x^*$, we denote $c_x^{**} \equiv (a-2c)\left[\frac{\sqrt{\gamma}(n+2)}{\sqrt{\gamma(n+2)^2-8}} - 1\right]$ and $\tilde{c} \equiv ([an(\gamma(n+2)^2-4) + a(\gamma(n+2)^2(n+1) - 6n - 8)][(\sqrt{\gamma}(n+2)/\sqrt{\gamma(n+2)^2-8}) - 1])/([-16(n+1)+\gamma(n+2)^2(3n+2)+2(\gamma(n+2)^2(n+1)-6n-8)][(\sqrt{\gamma}(n+2)/\sqrt{\gamma(n+2)^2-8}) - 1])$.

Accordingly, we compare the profit of each domestic firm with R&D investment under FDI by the foreign firm (i.e., $(\gamma(a-2c)^2)/(\gamma(n+2)^2-8)$) and the counterpart without R&D

investment under exporting by the foreign firm (i.e., $(a-2c+c_x)^2/(n+2)^2$). This comparison shows that domestic firms prefer to invest in R&D that attracts FDI by the foreign firm if $c_x < c_x^{**}$. Further, to be consistent with the conditions in Proposition 1, we consider the following two situations:

(a) If $c \in (c^*, \bar{c})$, then FDI-attracting domestic R&D investment (i.e., $K \in (K^N, K^I)$) implies that $c_x < \overline{c_x} < c_x^*$ and $c_x^{**} < \overline{c_x}$. Therefore, domestic R&D incentives increase when $c_x < c_x^{**}$.

(b) If $c \in (\underline{c}, c^*)$, then FDI-attracting domestic R&D investment (i.e., $K \in (K^N, K^I)$) implies $c_x < c_x^*$. Given that $\partial(c_x^* - c_x^{**})/\partial c > 0$, $c_x^* < c_x^{**}$ for $c = \underline{c}$ and $c_x^* > c_x^{**}$ for $c = c^*$, there exists a threshold $\tilde{c} \in (\underline{c}, c^*)$ such that $c_x^{**} < c_x^*$ if $c > \tilde{c}$. Therefore, domestic R&D incentives increase when either $c \in (\tilde{c}, c^*)$ and $c_x < c_x^{**}$ or $c \in (\underline{c}, \tilde{c})$ and $c_x < c_x^*$.

Hence, we can conclude that when domestic R&D investment attracts the foreign firms to undertake FDI–that is $K \in (K^N, K^I)$ for $c \in (\underline{c}, \bar{c})$, domestic R&D incentives always increase if $c_x < \min\{c_x^*, c_x^{**}\}$.

Taking together the Cases A.1-A.3, if $c \in (\underline{c}, \tilde{c})$, then domestic R&D incentives always increase regardless of whether or not FDI is encouraged (namely, independent of the size of c_x).

Derivation for Proposition 3

First, if domestic cost reduction does not affect the foreign firm's production strategy, then the analysis of domestic welfare is presented by the following two cases.

Case 3.1. Assume that the foreign firm always chooses exporting regardless of domestic cost reduction, namely, $K > \max\left\{K^N, K^I\right\}$ for $c_x \in (0, \overline{c_x})$ and $c \in (\underline{c}, \bar{c})$. Therefore, the domestic welfare under no domestic R&D investment is given by

$$W_N^{x*} = \frac{[a(n+1) - 2c_x - cn]^2 + 2n(a - 2c + c_x)^2}{2(n+2)^2}, \tag{10}$$

whereas the domestic welfare under domestic R&D investment is

$$W_I^{x*} = \left[\begin{array}{l} 2n\gamma(\gamma(n+2)^2 - 8)(a - 2c + c_x)^2 \\ +(4c_x - \gamma(n+2)(c_x + cn) + a(\gamma(n+1)(n+2) - 4))^2 \end{array} \right] / \left[2(\gamma(n+2)^2 - 8)^2\right]. \tag{11}$$

Denote $H_1 = W_I^{x*} - W_N^{x*}$. It can be shown that for $c \in (\underline{c}, \bar{c})$, H_1 is concave in c_x, $H_1|_{c_x=0} > 0$ and $H_1|_{c_x=\overline{c_x}} > 0$. Hence, the level of domestic welfare becomes higher under domestic R&D investment as compared to under no domestic R&D investment.

Case 3.2. Assume that the foreign firm always undertakes FDI regardless of domestic cost reduction, namely, $K < \min\left\{K^N, K^I\right\}$ for $c_x \in (0, \overline{c_x})$ and $c \in (\underline{c}, \bar{c})$. Therefore, the domestic welfare under no domestic R&D investment is given by

$$W_N^{f*} = \frac{2n(a - 2c)^2 + [a(n+1) - cn]^2}{2(n+2)^2}, \tag{12}$$

whereas the domestic welfare under domestic R&D investment is

$$W_I^{f*} = \frac{2n\gamma(\gamma(n+2)^2 - 8)(a - 2c)^2 + [cn\gamma(n+2) - a(\gamma(n+1)(n+2) - 4)]^2}{2(\gamma(n+2)^2 - 8)^2}. \tag{13}$$

Denote $H_2 = W_I^{f*} - W_N^{f*}$. It can be shown that for $c \in (\underline{c}, \bar{c})$, H_2 is convex in c, and H_2 reaches the minimum level at $c_f^{min} = \frac{a[\gamma(n+2)^2(3n+10) - 16(n+5)]}{4[\gamma(n+2)^2(n+4) - 4(n+8)]}$, which is greater than $\bar{c}$. Moreover, $H_2|_{c=\underline{c}} > 0$ and $H_2|_{c=\bar{c}} = 0$. Hence, the level of domestic welfare becomes higher under domestic R&D investment as compared to under no domestic R&D investment.

Next, let us suppose that domestic cost reduction changes the foreign firm's production strategy. Then, the analysis of domestic welfare is given by the following two cases.

Case 3.3. Assume that the foreign firm chooses exporting under no domestic R&D, while it changes to undertake FDI under domestic R&D. According to Case A.3 in the proof for Proposition 2, we have $K^N < K < K^I$ for $c_x \in (0, \min\{c_x^*, c_x^{**}\})$ and $c \in (\underline{c}, \bar{c})$ in this case. Therefore, we compare the domestic welfare between under "domestic R&D and FDI by the foreign firm" and under "no domestic R&D and exporting by the foreign firm," namely, (13) and (10). Denote $H_3 = W_I^{f*} - W_N^{x*}$. It can be shown that for $c \in (\underline{c}, \bar{c})$, H_3 is concave in c_x. In addition, $H_3|_{c_x=0} > 0$, $H_3|_{c_x=c_x^*} > 0$, and $H_3|_{c_x=c_x^{**}} > 0$. Hence, the level of domestic welfare becomes higher under domestic R&D investment as compared to under no domestic R&D investment.

Case 3.4. Assume that the foreign firm undertakes FDI under no domestic R&D, whereas it chooses exporting under domestic R&D. According to Case A.2 in the proof for Propsoition 2, we have $K \in (K^I, K^N)$ for $c \in (\underline{c}, c^*)$ and $c_x > c_x^*$ in this case. Therefore, we compare the domestic welfare between under "domestic R&D and exporting by the foreign firm" and under "no domestic R&D and FDI by the foreign firm," that is (11) and (12). Denote $H_4 = W_I^{x*} - W_N^{f*}$. It can be shown that for $c \in (\underline{c}, \bar{c})$, H_4 is convex in c_x, and H_4 reaches the minimum level at $c_x^{min} = \frac{c\gamma n(4(n-6)+3\gamma(n+2)^2)+a(16-\gamma(\gamma(n-1)(n+2)^2+4(n^2+4)))}{\gamma^2(n+2)^2(2n+1)-8\gamma(3n+2)+16}$. Denote the roots for $H_4(c)|_{c_x=c_x^{min}} = 0$ as c^{min} and c^{max} where $c^{min} < c^{max}$. Since $H_4(c)|_{c_x=c_x^{min}}$ is concave in c, if $\gamma \in \left(\frac{4}{n+2}, \bar{\gamma}\right)$, then we find that $c^{min} \leq \underline{c} < c^* \leq c^{max}$ so that $H_4(c)|_{c_x=c_x^{min}} > 0$. Accordingly, given the condition on γ, the level of domestic welfare becomes higher under domestic R&D investment as compared to under no domestic R&D investment. See the derivation of c^{min}, c^{max}, and $\bar{\gamma}$ in next subsection, which also provides a numerical example for this case.

Derivation for Case 3.4

In Case A.2, we know that the foreign firm undertakes FDI under no domestic R&D, while it chooses exporting under domestic R&D if $K \in \left(K^I, K^N\right)$ for $c \in (\underline{c}, c^*)$ and $c_x >$

c_x^*. Given that the domestic welfare difference between these two situations is denoted as $H_4 = W_I^{x*} - W_N^{f*}$, we obtain that H_4 is convex in c_x for $c \in (\underline{c}, c^*)$ and that H_4 reaches the minimum level at c_x^{min}. In this case, we only need to solve for the conditions that guarantee $H_4|_{c_x=c_x^{min}} > 0$ for $c \in (\underline{c}, c^*)$ and $c_x > c_x^*$, then domestic cost reduction always increases domestic welfare.

Substituting c_x^{min} into H_4 yields a quadratic function of c. We find that there exist two roots for $H_4|_{c_x=c_x^{min}} = 0$, which are denoted by c^{min} and c^{max} where $c^{min} < c^{max}$. Specifically, $c^{min} = (\mathcal{M} - \mathcal{N})/\mathcal{D}$ and $c^{max} = (\mathcal{M} + \mathcal{N})/\mathcal{D}$, where $\mathcal{M} = an(3\gamma^2(n-1)(n+2)^2+16(n+5)-8\gamma(n(n+9)+2))$, $\mathcal{N} = 4\sqrt{a^2n(n+2)^2(\gamma+\gamma n-2)(16+\gamma^2(n+2)^2(2n+1)-8\gamma(3n+2))}$, and $\mathcal{D} = n(9\gamma^2n(n+2)^2+16(n+8)-8\gamma(n(n+18)+8))$. Moreover, $H_4|_{c_x=c_x^{min}}$ is concave in c. Thus, we need to check if $H_4|_{c_x=c_x^{min}} > 0$ is an empty set when $(\underline{c}, c^*)$ is within (c^{min}, c^{max}).

Given that $\gamma > \frac{4}{n+2}$ and $n \geq 6$, we always have $c^{min} < \underline{c} < c^*$. Then the remaining task is to see if it is possible that $c^* < c^{max}$. Under the same conditions, it can be also shown that c^* is increasing in γ and that c^{max} is decreasing in γ.[1] Further, when $\gamma \to 4/(n+2)$, $c^* < c^{max}$, implying that there must exist a threshold level $\bar{\gamma}$ such that $c^* = c^{max}$. Hence, $c^* < c^{max}$ for $\gamma < \bar{\gamma}$. Finally, if the condition that $\gamma \in \left(\frac{4}{n+2}, \bar{\gamma}\right)$ holds, then $(\underline{c}, c^*)$ is within (c^{min}, c^{max}), so that $H_4|_{c_x=c_x^{min}}$ is always positive.

Example. Assume that $a = 1$ and $n = 10$. When domestic R&D prevents FDI, the domestic welfare difference H_4 is minimized at $c_x^{min} = \frac{16+10c\gamma(16+432\gamma)-\gamma(416+1296\gamma)}{16-256\gamma+3024\gamma^2}$. The two roots c^{min} and c^{max} satisfying $H_4|_{c_x=c_x^{min}} = 0$ are $\frac{(25+5\gamma(81\gamma-32))\pm2\sqrt{10(11\gamma-2)(1+\gamma(189\gamma-16))}}{30(1+\gamma(45\gamma-8))}$, respectively, where $c^{min} < c^{max}$. We also get that $\underline{c} = \frac{180\gamma-5}{576\gamma-22}$ and $c^* = \frac{3(9+\gamma(4356\gamma-379))}{44+6\gamma(4356\gamma-349)}$. Hence, domestic cost reduction is always welfare-enhancing (i.e., $H_4|_{c_x=c_x^{min}} > 0$) if $(\underline{c}, c^*)$ is within (c^{min}, c^{max}), which is achieved by $\gamma \in \left(\frac{4}{n+2}, \bar{\gamma}\right)$. The upper bound $\bar{\gamma}$ is given by the real root that satisfies the condition $c^* = c^{max}$ and is also greater than 1/3, implying that $\bar{\gamma}$ approximately equals 0.472 (see Footnote 1 in this appendix). This outcome suggests that domestic

[1] The proof can be seen in the complementary *Mathematica* files, which are available upon request.

cost reduction that prevents FDI is welfare-improving if the rate at which the marginal cost of R&D investment rises is lower than 47.2%.